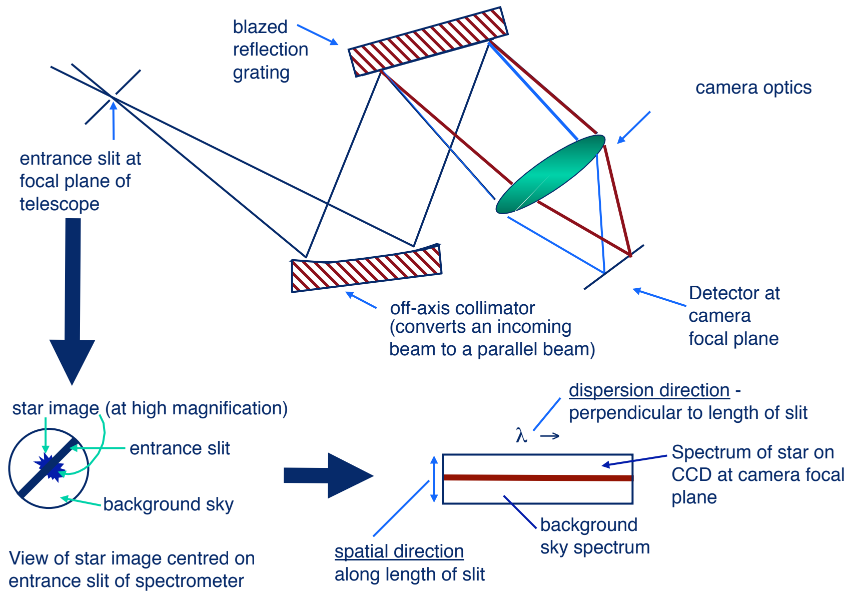
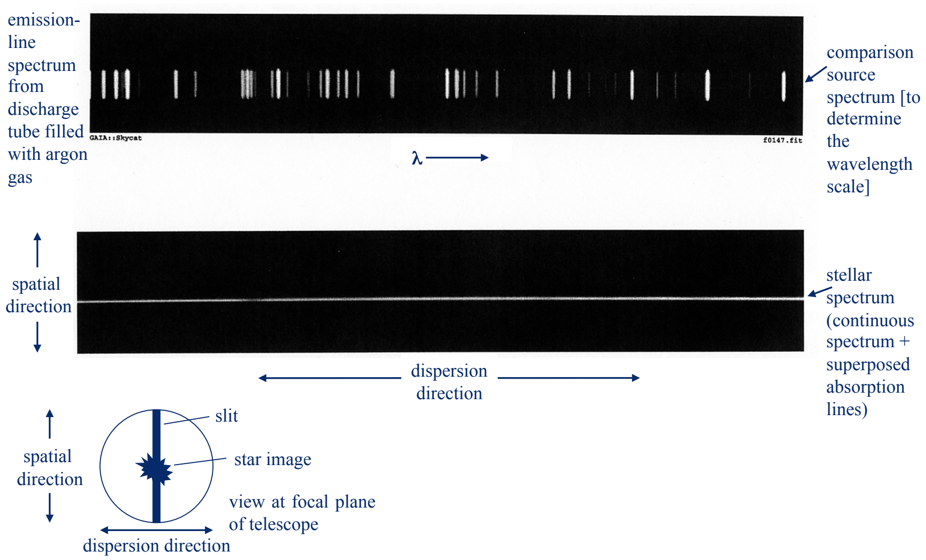
10. Spectroscopy

- **Slit spectroscopy: entrance aperture of the spectrometer is a narrow slit.**
- **Entrance slit must be**
  - **wide enough to allow light from the source through to spectrograph,**
  - **narrow enough to preserve a desired spectral resolution.**
- **Typical slit width ~1 arcsec, while seeing discs are usually >1 arcsec, so lose some light from source.**
- **Need to design spectrograph optics to give required spectral resolution for a given slit width.**
  - **Cannot use a slit spectrograph for spectrophotometry because light losses vary with seeing, unless you open the slit wide.**

