How far to a Star ?

- Fundamental problem …..

- How far away is a point of light ?

. Is it a 100-watt light bulb ?
. A star as bright as the Sun ?
. A galaxy of 10^{11} stars ?

- Brightness alone is not enough.

- How to measure astronomical distances ?

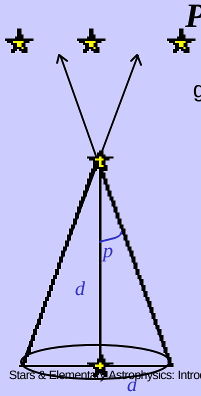
Astronomical Distances

- distance = speed x time
- light speed $c = 3\times10^{5}$ km/s
- 0.002 s Edinburgh-St.Andrews
- 0.1 s Earth circumference
- 1.2 s Earth-Moon distance
- 8 min Sun-Earth
- 40 min Jupiter
- 5 hr Pluto

Astronomical Distances

- 4.3 yr nearest star (Proxima Centauri)
- 25,000 yr centre of our Milky Way Galaxy
- 2×10^{6} yr nearest big galaxy (Andromeda, Messier 31, M31)
- 5×10^{7} yr nearest cluster of galaxies (Virgo cluster)
- $\approx 10^{10}$ yr edge of visible part of Universe (The Big Bang)

Parallax



geometrical method
p = parallax angle
a = 1 Astronomical Unit
(1 AU) = radius of Earth's
orbit around the Sun.

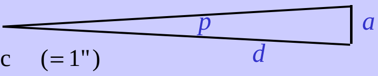
$$a = d \tan p$$

$$a \approx d\ p \quad \text{(small angle } p\text{)}$$

$$d \approx \frac{a}{p}$$

The Parsec -new distance unit

Example:



$p = 1$ arcsec $(= 1'')$

$$d = \frac{a}{p} = \frac{1\ \text{AU}}{1''} \times \frac{3600''}{1^\circ} \times \frac{180^\circ}{\pi\ \text{radian}}$$

$= 206265$ AU

$= 3.1\times10^{16}$ m

$= 3.26$ light yr

$= 1$ parsec (1 pc)

Fast Method:

$d = 1/p$

For p in arcsec
and d in parsec

Stellar Parallaxes are *tiny*

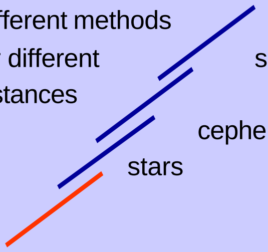
- Bessel (1838) First parallax:
  61 Cygni $p = 0.29$ arcsec ⇒
  $d = 1/0.29 =$ 3.42 pc
- also in 1838: Henderson (á Centauri)
  . Struve (Vega)
- The nearest star (beyond the Sun)
  Proxima Centauri $p = 0''.76$, $d = 1.31$ pc.

Limits of Parallax

- Ground-based CCDs 0.05” 20 pc
- Hubble Telescope 0.01” 100 pc
- Hipparcos (all sky) 0.005” 200 pc
  Today: Nearby Stars Only

- 2015: GAIA (whole galaxy) 10,000 pc

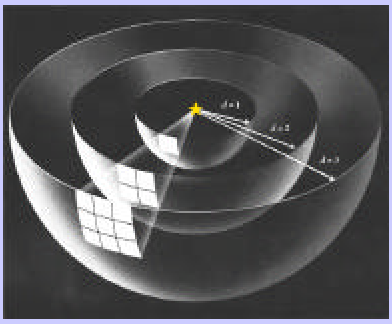
The Distance Ladder



- Different methods
- for different supernovae
- distances
- cepheid variables
- stars

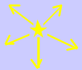
- ***PARALLAX IS THE FOUNDATION FOR ALL OTHER METHODS***

Inverse Square Law



Luminosity, Flux

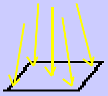
Luminosity $L = \frac{\text{energy}}{\text{time}}$



Units: Joule / sec = Watt (e.g. 100 W light bulb)

Solar luminosity: $L_{sun} = 3.8\times10^{26}\,\text{W}$

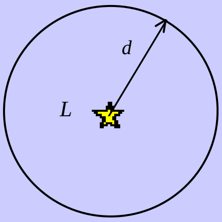
Flux $F = \frac{\text{energy}}{\text{time}\times\text{area}}$



Units: Watts / square metre

Sun viewed from Earth: $F_{sun} = 1380\ \text{W}/\text{m}^2$

Inverse Square Law



area of sphere $= 4\pi d^2$

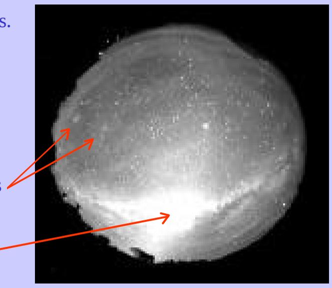
Flux viewed from distance d : $F = \frac{L}{4\pi d^2}$

http://concam.net

Continuous sky images.
fish-eye lens + CCD.

7 observatories

Sky from Australia:



Magellenic Clouds

Milky Way centre

Magnitudes

- Hipparcos (2nd century BC)
  stars visible by eye:
  1st mag = brightest
  6th mag = faintest
- Herschel (1780s) 1st mag stars are 100 times brighter than 6th mag stars.
- 1800s -- eye responds to flux ratios

true flux	1	:	10	:	100
perceived	1	:	2	:	3

Apparent Magnitude
(measures apparent brightness)

- Pogson (1856)
  . ***apparent magnitude:***

$$m = -2.5 \log(F / F_0)$$

$$F = F_0 \times 10^{-(m/2.5)}$$

$$F_0 = \text{flux of } 0 \text{ mag star (Vega)}$$

Naked Eye Stars

- brightest: Sirius (mag -1.5)

• bright	mag	-1	1 star
.		0	3
.		1	11
.		2	40
.		3	150
.		4	500
.		5	1600
faint	mag	6	4800 stars

Magnitude Difference
(measures brightness ratio)

$$m_1 - m_2 = -2.5 \log(F_1 / F_2)$$

$m_1 > m_2$ $F_1 < F_2$

$m_1 - m_2$	F_2 / F_1	
5	100	
10	10^4	
1	$\sqrt[5]{100} \cong$	2.512
0.1	≈ 1.1	(10%)
0.01	≈ 1.01	(1%)

Apparent Magnitude

• Sun	m = -26.8
• full moon	-12.5
• venus	-4
• Sirius	-1.5
• Vega	0.0
• faintest galaxies detected by HST	+30

$25 \text{ mag} = 10^{10}$

$30 \text{ mag} = 10^{12}$

Absolute Magnitude
(measures true brightness)

- *m* = apparent mag at distance *d.*
- *M* = aparent mag at 10 pc.

$$F(10 \text{ pc}) = F(d) \times \left(\frac{d}{10 \text{ pc}} \right)^2$$

$$M = -2.5 \log \left(F(10 \; pc) / F_0 \right)$$

$$= m - 5 \log (d / 10 \text{ pc})$$

- *d* known (e.g. parallax) for nearby stars.

Absolute Magnitude

- *m* = apparent mag at true distance d.
- *M* = aparent mag at 10 pc.
- *d* known (parallax) for nearby stars

star	*m*	*M*	*d*(pc)
Vega	0.0	+0.5	8.1
Sirius	-1.5	+1.4	2.7
Sun	-26.8	+4.5	1/206265

- Which star is truly brighter?

Distance Modulus
(measures distance)

$$\boxed{\begin{aligned} m - M &= 5\log\left(d / 10\,\mathrm{pc}\right) \\ &= 5\log\left(d / \mathrm{pc}\right) - 5 \end{aligned}}$$

Electromagnetic Radiation (EMR)

- ***EMR = Light*** *(of any wavelength)*
- speed of light in vacuum (slower in air, glass,…)

$$\boxed{c = 3\times10^{8}\,\mathrm{m/s}}$$

- ***wave properties*** (interference,diffraction)
  - frequency = speed / wavelength
  - Hz = cycles/s = (m/s) / m

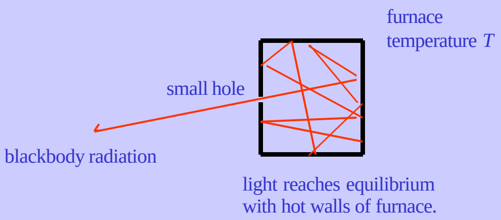
$$\boxed{f = c / l}$$

- ***particle properties*** (photons)
  - discovery by Planck (1900)
  - photon energy :
  - *h* = Planck's constant (6.6x10^(-34) Joule/Hz)

$$\boxed{E = h\, f}$$

Thermal (Blackbody) radiation

- ***Characterised by temperature T.***



furnace temperature *T*

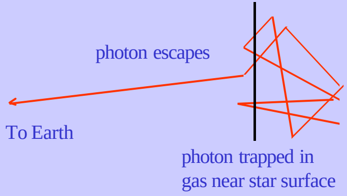
small hole

blackbody radiation

light reaches equilibrium with hot walls of furnace.

