Lecture 10: Chemical Evolution of Galaxies

<u>Metalicity evolution Z(t)   vs galaxy type</u>:

Processes that alter the metalicity:

1. Type-II SNe enrich the ISM.
2. Low-mass stars form from enriched ISM and "lock-up" metals.
3. Primordial gas falls in from IGM.
4. ISM ejected into IGM. (e.g. SN explosions, galaxy collisions)

Closed-box model: 1 and 2 only.

Metalicity Evolution: $Z(t)$

M_0 = total mass

$M_G(t)$ = mass of gas in ISM

$M_Z(t)$ = mass of metals in ISM

$M_S(t)$ = mass of the long-lived stars that lock up metals

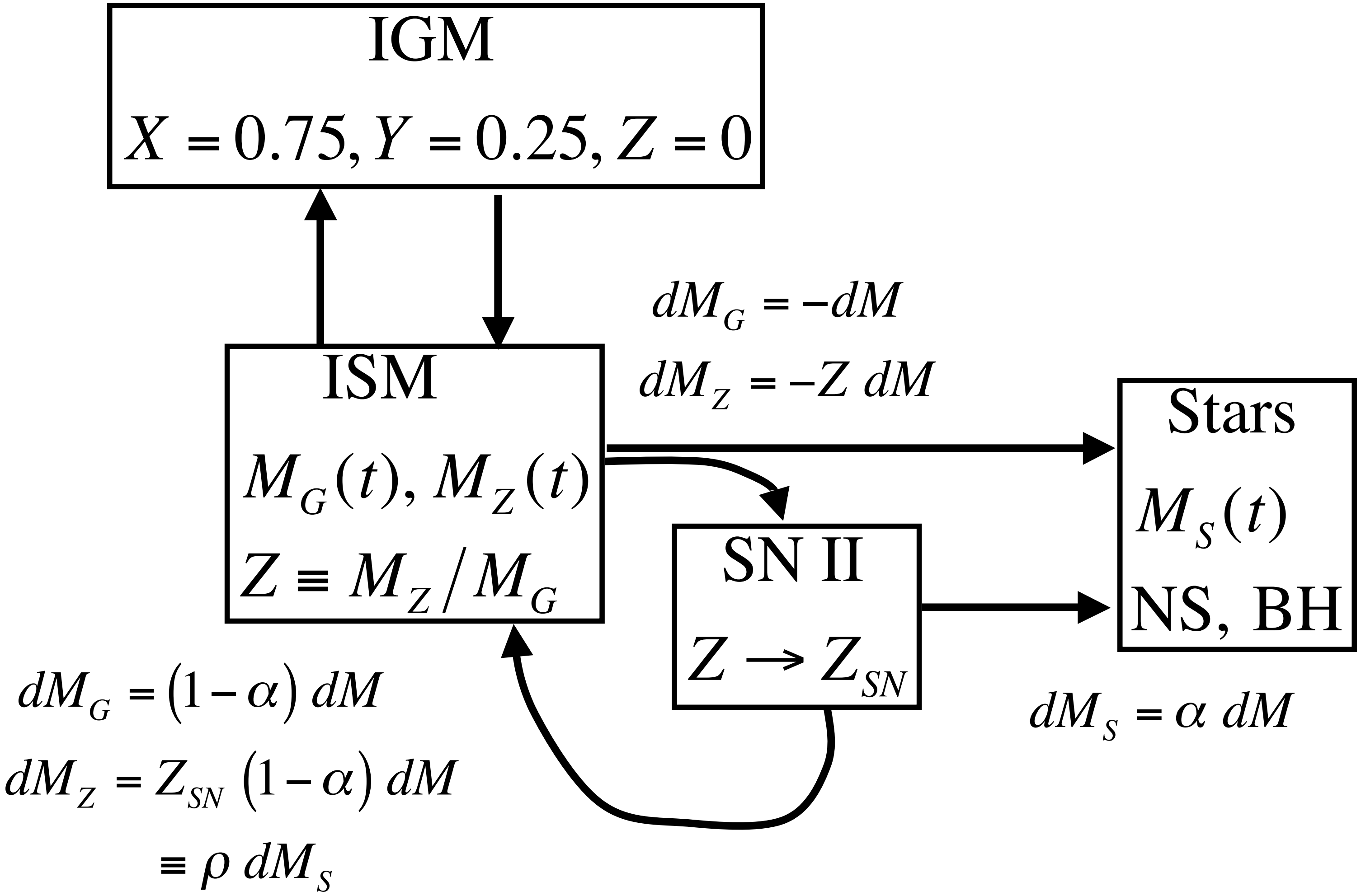
We know: $\mu(t) \equiv \frac{M_G(t)}{M_0}$ $\qquad \mu(0) = 1$

