The magnitude scale, parallax, the parsec, and Cepheid distances

Luminosity and flux
- Definitions
- The inverse square law

Measuring fluxes in astronomy
- The magnitude scale
- Absolute and apparent magnitudes

Measuring distance in astronomy
- Parallax
- The parsec (pc), kiloparsec (kpc), and Megaparsec (Mpc)

Extragalactic distances
- Cepheid Period-Luminosity relation
- Distance to Andromeda (M31)
Luminosity

- **Luminosity** is how much energy, in the form of light, a source emits per second
  - Luminosity $L$ has units of Joules sec$^{-1}$ or Watts

- Light is generally assumed to be emitted **isotropically** i.e. the same amounts in all directions (NB: some exceptions, like pulsars, quasars etc)

- From Earth, we only receive some part of the emitted light

image courtesy Jodrell Bank Observatory
Flux

• **Flux** is the amount of light an observer measures
  – sometimes called flux density

• The light from a star emitted in 1 second:
  – spreads out over the surface of a sphere
  – we intercept at distance \( d \), where the surface area is \( 4\pi d^2 \)
  – so in 1 m\(^2\) of collecting area:
    \[ f = \frac{L}{4\pi d^2} \]
  – so unit of flux is W m\(^{-2}\)

• Key concept: **Luminosity** is the total energy the source emits per second
  **Flux** is the energy received by the observer per m\(^2\) per sec
  **Flux is distance dependent, luminosity is not**
Light source emits radiant energy $L$ per unit time

- At twice the distance, the light has spread out to cover 4x as much area
- Therefore a unit collecting area (1 m$^2$) receives 1/4 as much light
• Using flux to get distance,
  – measure the flux $f$, for a source of known luminosity $L$
    • E.g., a Cepheid variable with a known period
  – apply $f = \frac{L}{4\pi d^2}$ to find $d$

• or to get Luminosity,
  – if you know the distance
    • e.g. by parallax
  – measure $f$ and derive $L$
  – e.g. for a new kind of astronomical object...
    e.g. discovery of brown dwarfs ('failed stars')
• The energy passing through a unit area decreases with the square of \( d \)

• At distances \( d_1 \) and \( d_2 \), the amounts of energy \( f_1 \) and \( f_2 \) received by a detector are in the proportion

\[
\frac{f_1}{f_2} = \frac{L}{4\pi d_1^2} \div \frac{L}{4\pi d_2^2} = \frac{4\pi d_2^2}{4\pi d_1^2} = \left(\frac{d_2}{d_1}\right)^2
\]

— the inverse-square law of light propagation

• Comparing the fluxes of two similar objects can give:
  — distance of a further one if distance to closest is known
  — relative distance if neither distance is known
Flux units

• SI flux/energy units
  i.e. f in W m\(^{-2}\)
  – or per unit of frequency interval (W m\(^{-2}\) Hz\(^{-1}\)), to compare data from different detectors

• Because, f is generally very small
  often use the Jansky
  \[ 1 \text{ Jy} = 10^{-26} \text{ W m}^{-2} \text{ Hz}^{-1} \]
  – Multiply Jansky by frequency of observation to get back to total energy, \(vf\)
• Flux is a linear scale
  – a source of 10 Jy is 10 times as bright as a source of 1 Jy

• But not convenient when comparing sources of very different brightness
  – e.g. if f ranges from $10^1$ to $10^{30}$ units

• Because of the large dynamic range (optical-near-IR) astronomers use a logarithmic-based scale magnitudes
  – a 'logical' scale would be 1 → 30
  – but we actually use a magnitude scale derived from ancient Greek astronomers…
magnitudes

• Hipparchos (2nd century BC) divided stars into 5 classes
  – no telescopes, so naked-eye visible
  – before invention of 'zero' so 5 classes, 1 → 5
  – class 1 was the brightest