Stellar Spectra

- Resemble blackbody spectra
- temperature *T* at star surface
- (hotter interior invisible)

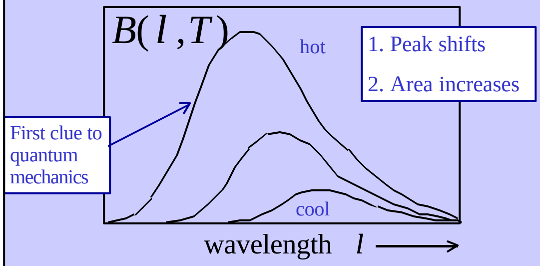


photon escapes

To Earth

photon trapped in gas near star surface

Blackbody Spectra
Planck function (1900)



$B(l, T)$

hot

cool

wavelength l →

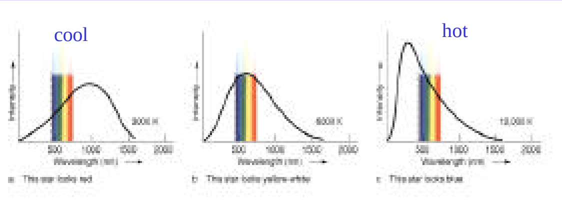
1. Peak shifts
2. Area increases

First clue to quantum mechanics

Wien's Law

$$\lambda_{\max} T = \text{constant}$$
$$\approx 0.003 \text{ m K}$$
$$\left(\frac{\lambda_{\max}}{300 \text{ nm}}\right) \approx \left(\frac{10^4 \text{ K}}{T}\right)$$

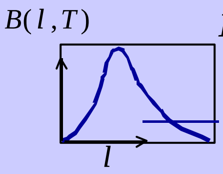
Star Colours



Wien's Law in action

- Sun: $T \approx 6000$ K, $\lambda_{\max} \approx 500$ nm
  - intensity peak at visible wavelength
  - looks yellow or white to us
- hot star: $T = 12000$ K, $\lambda_{\max} \approx 250$ nm
  - ultraviolet peak, looks blue
- cool star: $T = 3000$ K, $\lambda_{\max} \approx 1000$ nm
  - infrared peak, looks very red

Blackbody Flux



Area increases with T.

$$B(T) = \sigma T^4$$

$$\text{units:} \quad \frac{\text{energy}}{\text{time area}} \quad \frac{\text{W}}{\text{m}^2} = \left(\frac{\text{W}}{\text{m}^2\text{K}^4}\right)\left(\text{K}^4\right)$$

Stefan Boltzmann constant

$$\sigma = 5.7 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$$

Review

magnitudes: $m = -2.5 \log(F/F_0)$

$M = m - 5 \log(d/10 \text{ pc})$

light speed: $c = 3\times10^8 \text{ m/s}$

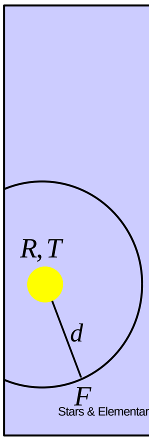
frequency: $f = c/\lambda$

photon energy: $E = h f$

blackbody peak: $\left(\frac{\lambda_{\max}}{300 \text{ nm}}\right) \approx \left(\frac{10^4 \text{K}}{T}\right)$

flux: $B(T) = \sigma T^4$

Review



temperature: T

flux at star surface: $B(T) = \sigma T^4$

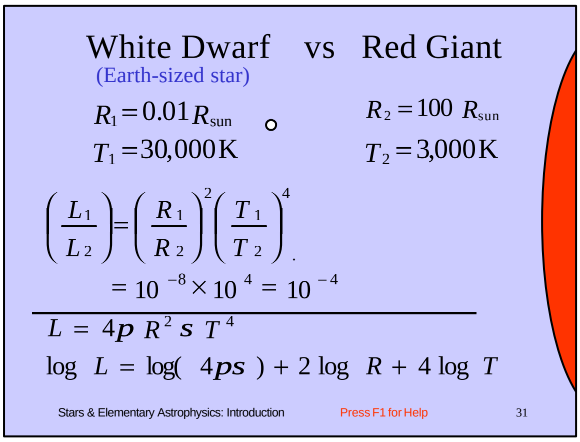
radius: R area: $4\pi R^2$

luminosity: $L = 4\pi R^2 \sigma T^4$

distance: d area: $4\pi d^2$

luminosity: $L = 4\pi d^2 F$

flux: $\boxed{F = \frac{L}{4\pi d^2} = \sigma T^4 \left(\frac{R}{d}\right)^2}$



White Dwarf vs Red Giant

(Earth-sized star)

$R_1 = 0.01\,R_{sun}$

$T_1 = 30{,}000\,\mathrm{K}$

$R_2 = 100\,R_{sun}$

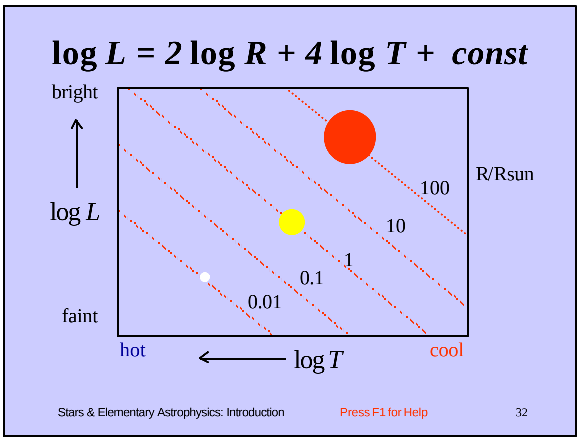
$T_2 = 3{,}000\,\mathrm{K}$

$$\left(\frac{L_1}{L_2}\right) = \left(\frac{R_1}{R_2}\right)^2 \left(\frac{T_1}{T_2}\right)^4 .$$

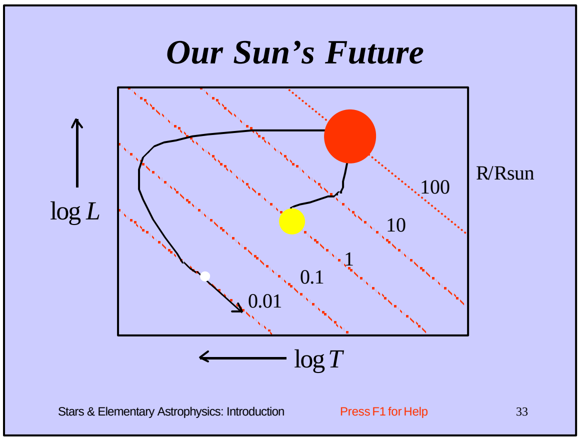
$$= 10^{-8} \times 10^{4} = 10^{-4}$$

$$L = 4\pi R^2 \sigma T^4$$

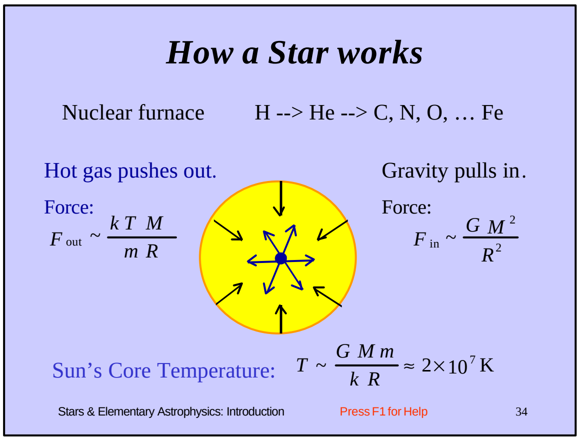
$$\log L = \log(4\pi\sigma) + 2\log R + 4\log T$$



$\log L = 2\log R + 4\log T + const$



Our Sun's Future



How a Star works

Nuclear furnace H --> He --> C, N, O, … Fe

Hot gas pushes out.

Force:

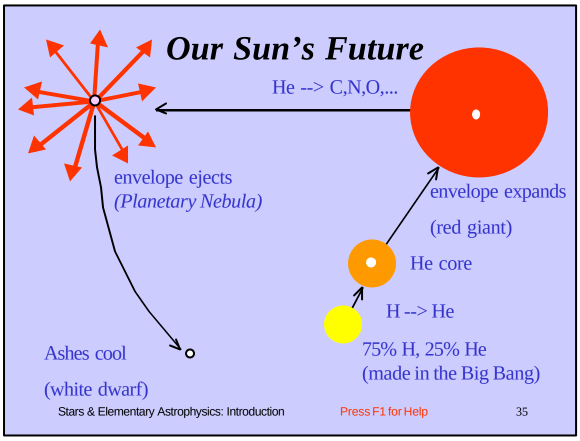
$F_{out} \sim \frac{k\,T\,M}{m\,R}$

Gravity pulls in.

Force:

$F_{in} \sim \frac{G\,M^2}{R^2}$

Sun's Core Temperature: $T \sim \frac{G\,M\,m}{k\,R} \approx 2\times10^7\,\mathrm{K}$



Our Sun's Future

He --> C,N,O,...

envelope ejects
(Planetary Nebula)

envelope expands

(red giant)

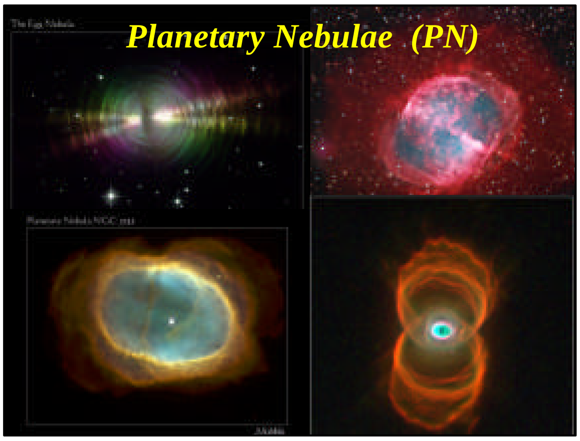
He core

H --> He

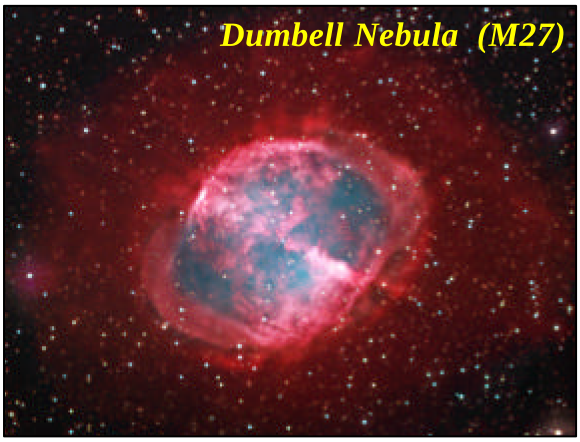
75% H, 25% He
(made in the Big Bang)