Grating spectrograph:schematic



‘Raw’ spectra recorded by a CCD on a spectrometer

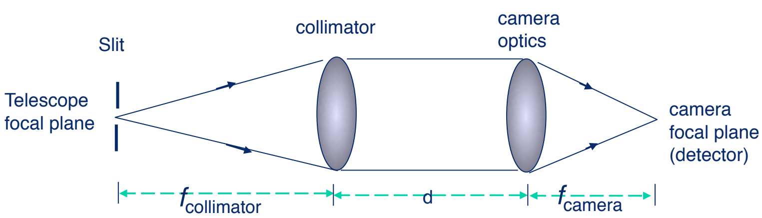


Collimator, camera and image size

- **Image scale at slit (telescope focal plane):**

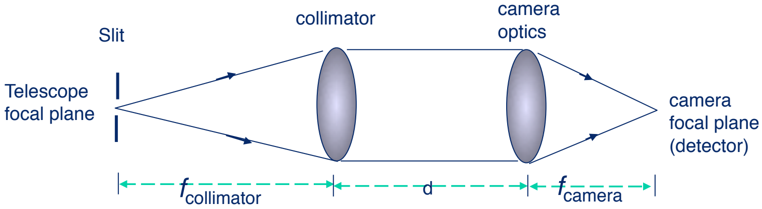
  $206265/f_t$ arcsec mm^{-1}

- **Image of slit is projected onto CCD in spectrograph camera focal plane by 2 optical elements: collimator and camera.**
  - **Need to match projected slit width to CCD pixel size.**



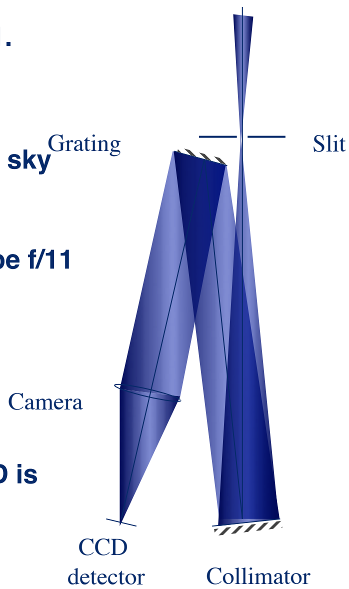
- **Think of *collimator* as telescope of focal length $f_{collimator}$ acting in reverse.**
  - **Slit size x_{slit} is related to spread θ in beam directions reflected off collimator by: $x_{slit} = f_{collimator}\,\theta$.**
- **The *camera* is a "telescope" of focal length f_{camera}.**
  - **Size of slit image on CCD is $x_{CCD} = f_{camera}\,\theta$.**
- **Hence image of slit projected on CCD is reduced in size by the magnification factor**

$$M_T = \frac{x_{CCD}}{x_{slit}} = \frac{f_{camera}}{f_{collimator}}.$$



Example: Leslie Rose Spectrograph

- **mirror diameter D = 50 cm; Cass. focus f/11.**
  - ∴ **Telescope focal length $f_{telescope}$ = 5500mm**
  - ∴ **image scale = 206265/ $f_{telescope}$ = 37.5 arcsec/mm**
- **seeing disc at St Andrews is ~2-2.5" on the sky**
- **(narrow!) physical width of slit = 53 - 67 μm**
- **Spectrograph collimator mirror must also be f/11**
  - **$D_{collimator}$ = 50 mm**
  - ∴ **$f_{collimator}$ = 550 mm**
- **Camera focal length f_{camera} = 208 mm**
  - ∴ **M_T = 208/550 = 0.378**
- **Hence width of slit image projected on CCD is**
  - **53 to 67 μm x M_T = 20 to 25 μm**
  - **the typical sizes of CCD pixels, as required.**