To derive: $Z(t) \equiv \frac{M_Z(t)}{M_G(t)}$ $\qquad Z(0) = 0$

Find:

$Z(\mu(t))$

ISM Recycling



The Yield ρ

Mass is conserved (gas- > stars)

$$dM_G = -\alpha\, dM \qquad dM_S = \alpha\, dM$$

Metals are lost to stars and enriched by SNe:

$$dM_Z = -Z\, dM \quad + \quad Z_{SN}\left(1-\alpha\right) dM$$

$$= \left[Z_{SN}\left(1-\alpha\right) - Z + \alpha Z - \alpha Z\right] \frac{dM_S}{\alpha}$$

$$= \left[\frac{\left(Z_{SN} - Z\right)\left(1-\alpha\right)}{\alpha} - Z\right] dM_S \quad \equiv \left(\rho - Z\right) dM_S$$

The Yield: $\rho = \dfrac{\left(Z_{SN} - Z\right)\left(1-\alpha\right)}{\alpha}$

$$= \frac{\text{mass of metals added to ISM by SN}}{\text{mass moved from ISM to stars}}$$

Metalicity Evolution Z(t)

$$Z(t) \equiv \frac{M_Z(t)}{M_G(t)}$$

Differentiate:

$$\delta Z = \delta\left(\frac{M_Z}{M_G}\right) = \frac{\delta M_Z}{M_G} + M_Z\,\delta\left(\frac{1}{M_G}\right)$$

But,

$$\delta\left(\frac{1}{M_G}\right) = -\frac{\delta M_G}{{M_G}^2}$$

Thus:

$$\delta Z = \frac{\delta M_Z}{M_G} - \frac{M_Z}{{M_G}^2}\,\delta M_G$$

$$= \frac{1}{M_G}\left(\,\delta M_Z - Z\,\delta M_G\,\right)$$

$$\delta Z = \frac{1}{M_G}\left(\delta M_Z - Z\,\delta M_G \right)$$

$$= \frac{1}{M_G}\left((\rho - Z)\,\delta M_S - Z\,\delta M_G \right)$$

$$= -\rho \frac{\delta M_G}{M_G}$$

Definition of yield:

$$\delta M_Z = (\rho - Z)\,\delta M_S$$

Conserve mass:

$$\delta M_S = -\delta M_G$$

Integrate (with ρ = constant) :

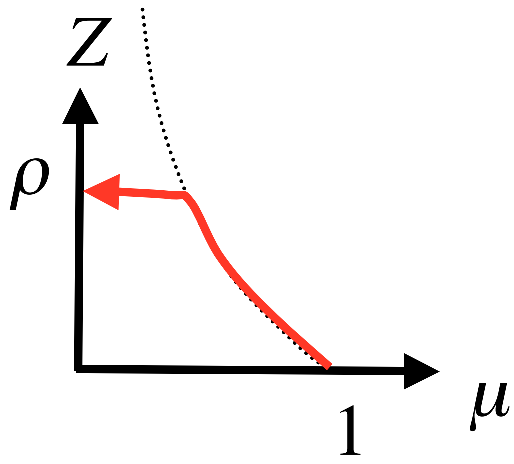
$$Z = -\rho\, \ln\left(M_G\right) + C$$

At $Z = 0$, $M_G = M_0$:

$$0 = -\rho \ln\left(M_0\right) + C \quad \Rightarrow \quad C = \rho \ln\left(M_0\right)$$

$$\therefore \boxed{Z = -\rho \ln\left(\frac{M_G}{M_0}\right)}$$

But, $$\mu = \frac{M_G}{M_0}$$



So finally,

$$\boxed{Z = -\rho \ln(\mu)}$$

As $\mu \rightarrow 0, \quad Z \rightarrow \infty$ impossible!