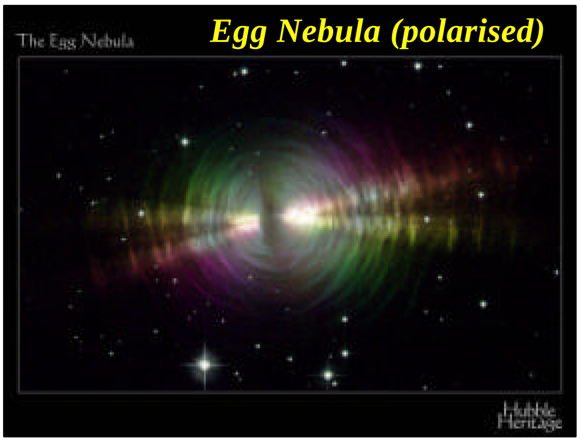
Ashes cool

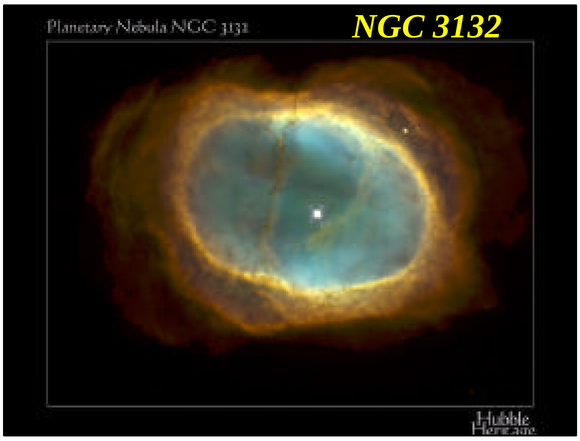
(white dwarf)

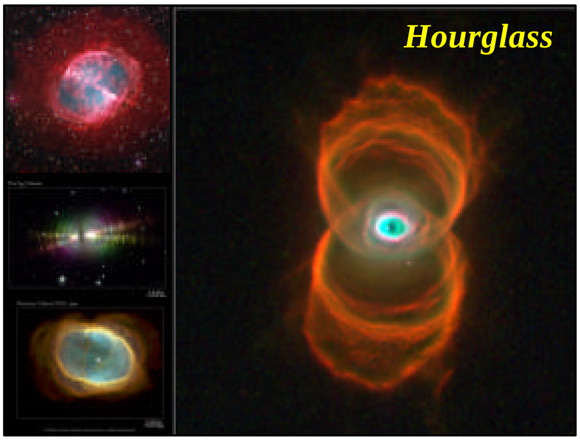


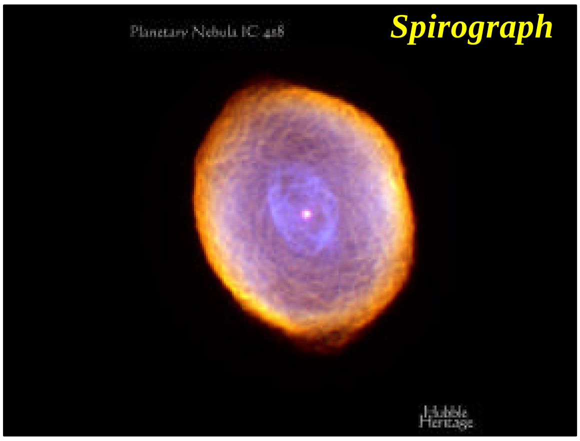
Planetary Nebulae (PN)


Dumbell Nebula (M27)


Egg Nebula (polarised)


NGC 3132

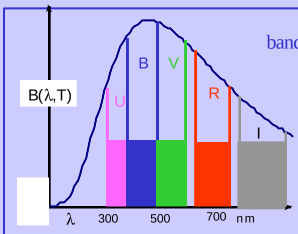

Hourglass


Spirograph


Ring (M57)

Death of a Star,
Birth of a White Dwarf

Broadband Photometry



bandpass filters

Flux ratios

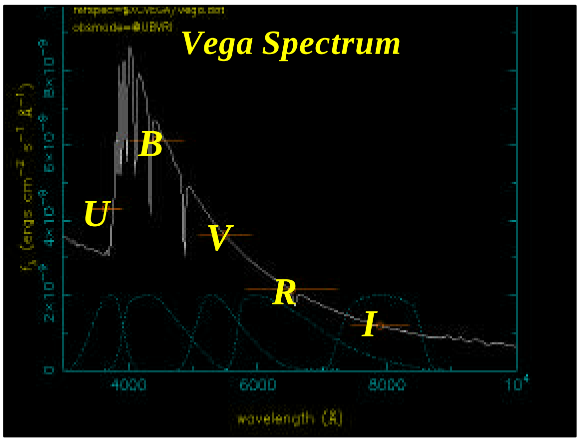
e.g. *F(B)/F(V)*

change with temperature *T*.

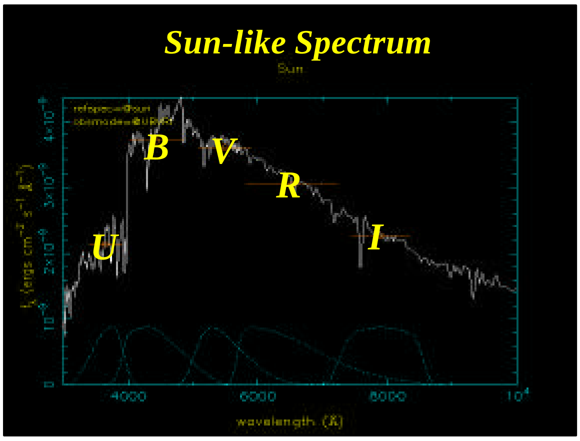
measure flux in each bandpass

$F(U) \quad F(B) \quad F(V) \quad F(R) \quad F(I)$

Vega Spectrum



Sun-like Spectrum



Colour Indices

apparent mag: $V = m_V = -2.5\log\left(\frac{F(V)}{F_0(V)}\right)$

absolute mag: $M_V = m_V - 5\log(d/10\text{ pc})$

(and similarly for U, B, R, I, etc.)

---

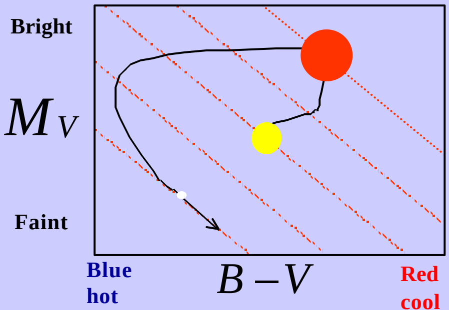
colour index:

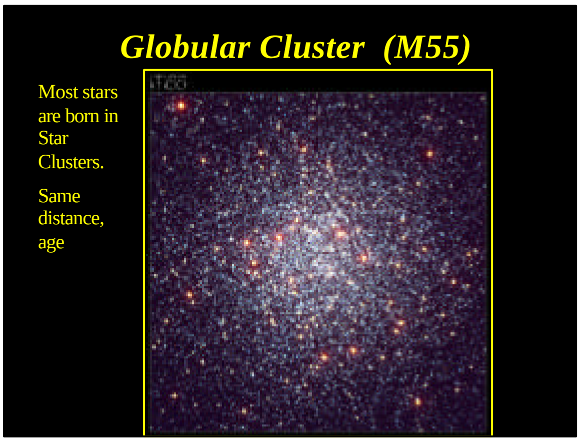
$$B - V = -2.5\log\left(\frac{F(B)/F_0(B)}{F(V)/F_0(V)}\right)$$

(and similarly for *U-B*, *V-R*, *R-I*, etc.)

Theory	***Observation***
flux	apparent mag
F	$U, B, V, \ldots$
luminosity	absolute mag
L	$M_V = V - 5\log(d/10\text{pc})$
temperature	colour index
T	$B - V$

Hertzsprung-Russell (or H-R) Diagram

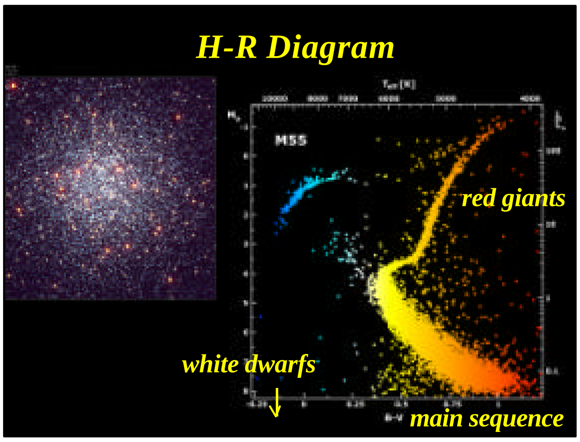




Globular Cluster (M55)

Most stars are born in Star Clusters.

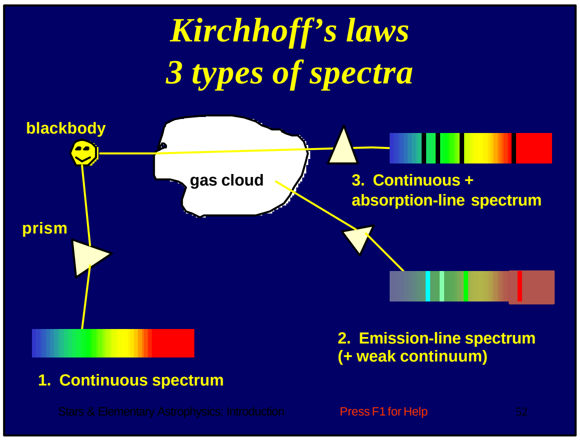
Same distance, age



H-R Diagram

Spectral Analysis

- light is dispersed into a spectrum using a diffraction grating (or prism) in a spectrograph
- Fraunhofer (1815) first extensive study of the Sun - identified about 600 dark lines
- Fraunhofer lines - strongest named A,B,...



