

DISCUSSION

LENS-LIKE ACTION OF A STAR BY THE
DEVIATION OF LIGHT IN THE
GRAVITATIONAL FIELD

SOME time ago, R. W. Mandl paid me a visit and asked me to publish the results of a little calculation, which I had made at his request. This note complies with his wish.

The light coming from a star A traverses the gravitational field of another star B , whose radius is R_0 . Let there be an observer at a distance D from B and at a distance x , small compared with D , from the extended central line \overline{AB} . According to the general theory of relativity, let α_0 be the deviation of the light ray passing the star B at a distance R_0 from its center.

For the sake of simplicity, let us assume that \overline{AB} is large, compared with the distance D of the observer from the deviating star B . We also neglect the eclipse (geometrical obscuration) by the star B , which indeed is negligible in all practically important cases. To permit this, D has to be very large compared to the radius R_0 of the deviating star.

It follows from the law of deviation that an observer situated exactly on the extension of the central line \overline{AB} will perceive, instead of a point-like star A , a luminous circle of the angular radius β around the center of B , where

$$\beta = \sqrt{\alpha_0 \frac{R_0}{D}}.$$

It should be noted that this angular diameter β does not decrease like $1/D$, but like $1/\sqrt{D}$, as the distance D increases.

Of course, there is no hope of observing this phenomenon directly. First, we shall scarcely ever approach closely enough to such a central line. Second, the angle β will defy the resolving power of our instruments. For, α_0 being of the order of magnitude of one second of arc, the angle R_0/D , under which the deviating star B is seen, is much smaller. Therefore, the light coming from the luminous circle can not be distinguished by an observer as geometrically different from that coming from the star B , but simply will manifest itself as increased apparent brightness of B .

The same will happen, if the observer is situated at a small distance x from the extended central line \overline{AB} . But then the observer will see A as two point-like light-sources, which are deviated from the true geometrical position of A by the angle β , approximately.

The apparent brightness of A will be increased by the lens-like action of the gravitational field of B in the ratio q . This q will be considerably larger than unity only if x is so small that the observed positions of A and B coincide, within the resolving power of our instruments. Simple geometric considerations lead to the expression

$$q = \frac{l}{x} \cdot \frac{1 + \frac{x^2}{2l^2}}{\sqrt{1 + \frac{x^2}{4l^2}}},$$

$$l = \sqrt{\alpha_0 D R_0}.$$

where

If we are interested mainly in the case $q \gg 1$, the formula

$$q = \frac{l}{x}$$

is a sufficient approximation, since $\frac{x^2}{l^2}$ may be neglected.

Even in the most favorable cases the length l is only a few light-seconds, and x must be small compared with this, if an appreciable increase of the apparent brightness of A is to be produced by the lens-like action of B .

Therefore, there is no great chance of observing this phenomenon, even if dazzling by the light of the much nearer star B is disregarded. This apparent amplification of q by the lens-like action of the star B is a most curious effect, not so much for its becoming infinite, with x vanishing, but since with increasing distance D of the observer not only does it not decrease, but even increases proportionally to \sqrt{D} .

ALBERT EINSTEIN

INSTITUTE FOR ADVANCED STUDY,
PRINCETON, N. J.