But $\delta M_Z = (\rho - Z)\, \delta M_S$

So, as $Z \rightarrow \rho, \quad \delta M_Z \rightarrow 0$

The equation above is valid when $Z < \rho$,
after which $Z = \rho$.

Insert $\mu(t)$ for each galaxy type into $$Z(t) = -\rho \ln\left(\mu(t)\right)$$

Ellipticals:

$$\mu(t) = e^{\left(-t/t_*\right)}$$

$$Z(t) = -\rho \ln\left(e^{-t/t_*}\right) = \rho \frac{t}{t_*} \quad \text{for} \quad Z \le \rho$$

$$Z(t) = \rho \quad \text{otherwise}$$

Spirals:

$$\mu(\mathrm{t}) = 1 - \frac{\alpha \dot{M} t}{M_0}$$

$$Z(t) = -\rho \ln\left(1 - \frac{\alpha \dot{M} t}{M_0}\right) \quad \text{for} \quad Z \le \rho$$

$$Z(t) = \rho \quad \text{otherwise}$$

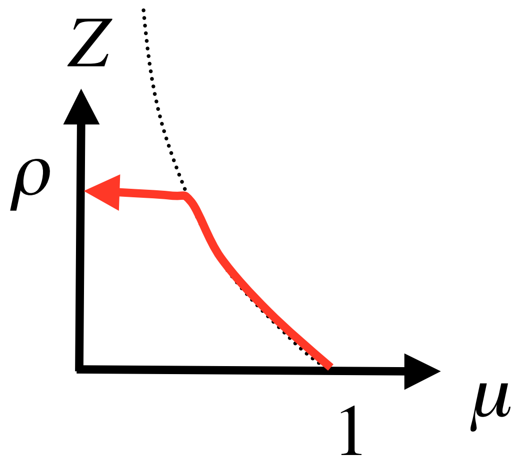
Irregulars:

$$\langle \mu(t) \rangle = f \frac{\alpha M t}{M_0}$$

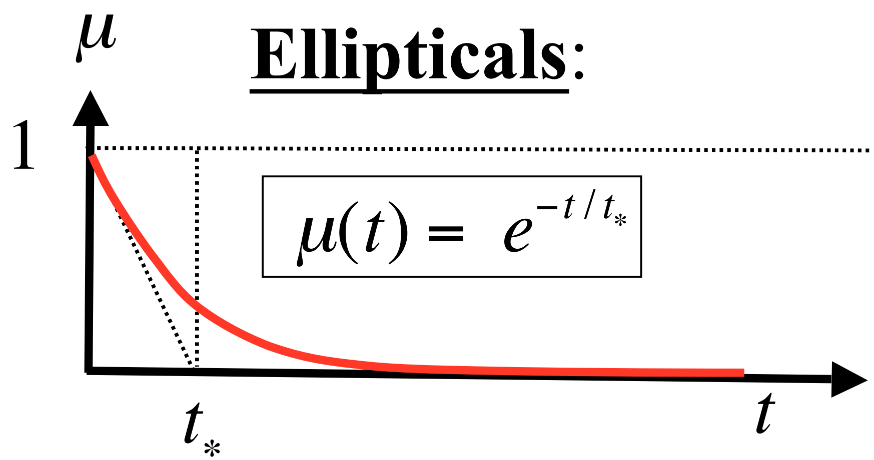
$$Z(t) = -\rho \ln\left(1 - f \frac{\alpha \dot{M} t}{M_0}\right) \quad \text{for} \quad Z \le \rho$$