• William Herschel (18th century) used a telescope to compare these classes
  – difference of 5 classes is 100x in flux
  – the class or magnitude scale came to be used... except that we now use 0 as the reference point
    • Vega is the standard magnitude-zero star at all wavelengths (UV-Near-IR)
• 19th century science found out that the human eye sees logarithmically
  – going from class 1 to 2, or 2 to 3... is roughly going to half as bright
• 1856, Norman Pogson formalised this:
  – 1 magnitude ≡ factor of $100^{1/5}$ difference in flux
    (so actually 2.512... not 2)
    \[ m \propto -2.5 \log_{10} f \quad \text{or} \quad f \propto 10^{-0.4m} \]
• If we are comparing the brightness of 2 objects the constant of proportionality cancels out.
  \[ m_A - m_B = -2.5 \log_{10} \left( \frac{f_A}{f_B} \right) = 2.5 \log_{10} \left( \frac{f_B}{f_A} \right) \]
  – Let $f_A / f_B$ be 100, so star A is brighter
  – then $m_A - m_B = -2.5 \times \log_{10} (100) = -2.5 \times 2 = -5$ mag
  – star A is five magnitudes lower (i.e., brighter)
• Alternatively if star A is 1 mag brighter than star B

\[
\frac{f_A}{f_B} = 10^{-0.4(m_A-m_B)}
\]

\[
\frac{f_A}{f_B} = 10^{-0.4(-1)} = 2.512
\]

i.e., consistent with Pogson's law

• Confusingly, log_{10} and linear systems are both in common usage
  – linear e.g. Jy, makes sense for detectors that respond linearly to light
    (typically used in Radio astronomy)
  – log_{10} makes sense for a logarithmic response, e.g., photographic film
    (typically used in UV, optical, and near-IR astronomy)
Absolute magnitudes

- **apparent magnitude** \( m \) depends on distance to the star
- so to compare stars fairly, **absolute magnitudes** \( M \) are used
  - apparent magnitude object would have if located 10 pc away from us
- using the inverse-square law
  \[
  \frac{f_d}{f_{10\text{pc}}} = (\frac{10}{d})^2
  \]
- using the magnitudes definition:

  \[
  d \text{ given in pc}
  \]

  or

  \[
  \text{[m-M=5log(d)+25 if } d \text{ given in Mpc]}
  \]

\[
\begin{align*}
m_1 - m_2 &= m - M \\
m - M &= -2.5 \log_{10} \left( \frac{f_d}{f_{10\text{pc}}} \right) \\
m - M &= -2.5 \log_{10} \left( (\frac{10}{d})^2 \right) \\
m - M &= -5 \log_{10} \left( \frac{10}{d} \right) \\
m - M &= 5 \log_{10} \left( \frac{d}{10} \right) \\
m - M &= 5 \log_{10} (d) - 5 \log_{10} (10) \\
m - M &= 5 \log_{10} (d) - 5
\end{align*}
\]
Example 1.

- If a star has an **apparent magnitude** (m) of +13 mag and lies at a distance of 20pc what is its **absolute magnitude** (M)?

\[
m-M=5\log_{10}(d)-5
\]
\[
M=m-5\log_{10}(d)+5
\]
\[
M=13.0-5\log_{10}(20)+5
\]
\[
M=11.5 \text{ mag}
\]

**Note:**
- Units are mag
- More negative = brighter (NB: this is counter-intuitive, beware!)
- Typical range of Abs. mag: stars -1 to +10, galaxies -24 to -6
- Typical range of app. mag: stars/galaxies, -27 (Sun) to +30 (faintest detectable galaxy)
- Objects with app mag < 5 visible to naked eye.
Distances to the stars

• Fundamental measurement:
  – how far away is a point of light?
    is it a 100-watt light bulb? as bright as the Sun? a galaxy of $10^{11}$ stars?

