Lecture 10: Chemical Evolution of Galaxies

Metalicity evolution Z(t) vs galaxy type:

Processes that alter the metalicity:

1. Type-II SNe enrich the ISM.
2. Low-mass stars form from enriched ISM and "lock-up" metals.
3. Primordial gas falls in from IGM.
4. ISM ejected into IGM. (e.g. SN explosions, galaxy collisions)

Closed-box model: 1 and 2 only.

Metalicity Evolution: Z(t)

M_0 = total mass

$M_G(t)$ = mass of gas in ISM

$M_Z(t)$ = mass of metals in ISM

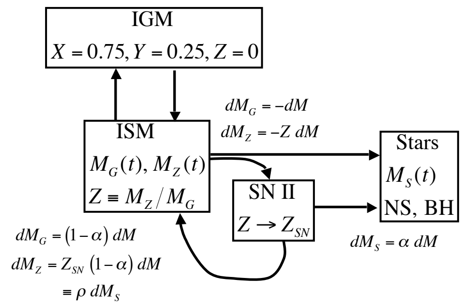
$M_S(t)$ = mass of the long-lived stars that lock up metals

We know: $\mu(t) \equiv \frac{M_G(t)}{M_0}$ $\quad \mu(0) = 1$

To derive: $Z(t) \equiv \frac{M_Z(t)}{M_G(t)}$ $\quad Z(0) = 0$

Find: $Z(\mu(t))$

ISM Recycling



The Yield ρ

Mass is conserved (gas- > stars)

$dM_G = -\alpha\, dM \quad dM_S = \alpha\, dM$

Metals are lost to stars and enriched by SNe:

$$dM_Z = -Z\, dM \;+\; Z_{SN}(1-\alpha)\, dM$$

$$= \left[Z_{SN}(1-\alpha) - Z + \alpha Z - \alpha Z\right]\frac{dM_S}{\alpha}$$

$$= \left[\frac{(Z_{SN}-Z)(1-\alpha)}{\alpha} - Z\right] dM_S \equiv (\rho - Z)\, dM_S$$

The Yield: $\rho = \frac{(Z_{SN}-Z)(1-\alpha)}{\alpha}$

$$= \frac{\text{mass of metals added to ISM by SN}}{\text{mass moved from ISM to stars}}$$

Metalicity Evolution Z(t)

$$Z(t) \equiv \frac{M_Z(t)}{M_G(t)}$$

Differentiate:

$$\delta Z = \delta\left(\frac{M_Z}{M_G}\right) = \frac{\delta M_Z}{M_G} + M_Z\, \delta\left(\frac{1}{M_G}\right)$$

But,

$$\delta\left(\frac{1}{M_G}\right) = -\frac{\delta M_G}{M_G^{\,2}}$$

Thus:

$$\delta Z = \frac{\delta M_Z}{M_G} - \frac{M_Z}{M_G^{\,2}}\,\delta M_G$$

$$= \frac{1}{M_G}\left(\delta M_Z - Z\,\delta M_G\right)$$

$$\delta Z = \frac{1}{M_G}\left(\delta M_Z - Z\,\delta M_G\right)$$

$$= \frac{1}{M_G}\left((\rho - Z)\delta M_S - Z\,\delta M_G\right)$$

$$= -\rho\,\frac{\delta M_G}{M_G}$$

Definition of yield: $\delta M_Z = (\rho - Z)\,\delta M_S$

Conserve mass: $\delta M_S = -\delta M_G$

Integrate (with ρ = constant) :

$$Z = -\rho\,\ln(M_G) + C$$

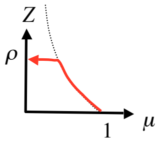
At $Z = 0$, $M_G = M_0$:

$$0 = -\rho\,\ln(M_0) + C \;\Rightarrow\; C = \rho\,\ln(M_0)$$

$$\therefore \boxed{Z = -\rho\,\ln\left(\frac{M_G}{M_0}\right)}$$

But, $\mu = \dfrac{M_G}{M_0}$

So finally, $\boxed{Z = -\rho \ln(\mu)}$



As $\mu \to 0$, $Z \to \infty$ impossible!

