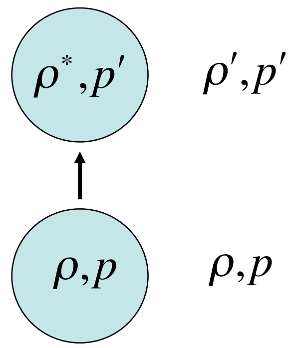
15. Convective Instability

- Perfect gas in equilibrium in a (uniform) gravitational field
- Blob displaced upwards
- Acoustic waves equalise pressure with outside, but no time to lose/ gain heat (i.e. adiabatic)



- Same p as environment, different ρ

- If ρ* < ρ’ blob continues upwards (buoyant)
- If ρ* > ρ’ blob falls back

<u>What is ρ*?</u>

- Adiabatic => $$\rho^* = \rho\left(\frac{p'}{p}\right)^{\frac{1}{\gamma}}$$

- New pressure $$p' = p + \frac{dp}{dz}\delta z$$

- Binomial expansion gives

$$\rho^* = \rho + \frac{\rho}{\gamma p}\frac{dp}{dz}\delta z$$

- But

$$\rho' = \rho + \frac{d\rho}{dz}\delta z$$

$$\frac{\rho}{p}\frac{dp}{dz} - \frac{\rho}{T}\frac{dT}{dz} \quad \text{(since } \rho = mp/kT\text{)}$$

- So the change in density is

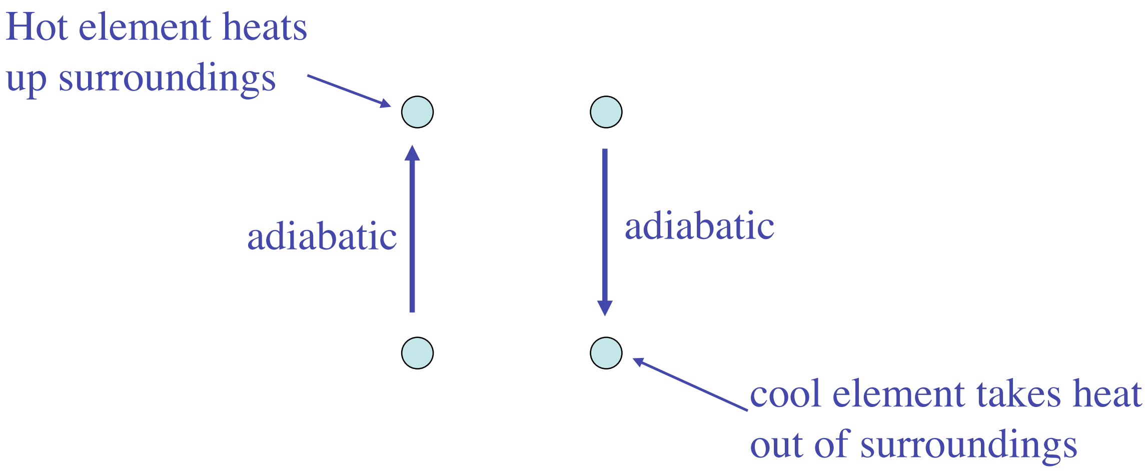
$$\rho^* - \rho' = \left[-\left(1 - \frac{1}{\gamma}\right)\frac{\rho}{p}\frac{dp}{dz} + \frac{\rho}{T}\frac{dT}{dz} \right]\delta z \qquad (15.1)$$

- Since both dp/dz and dT/dz are negative, the fluid is stable against convection if

$$\rho^* > \rho' \quad \Rightarrow \quad \left|\frac{dT}{dz}\right| < \left(1 - \frac{1}{\gamma}\right)\frac{T}{p}\left|\frac{dp}{dz}\right| \qquad (15.2)$$

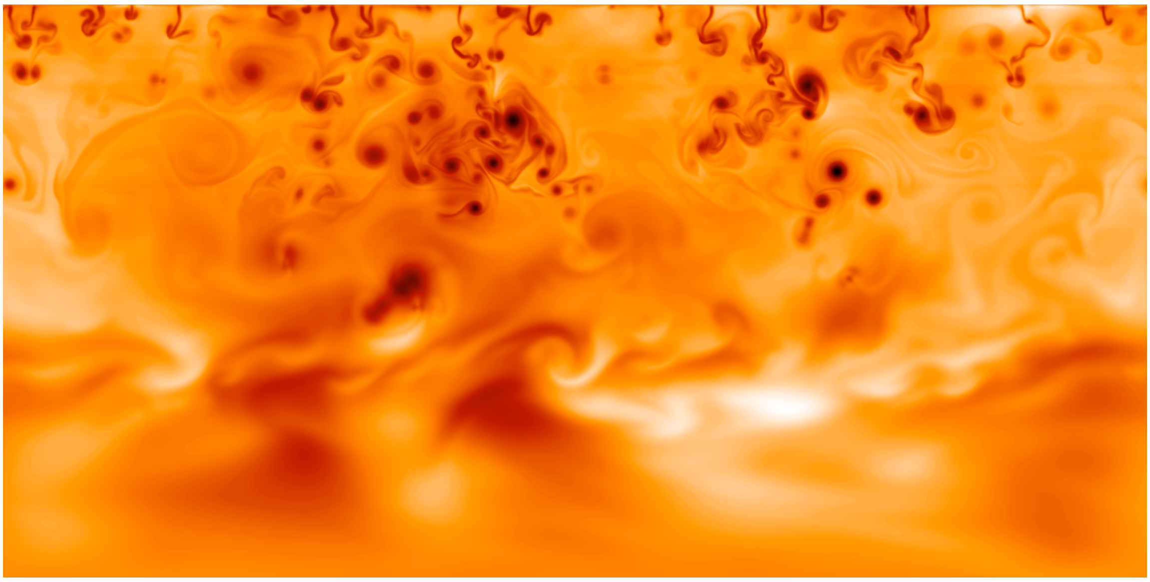
Schwarzschild stability criterion

- Blobs displaced upwards are hotter than their surroundings so heat exchange => energy is deposited in surroundings.



- Hence convection transports heat upwards by carrying it in the displaced elements and then releasing it to the surroundings at the top.

- Size of convective cells set by length scale over which elements cease being adiabatic (i.e. start to exchange heat with surroundings)



- http://www.solarviews.com/cap/misc/convect1.htm

Aside: Internal gravity waves

- Equation of motion* inside displaced blob:

$$\rho \frac{d^2}{dt^2}(\delta z) = -(\rho^* - \rho')g$$

- i.e. $$\frac{d^2}{dt^2}(\delta z) + N^2(\delta z) = 0$$

- Where from (15.1) $$N = \sqrt{\frac{g}{T}\left[\frac{dT}{dz} - \left(1 - \frac{1}{\gamma}\right)\frac{T}{p}\frac{dp}{dz}\right]}$$

* Neglect any motions induced by the blob in the surrounding fluid

- N (the *Brunt-Vaisala frequency*) is real if the stability criterion (15.2) is satisfied.

- A blob displaced vertically in a stably-stratified fluid will oscillate.

- check: $\delta z \sim e^{i(kx-\omega t)}$

$$\Rightarrow \frac{d^2}{dt^2}(\delta z) = -\omega^2(\delta z)$$

<u>Note:</u>

- If the stability criterion is *not* satified, N is imaginary and so the displacement grows exponentially with time.