• we need a method of distance measurement that is *independent of flux*

\[ m - M = 5 \log_{10}(d) - 5 \]

or

\[ d = 10^{0.2(m-M+5)} \]

where $d$ is in parsecs
Example 2

• The faintest detectable object in our deepest images are at about $\sim 25^{\text{th}}$ mag. What distance would this equate to for:

  i) a bright star ($M = -1$ mag)
  \[ d = 10^{0.2(m-M+5)} = 10^{0.2(25-(-1)+5)} = 1585 \text{kpc} \]

  ii) a bright galaxy ($M = -21$ mag)
  \[ d = 10^{0.2(m-M+5)} = 10^{0.2(25-(-21)+5)} = 15849 \text{Mpc} \]

• Typically use parsec (pc) or kiloparsec (kpc) for distances to stars and Megaparsecs for distances to galaxies (Mpc)
  – Nearest major galaxy = Andromeda (M31) = $\sim 1 \text{ Mpc}$
Magnitude equation in Mpc

\[ m - M = 5 \log_{10}(d) - 5 \]
\[ m - M = 5 \log_{10}(d \times 10^6) - 5 \]
\[ m - M = 5 \log_{10}(d) + 5 \log_{10}(10^6) - 5 \]
\[ m - M = 5 \log_{10}(d) + 25 \]

Above equation typically used for galaxies where distances are generally specified in Mpc
Example 3

• Two galaxies collide, each has an apparent magnitude of 12 mag assuming flux is conserved what is the final magnitude of the merger product.

\[
\begin{align*}
  f_{\text{final}} &= f_1 + f_2 = 2f_1 \\
  10^{-0.4m_{\text{final}}} &= 2 \times 10^{-0.4m_1} \\
  -0.4m_{\text{final}} &= \log_{10}(2 \times 10^{-0.4m_1}) \\
  m_{\text{final}} &= -2.5 \log_{10}(2 \times 10^{-0.4m_1}) \\
  m_{\text{final}} &= -2.5 \log_{10}(2) + m_1 \\
  m_{\text{final}} &= 12 - 0.75 \\
  m_{\text{final}} &= 11.25\text{mag}
\end{align*}
\]

• NB: Answer is not 12+12 !!!
Parallax and the Parsec

- Triangulation during the course of Earth’s orbit around Sun:
  - Based purely on geometry
  - \( p = \angle \) angle of parallax
  - by measuring the angles of the star relative to the Sun then given a known baseline we can recover the distance

- As stars are very distant \( p \) is very small
  - use the Earth's orbit as the baseline!
  - note \( p \) is \textit{half} the full angle
• because \( p \) is small we can use the small angle formula
  
  \[ \tan p = \frac{s}{d}, \text{ therefore } d = \frac{s}{\tan(p)} \]
  
  \[ \text{but } \tan(p) \sim p, \text{ so: } d = \frac{s}{p} \text{ if } s \text{ is in m and } p \text{ is in radians} \]
  
  [1" = 1 second of arc = 1/3600th of one degree]
  
  – However we can also use this equation to define a new distance unit, the Parsec
Semi-major axis ("radius") of Earth’s orbit

= 1 Astronomical Unit (1 A.U.)
= $1.5 \times 10^{11}$ m

The distance ($d$) to an object is defined to be 1 parsec when the parallax angle ($p$) is 1 arcsecond

In general distance (in pc) = $1/p$ where $p$ is specified in arcseconds

Note: $p$ is half the annual parallax
When the observed object is further away relative to the background, the parallax is much smaller.
distances to the stars

• closest star system is at 1.3 pc
  – the Centauri system with \( p = 0.75" \)

• In metric units:

\[
p \approx \tan(p) = \frac{1 \text{A.U.}}{d \text{(in pc)}} = \frac{1.5 \times 10^{11}}{d \text{(in m)}}
\]

\[
d \text{(in m)} = \frac{1.5 \times 10^{11}}{p} = \frac{1.5 \times 10^{11}}{\pi / 180 / 3600}
\]

\[
d \text{(in m)} = 3.1 \times 10^{16} \text{ m}
\]

One parsec = \( 3.1 \times 10^{16} \text{ m} \)
Parallax Limit

• measurements of positions of stars on the sky (astrometry) can find parallaxes of stars to:
  – \( \sim 0.05" \) (\( d = 20 \text{ pc} \)) with ground-based telescopes
  – \( \sim 0.005" \) (\( d = 200 \text{ pc} \)) with satellites such as Hipparcos (1997)
  – \( \sim 0.001" \) with GAIA due for launch in 2013 by ESA
    • will survey 1 billion stars
    • produce 3D map of galaxy
    • will find 10,000+ extra-solar planets
    • Dwarf planets, TNOs, Kuiper Belt and Oort cloud