But $\delta M_Z = (\rho - Z)\, \delta M_S$

So, as $Z \to \rho$, $\delta M_Z \to 0$

The equation above is valid when $Z < \rho$, after which $Z = \rho$.

Insert $\mu(t)$ for each galaxy type into $Z(t) = -\rho \ln(\mu(t))$

Ellipticals:

$\mu(t) = e^{(-t/t_*)}$

$Z(t) = -\rho \ln\left(e^{-t/t_*}\right) = \rho \dfrac{t}{t_*}$ for $Z \le \rho$

$Z(t) = \rho$ otherwise

Spirals:

$\mu(t) = 1 - \dfrac{\alpha \dot{M} t}{M_0}$

$Z(t) = -\rho \ln\left(1 - \dfrac{\alpha \dot{M} t}{M_0}\right)$ for $Z \le \rho$

$Z(t) = \rho$ otherwise

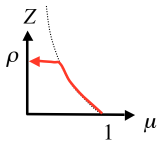
Irregulars:

$\langle \mu(t) \rangle = f \dfrac{\alpha M t}{M_0}$

$Z(t) = -\rho \ln\left(1 - f \dfrac{\alpha \dot{M} t}{M_0}\right)$ for $Z \le \rho$

$Z(t) = \rho$ otherwise

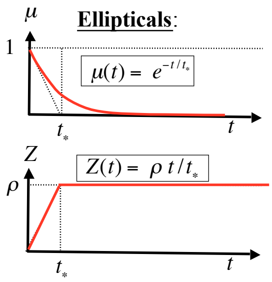
$\boxed{Z(t) = -\rho \ln(\mu(t))}$



Ellipticals:

$\mu(t) = e^{-t/t_*}$

$Z(t) = \rho\, t/t_*$

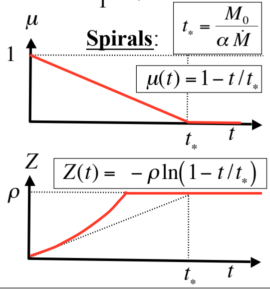


Spirals:

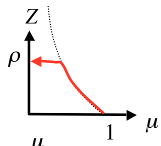
$t_* = \dfrac{M_0}{\alpha \dot{M}}$

$\mu(t) = 1 - t/t_*$

$Z(t) = -\rho \ln(1 - t/t_*)$



$\boxed{Z(t) = -\rho \ln(\mu(t))}$

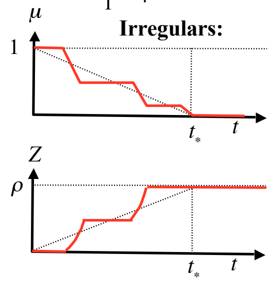


Irregulars:

$\langle \mu(t) \rangle \approx 1 - t/t_*$

$t_* = f \dfrac{M_0}{\alpha \dot{M}_{\text{burst}}}$ $f < 1$

$Z(t) \approx -\rho \ln(1 - t/t_*)$



The Yield ρ

$$\rho = \frac{(Z_{SN} - Z) \times \text{mass returned}}{\text{mass retained in old stars}}$$

First generation: $Z=0$ later generations $Z << Z_{SN}$:

From Salpeter IMF and SN 1987A: $\alpha = 0.93$

From SN 1987A: $Z_{SN} = 0.13$

$$\rho = \frac{(Z_{SN} - Z)\,(1-\alpha)}{\alpha}$$

$\Rightarrow$ $\boxed{\text{yield} = \rho \approx 0.01}$

However, note that $Z_{\odot} \approx 0.02$ which is larger than the calculated yield, implying $\rho > 0.02$! (remember $Z \le \rho$)