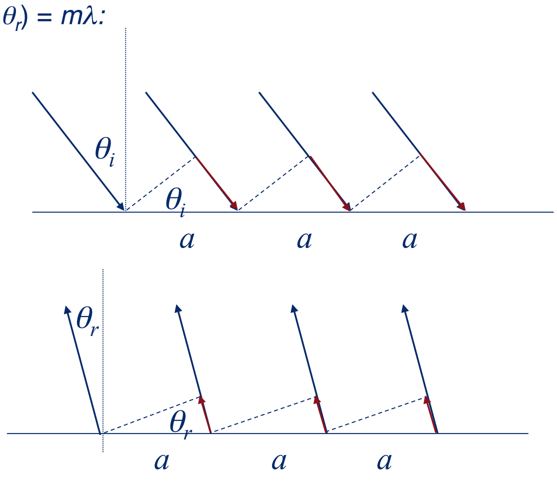
Diffraction Gratings

- **a repeating pattern of diffracting elements, apertures or obstacles.**
  - **reflection diffraction gratings common because *achromatic***
    - **parallel rules scratched in an aluminium surface on a plane glass plate at hundreds of rules per mm! (& plastic copies)**
  - **these act as reflection phase gratings since the interference pattern is caused by changes in phase of the light waves.**
- **Standard grating equation: $a \sin \theta_r = m\lambda$**
  - **for grating spacing a (distance between adjacent rules - e.g. a ~ 0.001 mm) & incident beam perpendicular to the grating.**
  - **Zeroth order (m = 0) is reflected straight back (θr=0). First order (m = ± 1) gives a different θr, clearly dependent on λ, a.**

More generally, for incident angle $\theta_i \neq 0$ the grating equation is modified to: $a (\sin \theta_i + \sin \theta_r) = m\lambda$:

Incident beam:
Path difference between adjacent grooves = $a \sin \theta_i$

Reflected beam:
Additional path difference between adjacent grooves = $a \sin \theta_r$

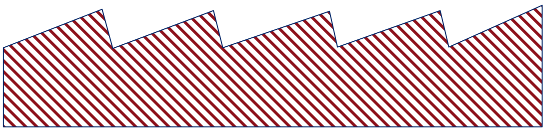


Total path difference between adjacent grooves = $a (\sin \theta_i + \sin \theta_r)$ - if diffracted beam is on same side of grating normal as incoming beam.

Otherwise path difference = $a (\sin \theta_i - \sin \theta_r)$.

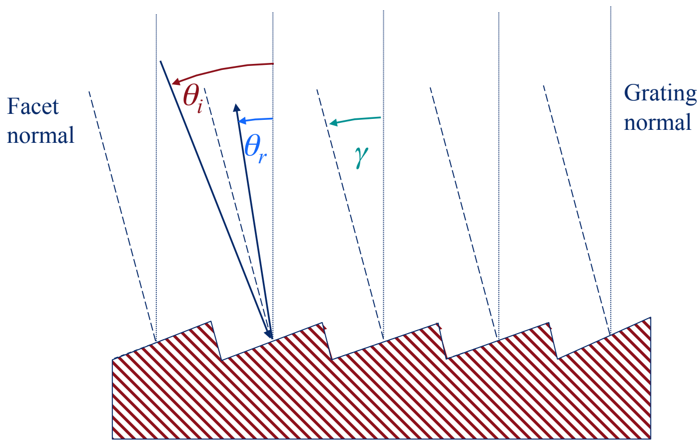
Interference

- **Outgoing beams from adjacent grooves interfere constructively when path lengths differ by an integer number of wavelengths:**
  - **m = ±1, ±2, … is constructive but ±1/2, ±3/2 … is destructive**
  - **for given m, θ_r depends on λ so each order is spread into a spectrum.**
- **but such a grating reflects most of the light into m = 0 order where all wavelengths overlap - useless for spectroscopy.**
  - **solution is a *blazed* reflection grating**
  - **grooves: inclined wedges in plane surface**



