Galaxies

Tutorial Sheet 3: Answers

Q1)

a) Let N=total number of galaxies per Mpc^3, then:

$N = \int_0^\infty \phi(L) dL$

$N = \int_0^\infty \phi_* (\frac{L}{L_*})^\alpha e^{-(\frac{L}{L_*})} dL$

$N = \phi_* \int_0^\infty (\frac{L}{L_*})^\alpha e^{-(\frac{L}{L_*})} dL$

$N = \phi_* \Gamma(\alpha + 1)$

b) The implication for $\alpha < -1$ is that the total number is infinite. The reason this is not a paradox is because there is presumably some finite luminosity limit for the luminosity function and at some point it turns down. But where ?

Q2) Let N be the total number of galaxies to some app mag limit, l:

$N = \int_{-\infty}^\infty \phi(L) V(L, l_{lim}) dL$

$V = \frac{\Omega}{360^2/\pi} \frac{4}{3} \pi d^3$

$d^2 = 10^{-10} \frac{L}{l}$, d in Mpc (from $m - M = 5\log(d) + 25 = -2.5\log(l/L)$)

$d^3 = 10^{-15} (\frac{L}{l})^{\frac{3}{2}}$

$V = \frac{\Omega}{360^2} \frac{4}{3} 10^{-15} (\frac{L}{l})^{\frac{3}{2}}$

$N = \int_{-\infty}^\infty \phi(L) \frac{\Omega}{360^2} \frac{4}{3} 10^{-15} (\frac{L}{l})^{\frac{3}{2}} dL$

$N = \frac{\Omega}{360^2} \frac{4}{3} 10^{-15} \phi_* \int_{-\infty}^\infty (\frac{L}{L_*})^\alpha e^{-(\frac{L}{L_*})} (\frac{L}{l})^{\frac{3}{2}} dL$

$N = \frac{\Omega}{360^2} \frac{4}{3} 10^{-15} \phi_* \int_{-\infty}^\infty (\frac{L}{L_*})^\alpha e^{-(\frac{L}{L_*})} (\frac{L}{L_*})^{\frac{3}{2}} (\frac{L_*}{l})^{\frac{3}{2}} dL$

$N = \frac{\Omega}{360^2} \frac{4}{3} 10^{-15} \phi_* (\frac{L_*}{l})^{\frac{3}{2}} \int_{-\infty}^\infty (\frac{L}{L_*})^{\alpha + \frac{3}{2}} e^{-(\frac{L}{L_*})} dL$ using Euler's integral (i.e., Gamma function).

$N = \frac{\Omega}{360^2} \frac{4}{3} 10^{-15} \phi_* (\frac{L_*}{l})^{\frac{3}{2}} \Gamma(\alpha + \frac{5}{2})$

$N = \frac{\Omega}{360^2} \frac{4}{3} 10^{-15} \phi_* 10^{-0.4\frac{3}{2}(M_* - M)} \Gamma(\alpha + \frac{5}{2})$

$N = 5 \times 10^{-19} .0.001.10^{-0.4\frac{3}{2}(M_* - M)} \Gamma(\alpha + 2.5)$

$N = 5 \times 10^{-22} 10^{-0.6(-21-19.5)} \Gamma(1)$

$N = 998(0)!$

$N = 998.$

Hence we expect to find $\sim 1,000$ galaxies in this survey (this is harder than anything I would ask in the exam).

Q3) From the notes:

$\rho = \frac{M}{L} L_* \phi_* \Gamma(\alpha + 2) = 10 \frac{M_\odot}{L_\odot} h_{0.5} 0.001 h_{0.5}^3 10^{10.64} L_\odot h_{0.5}^{-2} \frac{1}{(3 \times 10^{22})}^3 \sqrt{\pi}$

As $\Gamma(0.5) = \sqrt{\pi}$

This assumed $L_* = 10^{10.64} L_\odot$ (i.e., $(\frac{L_*}{L_\odot}) = 10^{-0.4(M-M_\odot)} = 10^{10.64}$, where $M_\odot = +5.6$) , $\text{Mass}_\odot = 2 \times 10^{30}$kg, 1Mpc$= 3 \times 10^{22}$m and chomping through on the calculator should give $\rho = 5.73 \times 10^{-29} h_{0.5}^2$kg m^{-3}.

$\Omega_{Crit} = \frac{3H_o^2}{8\pi G} = 5 \times 10^{-27} h_{0.5}^2$kg m^{-3}.

$\rho_{Total} = \rho_{Dark} + \rho_{Baryon} = \rho_{Baryon} \frac{M_{Dark}}{M_{Baryon}} + \rho_{baryon}$ hence $\frac{M_{Baryon}}{M_{Dark}} = \frac{1}{87}$.

Dark matter needs to outweigh the baryons by a factor of 87 to close the universe.

Q4) Galaxies were brighter in the past but fewer. Implies massive galaxies formed first and lower mass galaxies later, i.e., downsizing.

$J = \phi^* L^* \Gamma(2+\alpha)$ if α stays the same $J_z/J_0 = (1+z)^{\beta+\gamma} = (1+z)^{(-0.2)}$, i.e., luminosity density was lower in the past and at $z = 2$ was 80% the curent value.

Q5) $m = 24$ mag, $z = 1$, $A_v = 0.0$ mag.
$M = m - 5\log(d) - 25 - k(z)$, $d = \frac{cz}{H_o}$, adopt $H_o = 72$ km/s/Mpc
$M = 24 - 5\log(3 \times 10^5/72) - 25 - k(z) = -19.1 - k(z)$
i) $k(z) = az + bz^2 = 3.13 + 0.24 = 3.37$, therefore $M(E/S0) = -22.5$ mag
ii) $k(z) = az + bz^2 = 2.63 - 0.107 = 2.523$, therefore $M(E/S0) = -21.6$ mag
iii) $k(z) = az + bz^2 = 0.62 + 0.14 = 0.76$, therefore $M(E/S0) = -19.9$ mag
iv) We need to know the type to get the K(z) right.

Q6) At $z = 1$, $d = cz/H_o = 3 \times 10^5/72 = 4167$ Mpc
If A_v=0.2 mag then A_v per Mpc $= 4.8 \times 10^{-5}$ mag

Q7)
$m = M + 5\log(cz/H_o) + 25 + A_v + k(z)$
Adopt E/S0 k-correction:
$k(z) = 3.13 \times 0.5 + 0.24 \times 0.5^2 = 1.625$ mag
$A_v = -2.5\log(0.80) = 0.25$ mag
$m = -19.0 + 25.0 + 16.6 + 0.25 + 1.625 \approx 24.5$ mag

Ellipticals typically do not contain dust.