Kirchhoff's laws
3 types of spectra

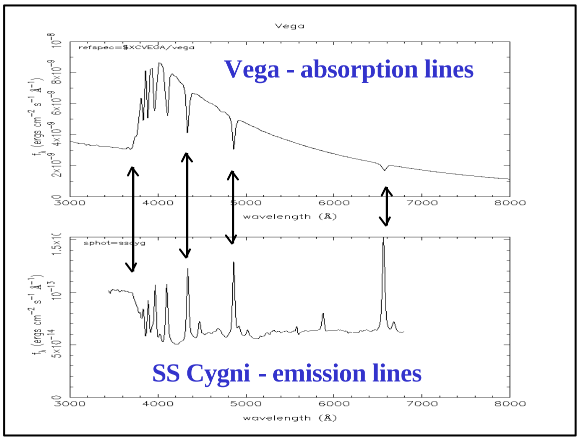
Kirchhoff's laws

A *hot opaque body*, e.g. a hot dense gas, produces a *continuous spectrum*, e.g. a "black-body" spectrum.

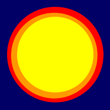
A *hot transparent gas* emits an *emission-line spectrum* , bright spectral lines, sometimes with a faint continuous spectrum.

A *cool transparent gas* in front of a continuous spectrum source produces an *absorption-line spectrum* - a series of dark spectral lines.

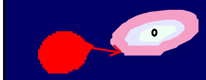
(Kirchoff and Bunsen laboratory experiments, 1850s)



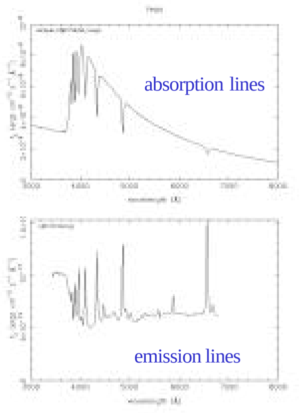
Kirchoff's Laws



hot opaque interior,
cool transparent atmosphere



hot transparent gas
(accretion disc)



Spectral Fingerprints

- Each element / molecule absorbs and emits only certain specific wavelengths of light.

- Spectral lines are diagnostic of the chemical composition and
- physical conditions (temperature, pressure)

Atoms and Ions

- atoms: **nucleus (protons + neutrons) + electrons**

	mass	**charge**
proton	**1**	**+1**
neutron	**1**	**0**
electron	**1/1836**	**-1**

atom - <u>neutral</u> : equal numbers of protons (+ve) and electrons (-ve)

ion - <u>ionised</u> : electrons removed, positive net charge

---

Atoms and Ions

- Examples:
  - **Hydrogen (H) (1 proton) + 1 electron**
  - **H II (1p) charge +1**

  - **Helium (He) (2p + 2n) + 2 e- 0**
  - **He II (2p + 2n) + 1 e- +1**
  - **He III (2p + 2n) +2**

  - **Oxygen (O) (8p + 8n) + 8 e- 0**
  - **OIII (8p + 8n) + 6 e- +2**
  - **OVI (8p + 8n) + 3 e- +5**

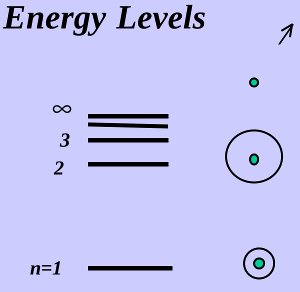
Atomic Energy Levels

e.g. Hydrogen:

$$E_n = -\frac{e^2}{r_n} = -\frac{I}{n^2}$$

$$E_1 = -13.6\ \text{eV}$$

$$E_\infty = 0$$

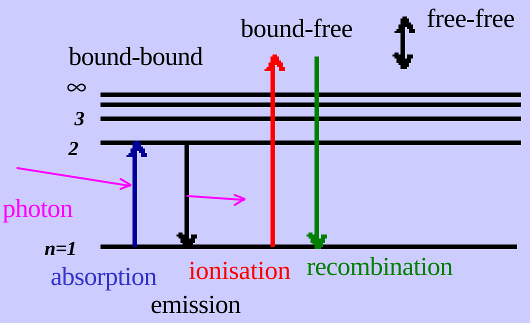


$I = 13.6\,\text{eV} = \text{Ionisation Potential}$

$e = \text{proton charge}$

$r = \text{size of electron orbit}$

Atomic Transitions



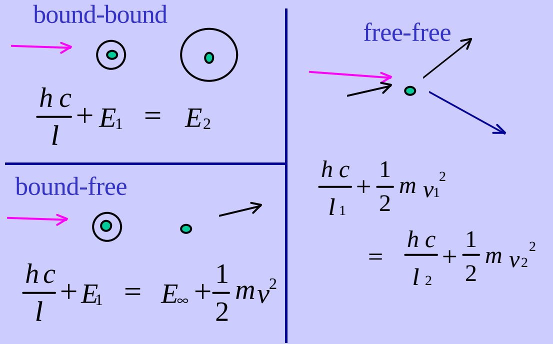
- **Energy change associated with each transition is**

$$E = h\,f = \frac{h\,c}{l}$$ **(*h* is Planck's constant)**

higher *E* ® higher frequency (shorter wavelength) photon is absorbed/emitted

- **energy changes are very small**
  - **measured in ELECTRON VOLTS (eV)**
    1 eV = 1.602 ´ 10^{-19} J

Energy Conservation



bound-bound

$$\frac{h\,c}{l} + E_1 = E_2$$

free-free

$$\frac{h\,c}{l_1} + \frac{1}{2} m\, v_1^2 = \frac{h\,c}{l_2} + \frac{1}{2} m\, v_2^2$$

bound-free

$$\frac{h\,c}{l} + E_1 = E_\infty + \frac{1}{2} m v^2$$

- Example:
  - H atom
    - (see handout: energy level diagram)
  - Energy difference between the ground state (n=1) and the first excited state (n=2) :

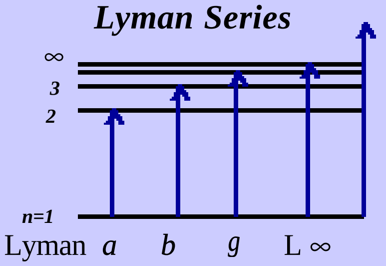
$$E = E_2 - E_1 = I \times \left(\frac{1}{1^2} - \frac{1}{2^2}\right) = (13.6\ \mathrm{eV})(0.75) = 10.2\,\mathrm{eV}$$

$$l = \frac{h\,c}{E} = \frac{(6.626\times10^{-34}\ \mathrm{J\ s})(3\times10^{8}\ \mathrm{m/s})(10^{9}\ \mathrm{nm/m})}{(10.2\,\mathrm{eV})(1.602\times10^{-19}\ \mathrm{J/eV})}$$

$$= 121.6\ \mathrm{nm}$$ in UV part of spectrum

Lyman α line (Lα)

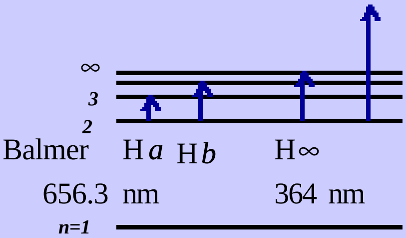
Lyman Series



91 .2 nm

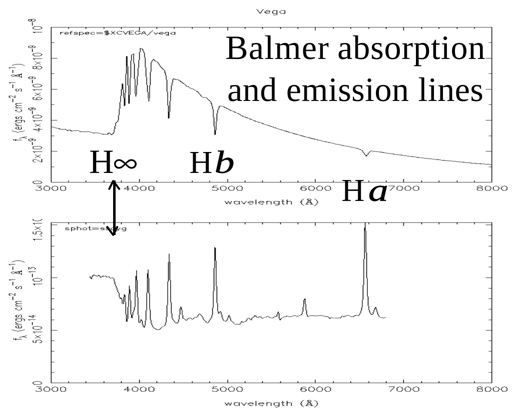
Lyman continuum photons ionise H from the ground state.

Balmer Series



656.3 nm 364 nm

Balmer continuum photons ionise from n=2



Rydberg formula

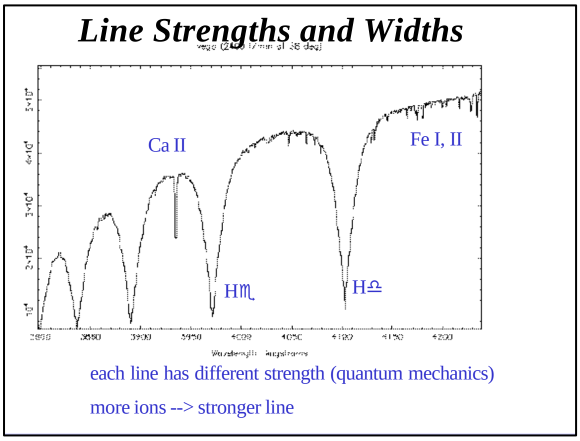
$$\boxed{\frac{1}{\lambda} = R\left(\frac{1}{n_\ell^2} - \frac{1}{n_u^2}\right)}$$

- Rydberg constant $R = 1097 \times 10^7\ \mathrm{m}^{-1}$ $\frac{1}{R} = 91.2\ \mathrm{nm}$

n_ℓ - principal quantum number of lower level
n_u - " " " of upper level

	n_ℓ	n_u	
LYMAN series	$n_\ell = 1$	$n_u = 2, 3, 4 \ldots$	UV
BALMER	2	3, 4, 5 …	visible
PASCHEN	3	4, 5 …	near IR
BRACKETT	4	5, 6 …	IR
PFUND	5	6, 7 …	IR