$$Z(t) = \rho \quad \text{otherwise}$$

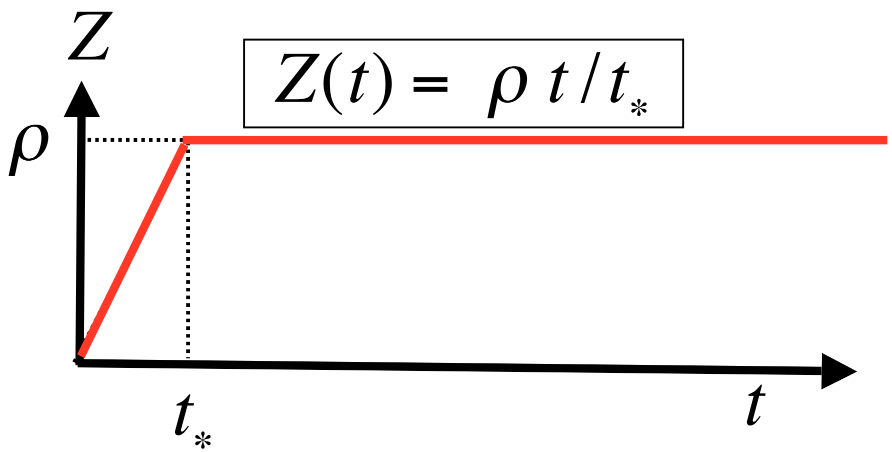
$$Z(t) = -\rho \ln\big(\mu(t)\big)$$



Ellipticals:



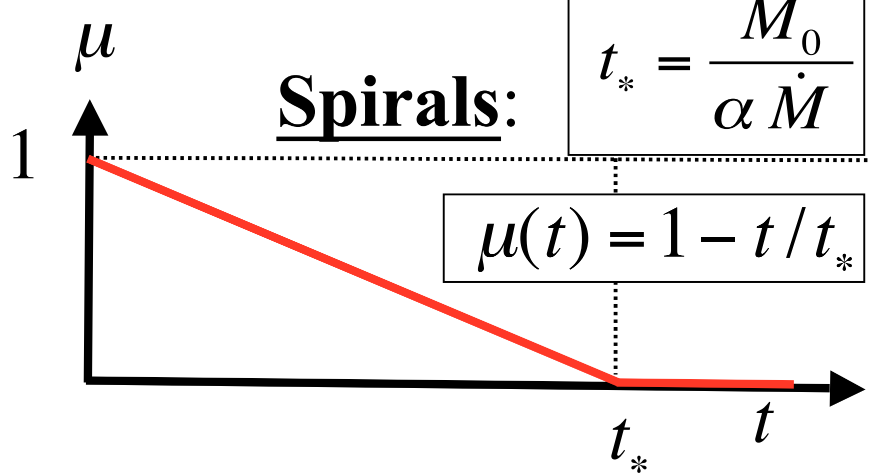
$$\mu(t) = e^{-t/t_*}$$



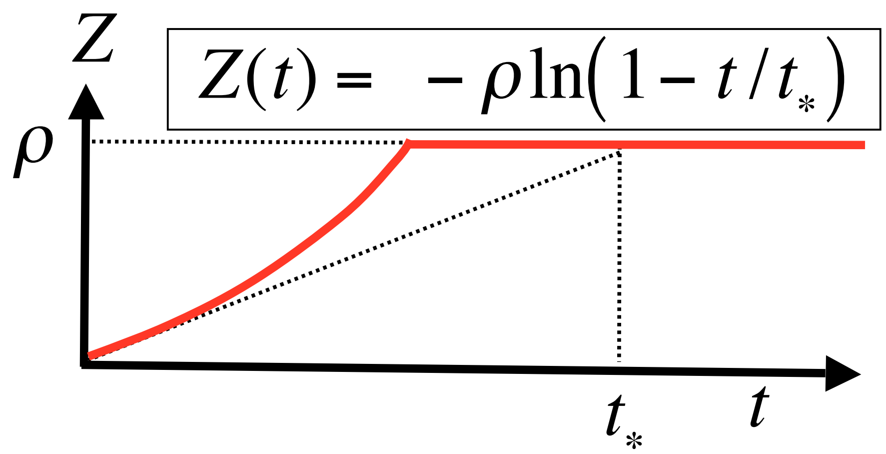
$$Z(t) = \rho\, t/t_*$$

Spirals:

$$t_* = \frac{M_0}{\alpha \dot{M}}$$

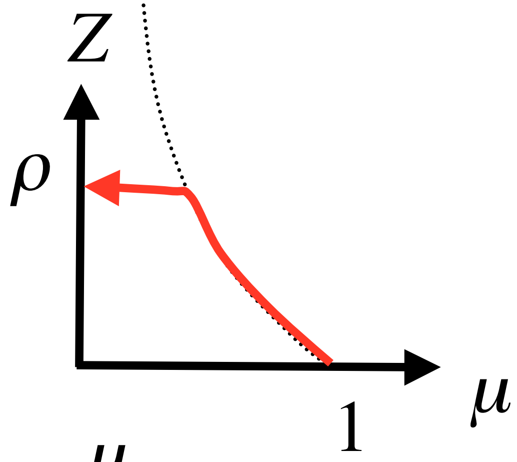


$$\mu(t) = 1 - t/t_*$$

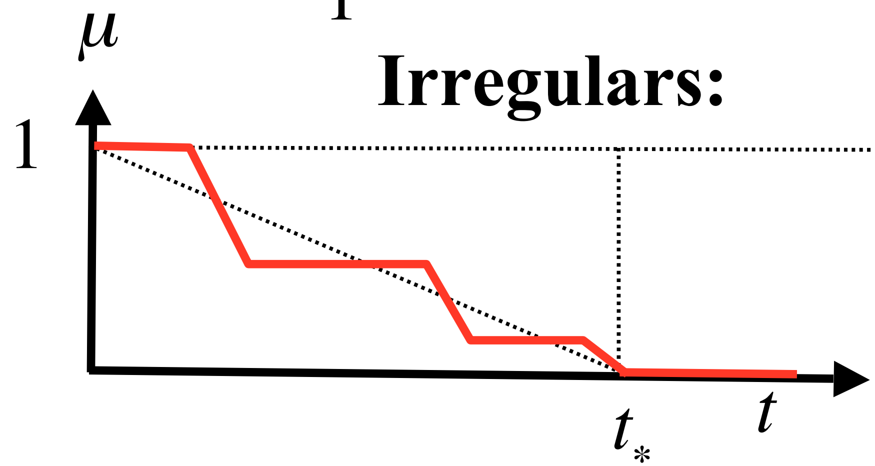


$$Z(t) = -\rho \ln\big(1 - t/t_*\big)$$

$$Z(t) = -\rho \ln\big(\mu(t)\big)$$



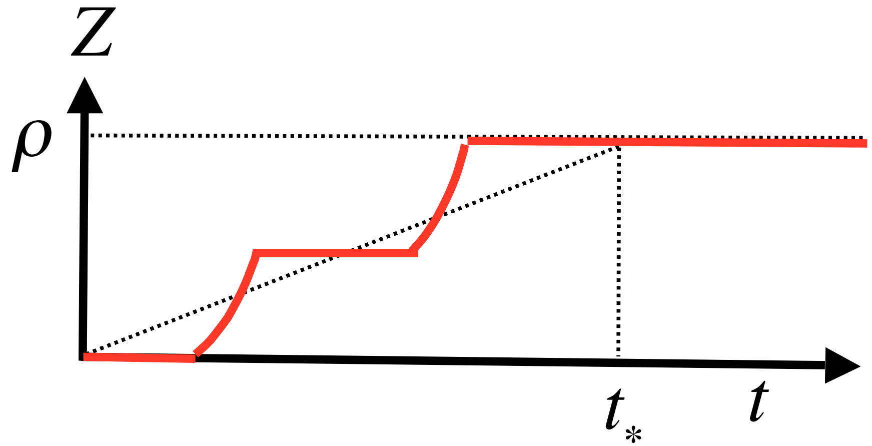
Irregulars:



$$\langle \mu(t) \rangle \approx 1 - t / t_*$$

$$t_* = f \frac{M_0}{\alpha \dot{M}_{\mathrm{burst}}} \qquad f < 1$$

$$Z(t) \approx -\rho \ln\big(1 - t / t_*\big)$$



The Yield ρ

$$\rho = \frac{(Z_{SN} - Z) \times \text{mass returned}}{\text{mass retained in old stars}}$$

First generation: $Z=0$ later generations $Z << Z_{SN}$:

From Salpeter IMF and SN 1987A: $\alpha = 0.93$

From SN 1987A: $Z_{SN} = 0.13$

$$\rho = \frac{(Z_{SN} - Z)\,(1-\alpha)}{\alpha}$$

$$\Rightarrow \quad \boxed{\text{yield} = \rho \approx 0.01}$$

However, note that $Z_{\odot} \approx 0.02$ which is larger than the calculated yield, implying $\rho > 0.02$! (remember $Z \leq \rho$)