Lecture 9: Supernova Rates
Star-Formation Efficiency, Yield
**Nucleosynthesis Flowchart**

Intergalactic medium

- Intergalactic medium
  - Galaxy formation inflow
  - Gal. winds, stripping, mergers

Interstellar medium

- Star formation
- Winds, PN, Novae He, 7Li, C, N
- Explosive r-process

- Small stars
  - D, Li
- Middling stars
- Big stars
- WD
- NS
- BH

Closed Box Model

Spallation 6Li, Be, B

Cosmic rays

BB

- 1,2H
- 3,4He
- 7Li
X-rays from Supernova remnants

Shocked gas hot enough to emit X-rays

Chandra X-ray Observatory

Supernova Portrait Gallery

http://chandra.harvard.edu
More ways to enrich the ISM

- *Stellar winds*: radiation pressure blows gas from the surfaces of hot massive stars.
- Can be very eruptive!
Planetary Nebulae
Universal IMF (Kroupa 2002)

\[ N(M) \propto M^\alpha \]

\[ \alpha \approx -\frac{7}{3} \quad M > M_{\text{SUN}} \]

\[ -\frac{4}{3} \quad 0.1 - 1 M_{\text{SUN}} \]

\[ -\frac{1}{3} \quad M < 0.1 M_{\text{SUN}} \]
The Salpeter IMF

How many supernovae per year for each galaxy type?

Use power-law IMF, Salpeter slope $-7/3 = -2.33$

$$N(M) \propto M^{-7/3}$$

Limits of validity, not well known

$\log N(M)$

$\log M$

Supernova limit

$0.1M_\odot$ $8M_\odot$ $20M_\odot$

Slope $= -\frac{7}{3}$
Integrating a Power-Law IMF

**Number of stars**:

\[
N = \int N(M) \, dM = A \int M^B \, dM = \frac{A}{B+1} M^{B+1} \quad \text{(if} \quad B \neq -1)\]

**Fraction of stars with** \( M > 8 M_\odot \)** for** \( B = -7/3 \)

\[
f_N = \frac{\int_{8}^{20} M^B \, dM}{\int_{0.1}^{20} M^B \, dM} = \frac{\text{number of SNe}}{\text{number of stars}}
\]

\[
f_N = \frac{A}{B+1} M_{B+1}^{B+1}\bigg|_{8}^{20} = \frac{M^{-4/3}}{B+1} M_{-4/3}^{B+1}\bigg|_{0.1}^{20} = \frac{0.018 - 0.063}{0.018 - 21.544} = 0.018 - 21.544
\]

Most stars at low-mass end!

\[
500 \text{ stars} \rightarrow 1 \text{ supernova!}
\]

\[
\Rightarrow \quad f_N = 0.2\%
\]
SN Mass Fraction

SNe are rare, but each is very massive.

What fraction of the mass goes into SNe?

\[ f_M = \frac{\int_{0.1}^{20} M \times M^{-\frac{7}{3}} dM}{\int_{8}^{20} M \times M^{-\frac{7}{3}} dM} \]

\[ = \left. \frac{M^{-\frac{1}{3}}}{M^{-\frac{1}{3}}} \right|_{0.1}^{20} = \frac{0.37 - 0.50}{0.37 - 2.15} \]

\[ \Rightarrow f_M = 7.2\% \]

Most of mass is still in low-mass stars.
Typical SN Mass

Median mass:
\[
\frac{1}{2} = \frac{\int_{8}^{\overline{M}_{SN}} M \times M^{-\frac{7}{3}} dM}{\int_{8}^{20} M \times M^{-\frac{7}{3}} dM} = \frac{\overline{M}_{SN}^{-1/3} - 0.50}{0.37 - 0.50}
\]

\[\Rightarrow \overline{M}_{SN} = 12.2 \, M_{\odot}\]

Mean mass:
\[
\langle M \rangle = \frac{\int_{8}^{20} M \times M^{-\frac{7}{3}} dM}{\int_{8}^{20} M^{-\frac{7}{3}} dM} = \frac{1}{M^{-1/3} \bigg|_{8}^{20}} - \frac{1}{M^{-4/3} \bigg|_{8}^{20}} = \frac{4 \times (20^{-1/3} - 8^{-1/3})}{20^{-4/3} - 8^{-4/3}} = \frac{4 \times (0.37 - 0.50)}{0.018 - 0.062} = 12 \, M_{\odot}
\]
**SN Rates**

**Spiral Galaxy:** SFR: $\sim 8 \, M_\odot \, \text{yr}^{-1}$. 7.2% have $M > 8M_\odot$.  
$\Rightarrow$ $(8 \, M_\odot \, \text{yr}^{-1}) \times 0.072 \sim 0.6 \, M_\odot \, \text{yr}^{-1}$ go into SNe  