Blazed gratings

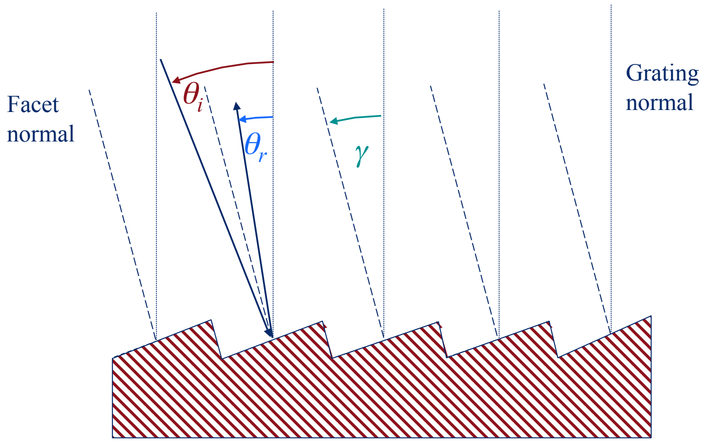
- **Shifts zeroth order light into first order.**
  - **θi - angle of incidence of incoming beam to the grating normal**
  - **θr - angle of diffracted beam to the grating normal**



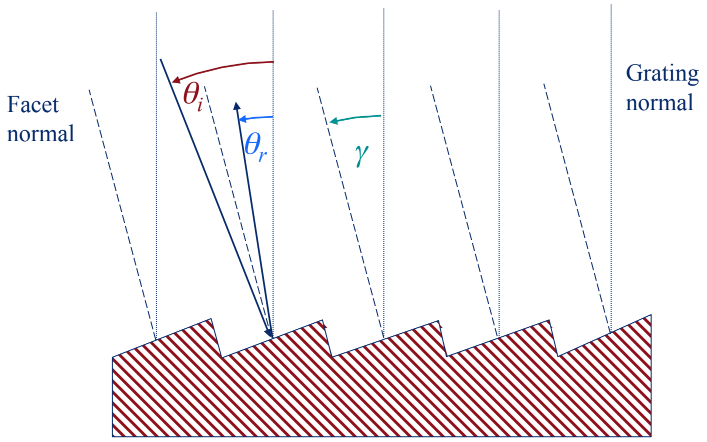
- θ_i, θ_r measured from normal to grating plane.
- Peak intensity in diffraction pattern corresponds to *specular reflection* off the groove facets.

(for very polished surface, single outgoing angle)

  – then $\theta_r = \theta_i = \gamma$ – the *blaze angle*.



- Blaze angle γ related to θ_i, θ_r by $\theta_i + \theta_r = 2\gamma$ for specular reflection - the diffraction peak.
  – When $\theta_i = \theta_r = \gamma$ and $m = 1$, grating equation defines *blaze wavelength*: $2a \sin \gamma = \lambda_b$



linear dispersion

- **FAQ: what range of wavelengths will fit on to the detector at the spectrometer's focal plane?**
  - θ_i **is the same for all** λ **(parallel, collimated, incoming beam).**
  - **For given order *m*, can differentiate the grating equation *w.r.t.* λ:**

$$m = a\cos\theta_r \frac{d\theta_r}{d\lambda}; \quad \frac{d\theta_r}{d\lambda} = \frac{m}{a\cos\theta_r}$$

  - ***linear dispersion* of the spectrum in focal plane (*x*-direction) of the camera is then:**

$$\frac{dx}{d\lambda} = f_{\mathrm{camera}} \frac{d\theta_r}{d\lambda} = \frac{f_{\mathrm{camera}} m}{a\cos\theta_r}$$

  - **e.g. Leslie Rose spectrograph**
    - **a = 1/1200 mm (1200 lines per mm), in first order m = 1, and f_{camera} = 208 mm, the linear dispersion is**

$$\frac{dx}{d\lambda} = \frac{208 \times 1}{{}^{1}\!/_{1200} \times \cos\theta_r} \approx \frac{249600}{\cos\theta_r}$$