Kepler’s Laws and our Solar System

- The Astronomical Unit, AU
- Kepler’s *Empirical* Laws of Planetary motion
- The mass of the Sun, $M_\odot$
- A very brief tour of the solar system
  - Major planets
  - Dwarf planets (definition)
  - Minor bodies
    - Asteroid Belt, Trojans, Centaurs
    - TNOs
    - Kuiper Belt
    - Oort cloud

- The mass distribution and abundance of the solar system
The Astronomical Unit (AU)

• **Approximately:** the mean Sun-Earth distance:
  – $1.495978707003 \times 10^{11}$ m (i.e., $\sim 1.5 \times 10^{11}$ m or 150 million kilometers)

• **Precisely:** Radius of a particle in a circular orbit around the Sun moving with an angular frequency of 0.01720209895 radians per Solar Day (i.e., $2\pi$ radians per year)

• **Measured via:** Transit of Venus (*trad.*), Radar reflections from the planets, or the time delay in sending radio signals to space missions in orbit around other planets and the application of Kepler’s Laws
The Earth-Sun distance

- Easy to measure relative distance of inner planets, i.e., Venus

- Angle $a$ can be observed (furthest angle Venus gets away from Sun) $AB=AC \cos(a)$

- Venus $= 0.72$AU

- [Can also use Kepler’s 3rd law to derive relative distances to all planets, see later in lecture]
Transit of Venus (1769, Tahiti)

1. Observe Transit of Venus from two or more locations.
2. Record displacement angle \(E\) of the Venus transit against the solar disc.
3. Bring data together.
4. Measure angle between transit points.
5. Use basic geometry to derive Earth-Venus distance.

\[
\tan \left( \frac{1}{2} \nu \right) = \frac{1}{2} \frac{d_{A-B}}{d_{\text{Earth-Venus}}} \\
\]

\[
d_{\text{Earth-Venus}} = (1 - 0.72)d_{\text{Earth-Sun}}
\]
2004 Transit

Next transit: 6th June 2012
Then: December 2117!

1761+1769 → 153 million kilometers
1874+1882 → 149 million kilometers
Microwave radar → 150 million kilometers +/- 30m!
Kepler’s *Empirical* Laws of Planetary Motion

- REMINDER: Definition of an ellipse.
  The set points whose sum of the distances from 2 fixed points (the foci) is a constant.

![Diagram of an ellipse with labeled axes and foci](image)

- **a**: semi-major axis
- **b**: semi-minor axis
- **e**: eccentricity

\[
e = \frac{f}{a} \quad \Rightarrow \quad b = a \cdot \sqrt{1 - e^2}
\]
Kepler’s *Empirical* Laws of Planetary Motion

- **REMINDER**: Definition of an ellipse.
  
The set points whose sum of the distances from 2 fixed points (the foci) is a constant.

\[ e = \frac{f}{a} \quad \Rightarrow \quad b = a \cdot \sqrt{1 - e^2} \]
Kepler’s First Law

1. Each planet moves about the Sun in an elliptical orbit, with the Sun at one focus of the ellipse.

<table>
<thead>
<tr>
<th></th>
<th>Eccentricity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>0.205</td>
</tr>
<tr>
<td>Venus</td>
<td>0.007</td>
</tr>
<tr>
<td>Earth</td>
<td>0.017</td>
</tr>
<tr>
<td>Mars</td>
<td>0.094</td>
</tr>
<tr>
<td>Jupiter</td>
<td>0.049</td>
</tr>
<tr>
<td>Saturn</td>
<td>0.057</td>
</tr>
<tr>
<td>Uranus</td>
<td>0.046</td>
</tr>
<tr>
<td>Neptune</td>
<td>0.011</td>
</tr>
</tbody>
</table>

**Perihelion** *(near Sun) distance:*

\[
d_{\text{perihelion}} = \frac{1}{2} (2a - 2f)
\]

\[
d_{\text{perihelion}} = a(1 - e)
\]

**Aphelion** *(away Sun) distance:*

\[
d_{\text{aphelion}} = d_{\text{perihelion}} + 2f
\]

\[
d_{\text{aphelion}} = a(1 + e)
\]

A comet has a *perihelion* distance of 3AU and an *aphelion* distance of 7AU. What is the *semi-major axis* of the ellipse? What is the *eccentricity*?
Kepler’s Second Law

2. The straight line (radius vector) joining a planet and the Sun sweeps out equal areas of space in equal intervals of time.

Area Sun-A-B = Area Sun-C-D if planet moves from C to D in same time as from A to B.
Kepler’s Second Law

- Area = 0.5 \( r \cdot v \cdot t \) = constant
- From consideration of areas being swept at Perihelion and Aphelion:

\[
\frac{v_{aph}}{v_{per}} = \frac{d_{per}}{d_{aph}}
\]
Kepler’s Second Law

- Forced via conservation of energy and angular momentum
- When an object is closer to the Sun the radial gravitational force felt is greater (inverse square law) which induces faster circular motions along the orbit path.

\[ v_{\text{perihelion}} = \sqrt{\frac{(1+e)GM}{(1-e)a}} , \quad v_{\text{aphelion}} = \sqrt{\frac{(1-e)GM}{(1+e)a}} \]
3. The squares of the sidereal periods (P) of the planets are proportional to the cubes of the semi-major axes (a) of their orbits:

\[ P^2 = ka^3 \quad with \quad k = \frac{4\pi^2}{GM_\odot} \]

If we choose the year as the unit of time and the AU as the unit of distance, then k=1.
Kepler’s Third Law -- verification.

