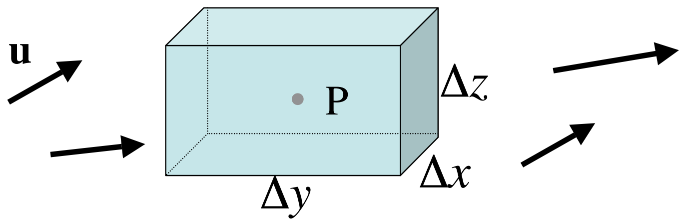
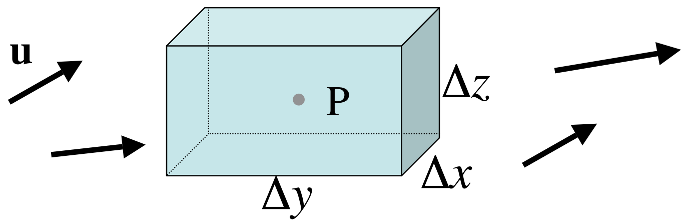
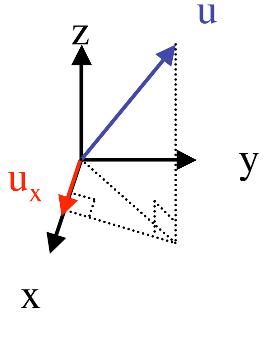
3. Aside on *divergence*

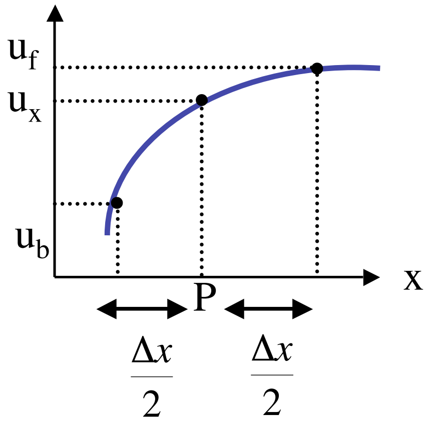
- Consider a gas flow u through a box with centre P





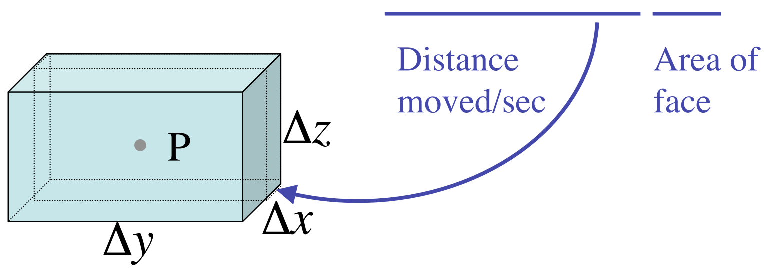
x-comp^t of u at P is u_x





- x-comp[t] of u at centre of back face $u_b \approx u_x - \frac{\Delta x}{2}\frac{\partial u_x}{\partial x}$
- " " " front face $u_f \approx u_x + \frac{\Delta x}{2}\frac{\partial u_x}{\partial x}$

- vol of gas crossing back face /sec = $\left(u_x - \frac{1}{2}\frac{\partial u_x}{\partial x}\Delta x\right)\Delta y \Delta z$



- " " front face/sec = $\left(u_x + \frac{1}{2}\frac{\partial u_x}{\partial x}\Delta x\right)\Delta y \Delta z$

- Net vol/sec flowing in x-direction = $\frac{\partial u_x}{\partial x}\Delta x \Delta y \Delta z$

- Similarly, net vol/sec in y-direction = $$\frac{\partial u_y}{\partial y}\Delta x\Delta y\Delta z$$

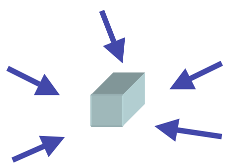
- z-direction = $$\frac{\partial u_z}{\partial z}\Delta x\Delta y\Delta z$$

- Total net vol/sec = $$\left(\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}+\frac{\partial u_z}{\partial z}\right)\Delta x\Delta y\Delta z$$

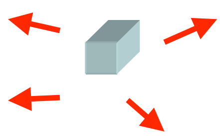
- But this is just $$(\underline{\nabla}\cdot\underline{u})\Delta x\Delta y\Delta z$$

- So $(\underline{\nabla}\cdot\underline{u})$ is just the volume of gas emerging per second from unit volume.

$(\underline{\nabla}\cdot\underline{u})<0 \Rightarrow$ Gas is flowing in

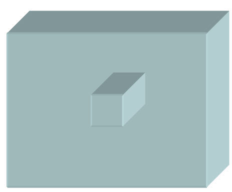


$(\underline{\nabla}\cdot\underline{u})>0 \Rightarrow$ Gas is flowing out

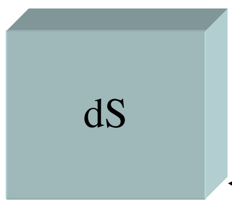


- So…this represents a “flux” of gas
- If we integrate over all the unit volumes in any shape, we get a total flux.

$$\iiint_V \underline{\nabla}\cdot\underline{u}\,dV$$



•Another way to look at this is to look at the amount of fluid that crosses each surface element dS in time dt:



•Volume of gas that flows through dS each second is $(\underline{u} \cdot \underline{n})dS$

•Add up all the surface elements dS to get the total volume per second emerging from a volume:

$$\iint_S \underline{u}.\underline{dS}$$

•But this is just the same as the divergence that we got before!

•So…

$$\iint_S \underline{u}.\underline{dS} = \iiint_V \underline{\nabla} \cdot \underline{u} dV \qquad (3.1)$$

•This is the divergence theorem..it applies to a flux of any quantity..heat, charge, sheep…

•The divergence of sheep integrated over a volume is equal to the flux of sheep through the surface of that volume.