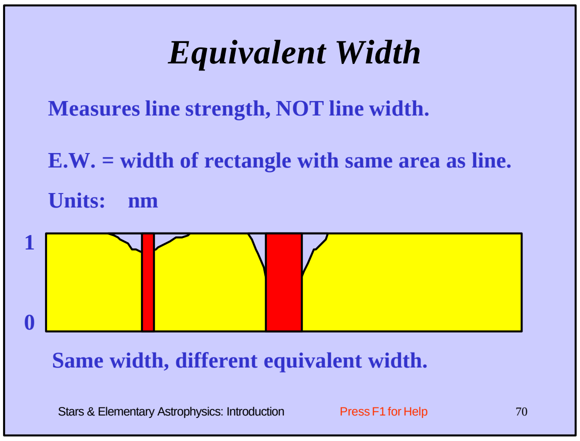
- Hydrogen is simplest.
- Multi-electron atoms have more complicated energy-levels.
- ions with 1 electron are like Hydrogen but with larger Ionisation Potential due to higher charge of nucleus



Line Strengths and Widths

each line has different strength (quantum mechanics)

more ions --> stronger line



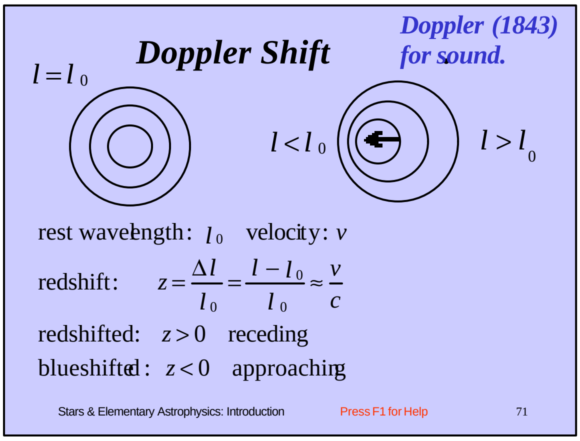
Equivalent Width

Measures line strength, NOT line width.

E.W. = width of rectangle with same area as line.

Units: nm

Same width, different equivalent width.



Doppler Shift

Doppler (1843) for sound.

rest wavelength: l_0 velocity: v

redshift: $z = \frac{\Delta l}{l_0} = \frac{l - l_0}{l_0} \approx \frac{v}{c}$

redshifted: $z > 0$ receding

blueshifted: $z < 0$ approaching

Doppler Shift

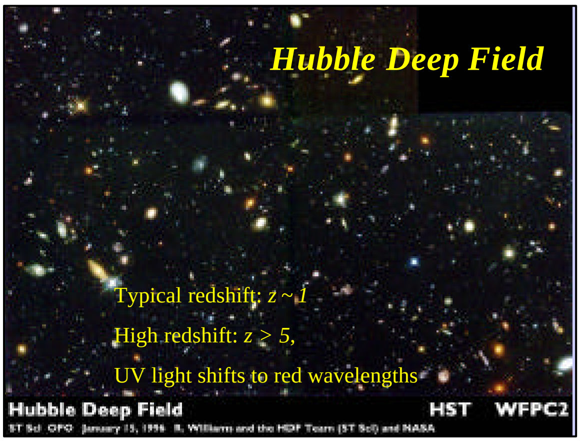
- ***Example: $v = 200\ \mathrm{km\ s^{-1}}$, $l_0 = 500\ \mathrm{nm}$***

$$\Delta l = \frac{v}{c}\, l_0 = \frac{200\ \mathrm{km\ s^{-1}}}{3\times10^5\ \mathrm{km\ s^{-1}}} \times 500\ \mathrm{nm} = 0.3\ \mathrm{nm}$$

- small shift, so no colour changes.
- unless $v \sim c$ (near a black hole, or relativistic jet)
- Cosmological redshifts can be large:

$$l = l_0\,(1+z) = (121\ \mathrm{nm})\ (1+6) = 848\ \mathrm{nm}$$

- Big Bang $T = \frac{T_0}{1+z} \approx \frac{3000\ \mathrm{K}}{1100} \approx 2.7\ \mathrm{K}$

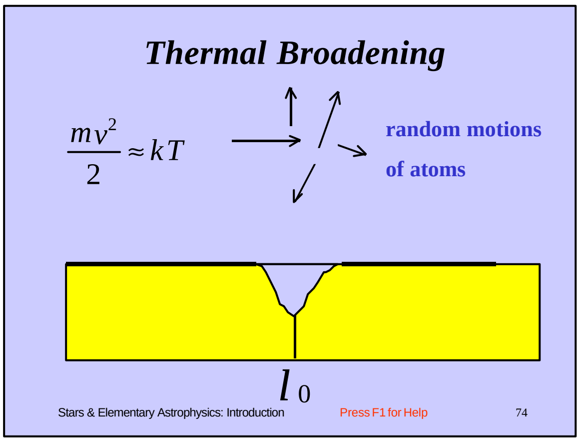


Hubble Deep Field

Typical redshift: $z \sim 1$

High redshift: $z > 5$,

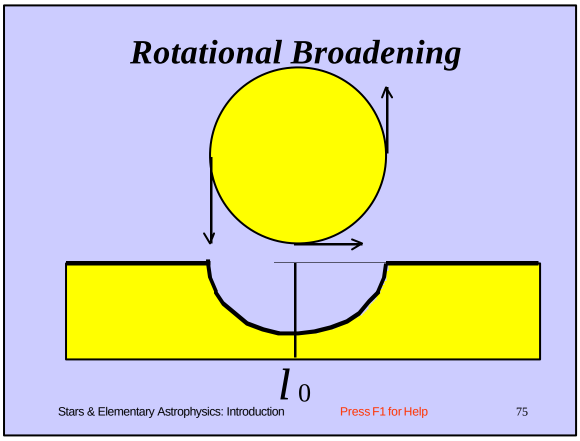
UV light shifts to red wavelengths



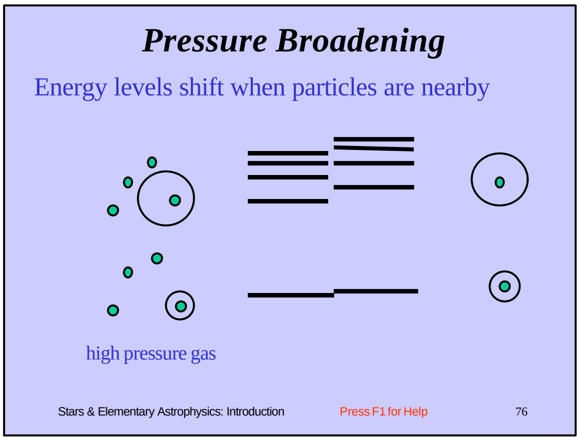
Thermal Broadening

$$\frac{mv^2}{2} \approx kT$$

random motions of atoms

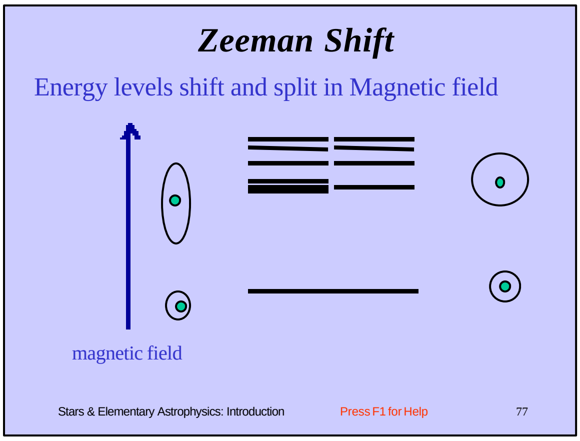


Rotational Broadening



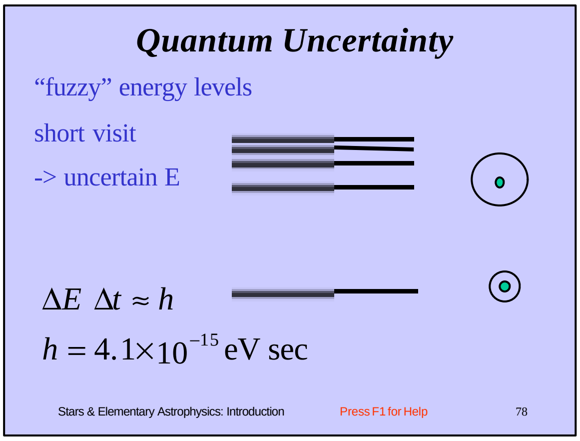
Pressure Broadening

Energy levels shift when particles are nearby



Zeeman Shift

Energy levels shift and split in Magnetic field



Quantum Uncertainty

"fuzzy" energy levels

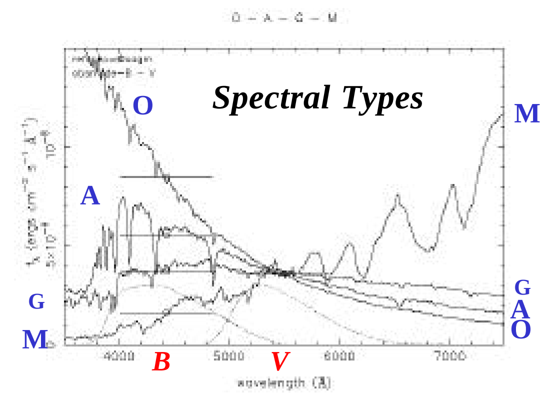
short visit

-> uncertain E

$$\Delta E \ \Delta t \approx h$$

$$h = 4.1 \times 10^{-15} \text{ eV sec}$$

Spectral Types



Why spectra differ

- Line strengths (EW ratios) change mainly due to SURFACE TEMPERATURE (hot-> high ionisation and excitation cool-> neutral atoms and molecules)
- Some line widths and ratios change with LUMINOSITY
- Very little range of abundances

  (74% H + 24% He 2% everything else)

Spectral Classification

- 1890s first photographic spectra
- 1918-24 Henry Draper Catalogue "spectral classes" of ~ 225,300 stars !!! (star names HD 35311, HD 209458, etc)
- original classification: A,B,… R,S from simple to complex lines
- many letters later dropped or merged.
- 1920s photometry (colour indices) revealed correct temperature sequence
- confirmed by atomic physics
- 1940s Morgan & Keenan (MK spectral types)

- handout:
  - spectral classes provide a "short-hand" description of the appearance of a stellar spectrum.