<table>
<thead>
<tr>
<th>Planet</th>
<th>a in AU</th>
<th>P in yr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>0.387</td>
<td>0.241</td>
</tr>
<tr>
<td>Venus</td>
<td>0.723</td>
<td>0.615</td>
</tr>
<tr>
<td>Earth</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Mars</td>
<td>1.524</td>
<td>1.881</td>
</tr>
<tr>
<td>Jupiter</td>
<td>5.203</td>
<td>11.86</td>
</tr>
<tr>
<td>Saturn</td>
<td>9.539</td>
<td>29.46</td>
</tr>
<tr>
<td>Uranus</td>
<td>19.18</td>
<td>84.01</td>
</tr>
<tr>
<td>Neptune</td>
<td>30.06</td>
<td>164.8</td>
</tr>
</tbody>
</table>
Kepler’s Third Law -- verification.

<table>
<thead>
<tr>
<th>Planet</th>
<th>a in AU</th>
<th>P in yr</th>
<th>$a^3$</th>
<th>$P^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>0.387</td>
<td>0.241</td>
<td>0.058</td>
<td>0.058</td>
</tr>
<tr>
<td>Venus</td>
<td>0.723</td>
<td>0.615</td>
<td>0.378</td>
<td>0.378</td>
</tr>
<tr>
<td>Earth</td>
<td>1</td>
<td>1</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Mars</td>
<td>1.524</td>
<td>1.881</td>
<td>3.540</td>
<td>3.538</td>
</tr>
<tr>
<td>Jupiter</td>
<td>5.203</td>
<td>11.86</td>
<td>140.9</td>
<td>140.7</td>
</tr>
<tr>
<td>Saturn</td>
<td>9.539</td>
<td>29.46</td>
<td>868.0</td>
<td>867.9</td>
</tr>
<tr>
<td>Uranus</td>
<td>19.18</td>
<td>84.01</td>
<td>7056</td>
<td>7056</td>
</tr>
<tr>
<td>Neptune</td>
<td>30.06</td>
<td>164.8</td>
<td>27162</td>
<td>27159</td>
</tr>
</tbody>
</table>
Kepler & Newton

- Newton’s Laws of Motion and Gravitation give Kepler’s Law:
- Consider the simple case of a circular orbit, Centrifugal=gravitational forces (in equilibrium) =>

\[
\frac{mv^2}{r} = \frac{GMm}{r^2}
\]

\[
P = \frac{2\pi}{\omega} = \frac{2\pi r}{v}
\]

=>

\[
\frac{4\pi^2mr}{P^2} = \frac{GMm}{r^2}
\]

=>

\[
P^2 \propto r^3
\]

Can be shown to hold for ellipses as well via conservation of energy and angular Momentum where \(r\) becomes \(a\) the semi-major axis.
Mass of the Sun, $M_\odot$

- Kepler’s third law allows us to derive the solar mass:

\[
\frac{4\pi^2}{P^2} = \frac{GM_\odot}{r^3}
\]

\[
M_\odot = \frac{4\pi^2r^3}{GP^2}
\]

\[
M_\odot = \frac{(4\pi^2)(1.5\times10^{11})^3}{(6.67\times10^{-11})(365.24 \times 24 \times 60 \times 60)^2}
\]

\[
M_\odot = 2.0 \times 10^{30} \text{ kg}
\]

- Using radar measurement of the AU
- The solar mass is the standard by which we measure all masses in astronomy, e.g., Milky Way central SMBH = $10^6 M_\odot$
Inner planets

- Rocky
Outer planets

- Gassy
Planet sizes to scale (not distances)
Our backyard: Geophysics
**Major planet Def’n:**
1. Orbits the Sun
2. Enough gravity to be spherical
3. Master of orbit

**Dwarf planet Def’n**
1 & 2 above
3. Not cleared orbit
4. Not a satellite

Dwarf planets
~9 known

All else: **Small solar System bodies.**

Ceres = planet (1801-1846), then asteroid (1847-2006), now dwarf planet (2006+)!
Largest known trans-Neptunian objects (TNOs)

- Eris
- Pluto
- 2005 FY$_9$
- 2003 EL$_{61}$
- Sedna
- Orcus
- Quaoar
- Varuna
Kuiper Belt

![Diagram of Kuiper Belt with planets](image-url)
Kuiper Belt’s around other stars
Oort Cloud: source of comets?
Sedna an Oort cloud object?
Mass of solar system

- Sun: 99.85%
- Planets: 0.135% (Jupiter 0.09%)
- Comets: 0.01%
- Satellites: 0.00005%
- Minor Planets: 0.0000002%
- Meteoroids: 0.0000001%
- Interplanetary Medium: 0.0000001%
Chemical abundance of Sun

Abundance of elements in the solar system, y-axis logarithmic
Only H, He and Li are formed in Big Bang...rest comes from stars
Our Galaxy
Herchel’s view of the Galaxy in 1860
Mapping the Galaxy from the stars

- Out of the plane of our Galaxy
- View of relatively few stars
- View of the Milky Way
- In the plane of our Galaxy
- Earth
- Disk of our Galaxy
Our Galaxy