• so geometrical distances can be found for nearby stars only
  – for larger distances, we need other methods, based on the properties of stars
Standard candles

• If an absolute magnitude is known, comparison to its apparent magnitude will give the distance:
  – E.g., 100W light bulb will look fainter if further away → can use mag equation (ratio of intrinsic to apparent flux) to derive distance d

• To do this we need a property that does not vary with distance which is correlated with the absolute magnitude, e.g:
  – Cepheid variables (discovered by Henrietta Leavitt 1912)
    • The Period-Luminosity relation use to prove galaxies are external and discover the expansion
  – Supernova Type Ia
    • Peak brightness of a supernova Ia light curve used to discover the dark energy

• These relations can be calibrated to an absolute scale via parallax:
  – Nearest Cepheid is Polaris (somewhat irregular) at 133pc
  – 220 trigometric parallaxes to Cepheids now measured via Hipparcos
Cepheid P-L relation

- Well studied stellar objects
- Very bright ($M_v \sim -2$)
- Pulsate regularly (~ few days)
- Pulsation period depends on luminosity
- P-L relation calibrated to 220 stars via Hipparcos parallax distances (1997)
Distances from Cepheids

\[ \log_{10} P + 0.394M_V = -0.657 \]

DAYS V Abs. Mag. (From Hipparcos)

Once this Equation has been calibrated (as above) we can use it to measure distances to other clusters.

STRATEGY: Observe a star cluster
Find Cepheids (via light curves)
Measure P
Measure V app. Mag peak
Calculate Mv
Calculate distance ( \[ m = M + 5\log_{10}(d) - 5 \] )
Andromeda: The nearest spiral
Example: Cepheid Distances

• If a Cepheid is observed to pulsate with a period of 2.5 days and has an apparent peak magnitude of 18.6 V mags, how far away is it?

USE:

\[ \log_{10} P + 0.394M_V = -0.657 \]

REARRANGE:

\[ M_V = \frac{-0.657 - \log_{10}(P)}{0.394} \]

GIVES:

\[ M_V = -2.68 \text{ mags} \]

USE:

\[ m = M + 5\log_{10}(d) - 5 \]

REARRANGE:

\[ d = 10^{0.2(m-M+5)} \]

GIVES:

\[ d = 0.18 \text{ Mpc} \]
The Distance to Andromeda

• Andromeda (M31) is our nearest giant galaxy. It lies 0.9Mpc away. What would be the apparent magnitude of a 3 day Cepheid?

AS BEFORE:

\[ M_V = \frac{-0.657 - \log_{10}(P)}{0.394} \]

\[ M_V = -2.88 \text{mags} \]

\[ m = -2.88 + 5\log_{10}(0.9) + 25 \]

\[ m = 21.9 \text{ mags} \]
Bolometric magnitudes and colour

• Filters
  – Typically magnitudes are measured through a specific filter or bandpass
  – Filters only allow light from a specific wavelength range through
  – Examples are ugriz or BVRI in the optical or YJHK in the near-IR
  – Use filter symbols as subscript, i.e., $m_K = K$-band magnitude
  – If magnitude is over entire wavelength range its called *bolometric*

• Adding magnitudes equivalent to multiplying fluxes
  – Typically only used for dust corrections where dust attenuates some fraction of the light, e.g., $m_V - M_V = 5 \log_{10} d + 5 + A_V$, $A_V$ = dust attenuation term

• Subtracting magnitudes equivalent to measuring a flux ratios
  – Used to derive *colours*, i.e., $(B-V) = m_B - m_V$
Typical filters

Most observations traditionally done in the optical band-passes, however over past decade observations now being made at all wavelengths via space missions and other Technological breakthroughs.
Colour, e.g., colour magnitude diagram

- Can plot (B-V), i.e., $M_B - M_V$ v $M_V$ for stars (not $M$ not $m$)
- Typically see galaxy population divide into two pops, red and blue