Spectral Types

hot cool

O B A F G K M

(Oh! Be A Fine Girl Guy, Kiss Me!)

("early-type" "late-type")

- sub-class 0 - 9 *e.g.* B0, B9, G2

Spectral Types

hot cool

O B A F G K M

(Oh! Be A Fine Girl Guy, Kiss Me!)

("early-type" "late-type")

- sub-class 0 - 9 *e.g.* B0, B9, G2

(N R S (No Romeo, Scram))

- one example of a luminosity criterion:
- H lines of Balmer series are affected strongly by pressure broadening
- pressure gradient ∝ surface gravity of star g

$$g = \frac{GM}{R^2}$$

(M = mass, R = radius, G = gravitational constant)

- dwarf star, small R, large $g \Rightarrow$ broadened H lines
- giant star, large R (~100×), small $g \Rightarrow$ narrow H lines
  - (handout: spectra showing luminosity effects)

Luminosity Classes

✶ – main-sequence	**V**	most common
– subgiants	**IV**	
– red giants	**III**	common
✶ – bright giants	**II**	rare
– supergiants (blue to red)	**I**a,b	very rare
– white dwarfs	DA	quite common
–	DB,DO	

MK Spectral Types

– *e.g.*	Sun	G2 **V**
	Vega	A0 **V**
.	Betelgeuse	M2 **I**ab
	Rigel	B8 **I**a
	Aldebaran	K5 **III**

Review

- Multicolour Photometry
  - – Use filters (e.g. *U B V R I*)
  - – measure *flux densities* : (f_B, f_V, …)
  - – apparent magnitudes : (*B*, *V*, …)
  - – colour indices :

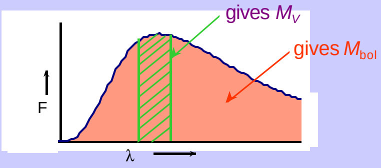
$$(B - V) = -2.5\log\left[f_B / f_V\right] + \text{constant}$$

  - – absolute magnitudes (*d* from parallax):

$$M_V = V - 5\log(d/10\ \text{pc})$$

Bolometric Magnitude

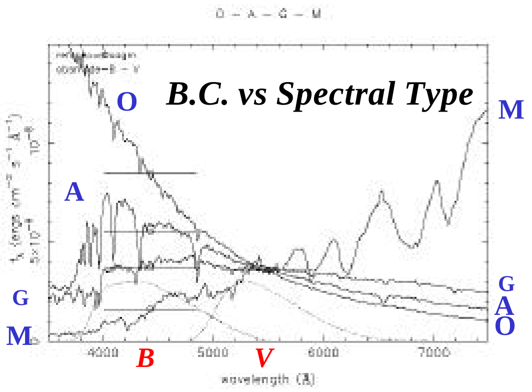
M_{bol} = absolute bolometric magnitude total flux over the entire spectrum.



Difficult to measure M_{bol}.

Easy to measure M_V .

B.C. vs Spectral Type



Bolometric Corrections

- B.C. = $M_{bol} - M_V$ < 0 to make star brighter.
- Sun: $M_V = 4.83$, B.C. = –0.14 $M_{bol} = 4.69$
- F0**V** B.C. = 0 (most optical)
  O5**V** = –3.8 (mostly UV
  M8**V** = –4.0 mostly IR)

$$\frac{L}{L(\mathrm{sun})} = 10^{-0.4\left(M_{bol} - M_{bol}(\mathrm{sun})\right)}$$

$$M_{bol} - M_{bol}(\mathrm{sun}) = -2.5 \log\left(\frac{L}{L(\mathrm{sun})}\right)$$

Calibrations

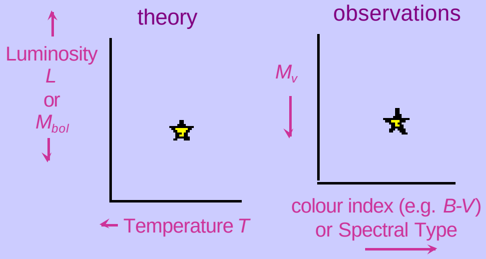
- Calibrations of colour indices, temperatures, absolute visual magnitude (M_V), and (less precise) spectral types
  - – very well defined for most main-sequence stars (5,000 ≤ T ≤ 30,000 K)
  - – less so for hotter O stars (> 40,000 K)
  - – and cooler M stars (< 2,500 K)

  (handout: example of relevant calibrations)

- The Hertzsprung-Russell (HR) Diagram
  - – first presented independently by H (1911) and R (1913) to show links between spectral types (or colours) of stars and absolute magnitudes
  - – now recognised as one of the most important diagrams for all astronomy, because of its importance for understanding the evolution (ageing) of stars
    - (handout: HR diagram)

Theory vs Observations

- Alternative versions of the H-R diagram:



theory

observations

↑ Luminosity L or M_{bol} ↓

M_v ↓

← Temperature T

colour index (e.g. B-V) or Spectral Type →

Stellar Radii

- To calculate R: $L = 4\pi R^2 \sigma T^4$
- Observe:
  . parallax p --> distance $d=1/p$.
  . spectral type or colour index --> T
  . apparent magnitude, e.g. V.

$$V - M_V = 5 \log(d/10\,\mathrm{pc}) \qquad M_{bol} = M_V + BC.$$

$$M_{bol} - M_{bol}(\mathrm{sun}) = -2.5 \log(L/L(\mathrm{sun}))$$

- Not highly accurate (10-50%)

Typical Radii

- Solar radius: Rsun=7x10^5 km

main-sequence stars:
R ~ 0.1 - 10 Rsun
giants: R ~ up to 100 Rsun
supergiants: red: R ~up to 1000 Rsun
blue: R ~20-50
white dwarfs: R ~ 0.01 Rsun

Accurate Radii

- Most accurate radii (<1%) from
  . ECLIPSING BINARY STARS
  and (for nearby stars)
  . INTERFEROMETRY
  and (for a few stars)
  . LUNAR OCCULTATIONS.
- Accurate R improves T via

$$T = \left(\frac{L}{4\pi R^2\sigma}\right)^{1/4}$$

- if distance (hence L) also known.

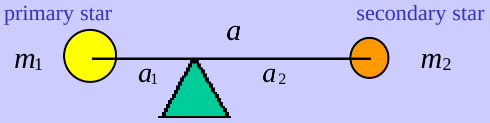
Binary Stars

-- two stars in mutual gravitational attraction, orbiting their common centre of mass

-- only source of empirical ***masses*** for stars

-- accurate sizes, shapes, temperatures, luminosities (hence distances)

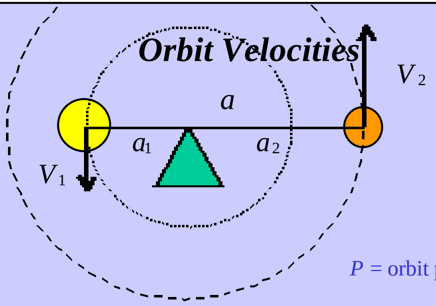
Centre of Mass



$$a_1\, m_1 = a_2\, m_2$$

$$\frac{a_1}{a} = \frac{m_2}{m_1 + m_2} \qquad \frac{a_2}{a} = \frac{m_1}{m_1 + m_2}$$

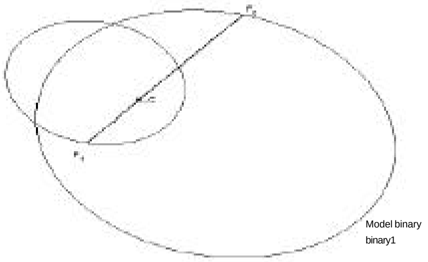
Orbit Velocities



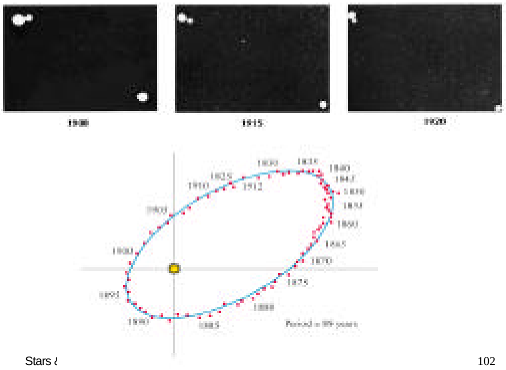
P = orbit period

$$\frac{V_1}{V_2} = \frac{a_1}{a_2} = \frac{m_2}{m_1} \qquad V_1 + V_2 = \frac{2\pi\, a}{P}$$