**SN rate:**  
\[
\frac{0.6 \, M_\odot \, \text{yr}^{-1}}{12.2 \, M_\odot} \sim \frac{1}{20} \, \text{yr}^{-1}
\]

**Irregular Galaxy:** $\sim 10x$ this rate during bursts (1 SN per 2 yr)!  
**Milky Way:** $\sim 1M_\odot \, \text{yr}^{-1} \Rightarrow$ 1 every 160 years  

No SNe between bursts.
SN Rates: Ellipticals

$t_* = 1$ Gyr       e-folding time
$t = 10$ Gyr       age
$\alpha = 0.95$       efficiency
$M_0 = 10^{11} M_\odot$    total mass = initial gas mass

Gas consumption:

\[ M_G(t) = M_0 e^{-t/t_*} = M_0 - \alpha M_S(t) \]

Star formation:

\[ M_S(t) = \frac{M_0}{\alpha} \left( 1 - e^{-t/t_*} \right) \]

\[ \frac{dM_S}{dt} \equiv \dot{M}_S = \frac{M_0}{\alpha} \frac{e^{-t/t_*}}{t_*} = \frac{\left(10^{11} M_\odot\right) e^{-10}}{(0.95) (10^9 \text{ yr})} = 5 \times 10^{-3} \text{ M}_\odot \text{yr}^{-1} \]

SN rate:

\[ \frac{(0.072) \left(5 \times 10^{-3} \text{M}_\odot \text{yr}^{-1}\right)}{12.2 \text{M}_\odot} \approx 3 \times 10^{-5} \text{ yr}^{-1} \]

3 SN per $10^5$ yr.   Negligible!
What Efficiency and Yield?

\[ M_G = 1 \]
\[ M_S = 0 \]
\[ X = 0.75 \]
\[ Y = 0.25 \]
\[ Z = 0.00 \]

\[ M_G = 0 \]
\[ M_S = 1 \]

\[ X = ? \]
\[ Y = ? \]
\[ Z = ? \]
Estimates for efficiency $\alpha$, yield in X, Y, Z

Assume:

1. Type-II SNe enrich the ISM.
   Ignore: Type-I SNe, stellar winds, PNe, ....

2. Ignore: Infall from the IGM (Closed Box)

3. SN 1987A is a typical Type-II SN.

Better models include these effects.

What do we know about SN 1987A?
**SN 1987A**

23 Feb 1987 in LMC

Brightest SN since 1604!

First SN detected in neutrinos.

Visible (14 → 4.2 mag) to naked eye, in southern sky.

Progenitor star visible:

~20 Msun blue supergiant.

3-ring structure (pre-SN wind)

Shockwave now hitting inner ring.
**Star Formation Efficiency**

Use SN 1987A to calculate $\alpha$ and yield.

SN 1987A:  initial mass $= 20 \, M_\odot$

NS mass $= 1.6 \, M_\odot$

From IMF, 7.2% of $M_S$ is in stars with $M > 8 \, M_\odot$

Fraction of $M_S$ returned to ISM:

\[
\beta = 0.072 \times \frac{20 - 1.6}{20} \approx 6.6\%
\]

Efficiency = fraction of $M_S$ retained in stars.

\[
\alpha = 1 - \beta = 1 - 0.066 = 93\%
\]
SN 1987A Lightcurve

Powered by radioactive decay of r-process nuclei. Use to measure metal abundances in ejected gas.

$^{56}\text{Ni} \Rightarrow ^{56}\text{Co}$ 6.1d half-life

$^{56}\text{Co} \Rightarrow ^{56}\text{Fe}$ 77d half-life
SNII yield from simulations integrated over Salpeter IMF

Note high in \(\alpha\)-elements (O,Mg, Si, S, Ca&Ti)
\textbf{X,Y,Z of ejecta from SN1987A}

Initial mass \quad \sim 20 \, M_{\odot}

NS mass \quad \sim 1.6 \, M_{\odot}

Mass ejected \quad \sim 18.4 \, M_{\odot}

\begin{align*}
&\text{in H} \quad 9.0 \, M_{\odot} \\
&\text{He} \quad 7.0 \, M_{\odot} \\
&Z \quad 2.4 \, M_{\odot}
\end{align*}

\begin{align*}
\Rightarrow \quad X &= \frac{9}{18.4} \approx 0.49 \\
Y &= \frac{7}{18.4} \approx 0.38 \\
Z &= \frac{2.4}{18.4} \approx 0.13
\end{align*}
**α-elements**

SNII produces a lot of <Fe elements (i.e., multi-shell burning)

SNIa predominantly produce Fe + r-process products

SNIa start ~1Gyr after SNII

Fast rapid SFR, i.e., low e-fold produce most SNII before SNIa kick-in

Galaxies therefore have a high incidence of stars with high α-elements

Enhanced α-elements seen as indicator of low e-fold values, e.g., Ellipticals, MW bulge
SNII yield from simulations integrated over Salpeter IMF

Note high in \( \alpha \)-elements (O, Mg, Si, S, Ca & Ti)
SNIIa Yield