Elliptical Orbits

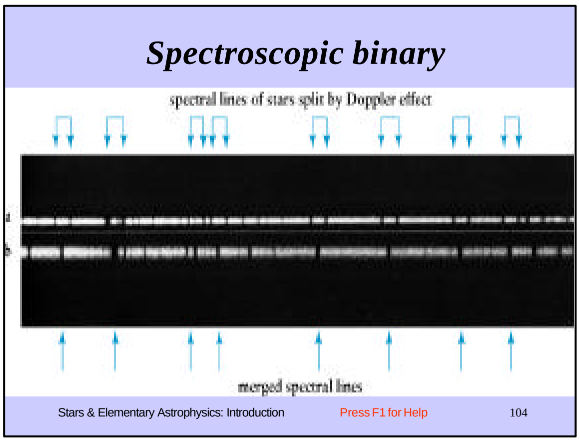


Visual binary



Types of Binaries

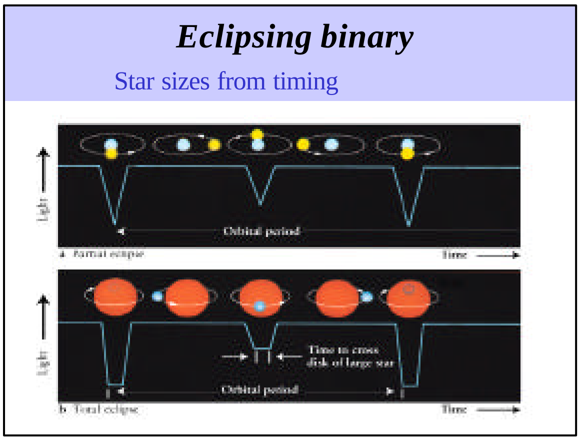
- visual binary
- spectroscopic binary
  . SB1 SB2 lines from 1 or 2 stars
- eclipsing binary



Spectroscopic binary

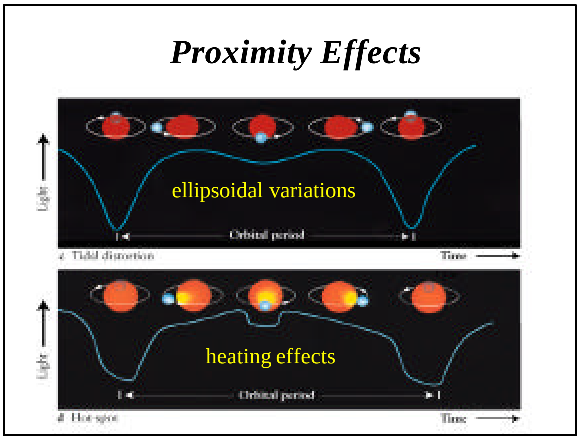
spectral lines of stars split by Doppler effect

merged spectral lines



Eclipsing binary

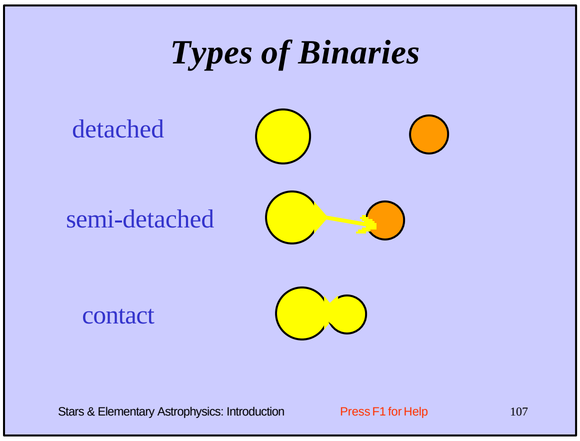
Star sizes from timing



Proximity Effects

ellipsoidal variations

heating effects



Types of Binaries

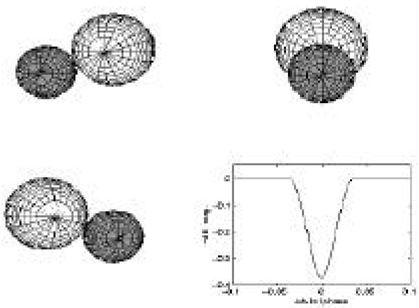
detached

semi-detached

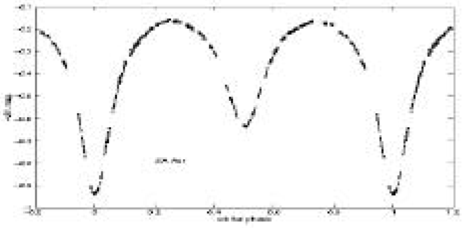
contact

Binary Star with Accretion Disc

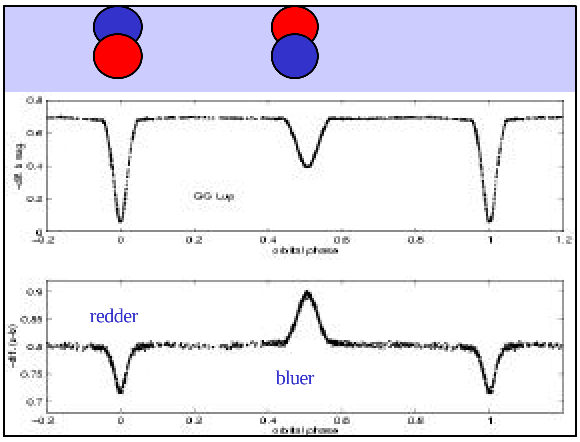
Computer Models of Eclipsing Binary Stars



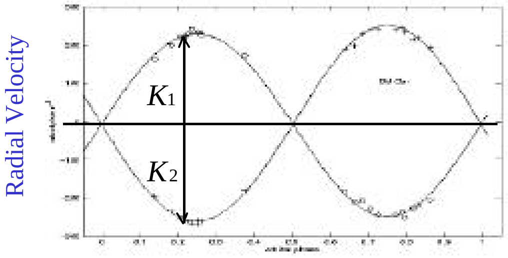
Light curve of Contact Binary



Orbital Phase



Velocity curve



Orbital Phase

Orbit inclination

i = 0 for face-on orbit

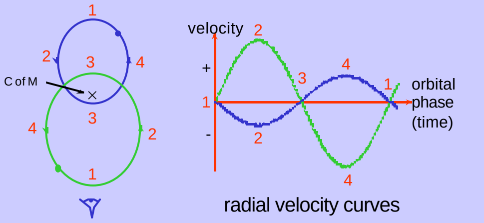
i = 90 degrees for edge-on orbit

Doppler shifts measure

. $K = V \sin i$

To measure masses:

- visual
– radial velocity and light variations - spectroscopic
- eclipsing



radial velocity curves

Masses

Observe: $K_1 = V_1 \sin i$ P

$K_2 = V_2 \sin i$

Calculate masses:

$$\frac{m_1}{m_2} = \frac{K_2}{K_1} \qquad 2\pi\ a \sin i = (K_1 + K_2)\, P$$

Kepler's Law:

$$\left(\frac{m_1 + m_2}{M_{\text{sun}}}\right)\left(\frac{P}{\text{yr}}\right)^2 = \left(\frac{a}{\text{AU}}\right)^3$$

- Analysis of RV curves gives "minimum masses"

  $(M_1 \sin^3 i), (M_2 \sin^3 i)$

  and projected sizes of orbits

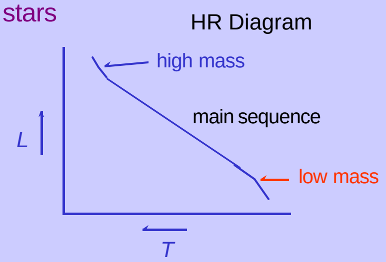
  $(a_1 \sin i), (a_2 \sin i)$

- Analysis of light curves of eclipsing binaries gives
  - orbital inclination i
  - radii of both stars, relative to the size of the orbit

$$\boxed{\left(\frac{r_1}{a}\right), \left(\frac{r_2}{a}\right)}$$

- Hence, for eclipsing, spectroscopic binaries, we obtain:
  - masses M_1 and M_2
  - radii R_1 and R_2
  - luminosities L_1 and L_2
    - (if T_1 or T_2 known)

- used as tests of theoretical models of stars

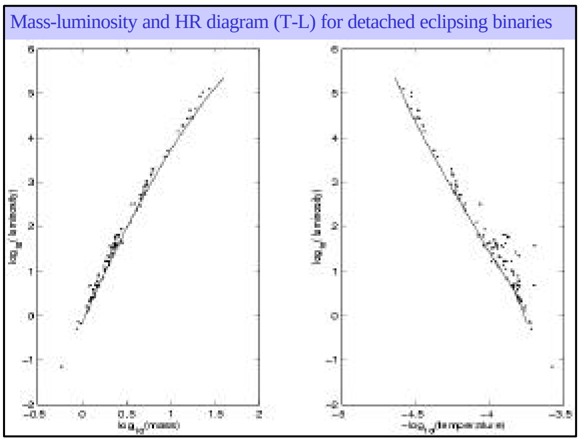


- Empirical MASS-LUMINOSITY relationship for main-sequence stars:

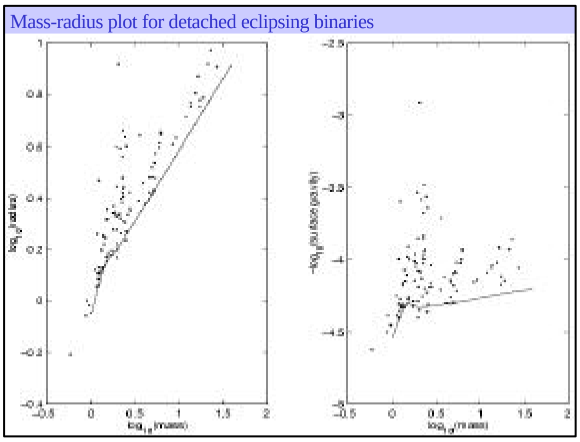
  · $L \propto M^4$ i.e.

$$\frac{L}{L_{sun}} = \left[\frac{M}{M_{sun}}\right]^4 \quad \text{for } 0.4\, M_{sun} < M < 10\, M_{sun}$$

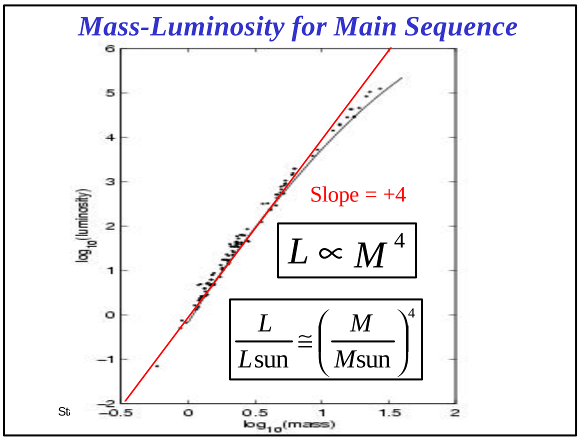
see $\log M$ vs. $\log L$ plots (handout)



Mass-luminosity and HR diagram (T-L) for detached eclipsing binaries



Mass-radius plot for detached eclipsing binaries



Mass-Luminosity for Main Sequence

Slope = +4

$$\boxed{L \propto M^4}$$

$$\boxed{\frac{L}{L\text{sun}} \cong \left(\frac{M}{M\text{sun}}\right)^4}$$

Star Lifetimes

- Energy supply: $E = \Delta M\, c^2$ (Joules)
- Rate of burning: $L \propto M^4$ (W = Joule/s)
- Lifetime:

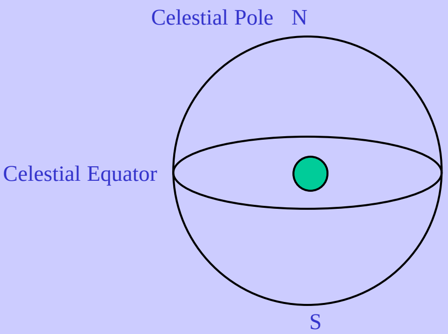
$$t \sim \frac{E}{L} = \frac{\Delta M\, c^2}{L} = \frac{\Delta M}{M}\,\frac{M\, c^2}{L}$$

$$\sim 10^{10}\ \text{yr}\left(\frac{\Delta M / M}{0.0015}\right)\left(\frac{M}{M_{\text{sun}}}\right)^{-3}$$

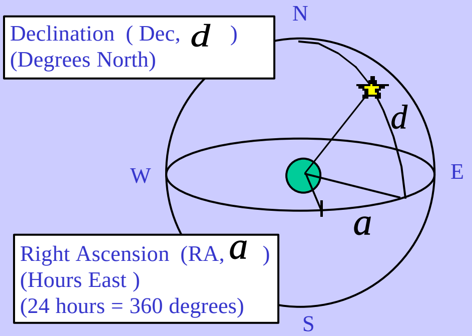
- Stars burn for a long time.
- Big stars burn out faster.

5. ***THE MOTIONS OF STARS IN SPACE***

The Celestial Sphere

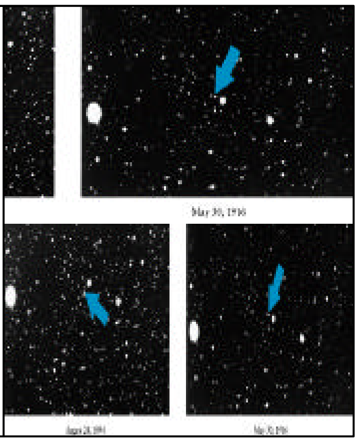


Star Coordinates

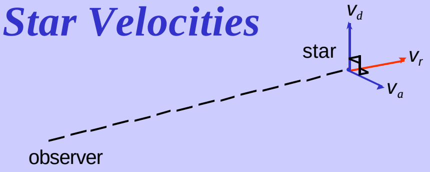


Declination (Dec, d)
(Degrees North)

Right Ascension (RA, a)
(Hours East)
(24 hours = 360 degrees)

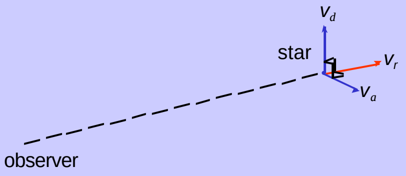


Star Velocities

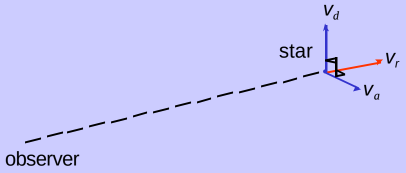


3 velocity components

(mutually perpendicular)

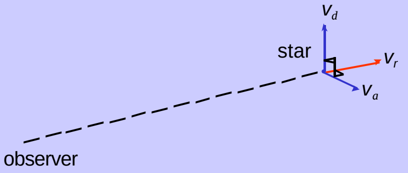


- in line-of-sight (radial direction)
RADIAL VELOCITY v_r (km s^{-1})



- 2 components perpendicular to the line of sight
TRANSVERSE VELOCITY v_t (km s^{-1})

$$V_t = \sqrt{V_a^2 + V_d^2}$$



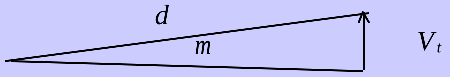
v_α and v_δ are the components of v_t in the directions of increasing RIGHT ASCENSION (α) and DECLINATION (δ) on the sky.

- v_r - from Doppler shift of spectral lines
- v_α, v_δ - projected onto the sky
  - we can measure only <u>angular changes</u> over time, called <u>PROPER MOTION</u>
    . μ (arcsec yr^{-1})
  - Two components of μ are
    $\mu_\alpha \cos\delta$, μ_δ
  - (see handout …)

Proper Motions tiny

Speeds of stars orbiting the Galaxy
$v \sim 250$ km s^{-1}

- but distances are in parsecs -
  1 pc = 3×10^{13} km
- thus proper motions μ are small
  (< 0.1 arcsec yr^{-1})



small angle

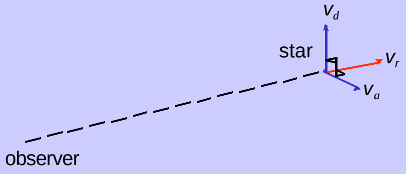
$$V_t = m \;\; d$$

$$= \frac{m\ (\text{arcsec}\,\text{yr}^{-1}) \times d\ (\text{pc})}{206{,}265\,(\text{arcsec/radian})} \times \left(\frac{3\times10^{15}\ \text{km}\,\text{pc}^{-1}}{3\times10^{7}\ \text{s yr}^{-1}} \right)$$

- Fast caclulation:

$$\boxed{V_t = 4.74 m \;\; d}$$

- ONLY for V_t in km s^{-1},
  μ in arcsec yr^{-1},
  d in parsecs



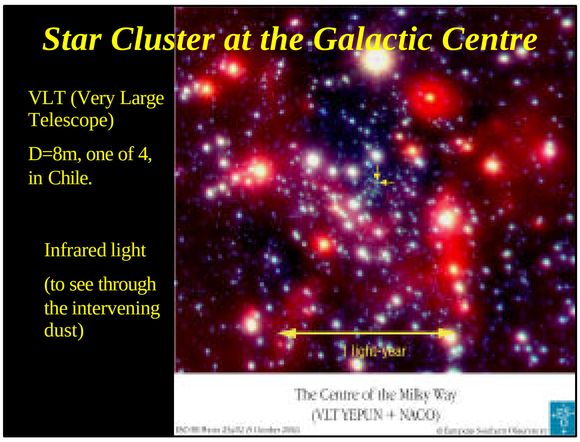
- <u>SPACE MOTION</u> = velocity vector

speed $V = \sqrt{V_r^2 + V_t^2}$

+ direction specified by v_α, v_δ, v_r

Astrometry Satellites

- ***<u>HIPPARCOS</u>*** (1997)
- Accurate parallax and proper motion for bright stars (V < 9)
  10^{-3} arcsec yr^{-1}
- stars to 200 pc
- ***<u>GAIA</u>*** planned ESA (2012 ...)
- parallax 10^{-5} arcsec
- proper motion 10^{-6} arcsec yr^{-1}
- distances and motions
  . of stars throughout the Galaxy



Star Cluster at the Galactic Centre

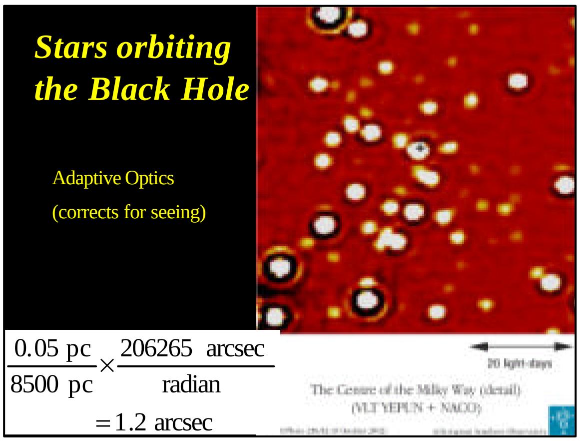
VLT (Very Large Telescope)

D=8m, one of 4, in Chile.

Infrared light

(to see through the intervening dust)

The Centre of the Milky Way
(VLT YEPUN + NACO)



Stars orbiting the Black Hole

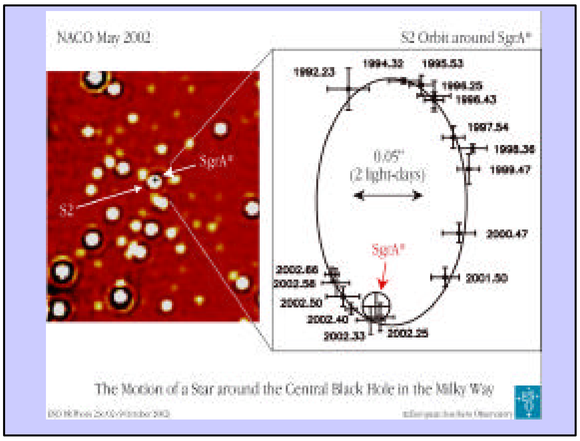
Adaptive Optics

(corrects for seeing)

$$\frac{0.05\ \mathrm{pc}}{8500\ \mathrm{pc}} \times \frac{206265\ \ \mathrm{arcsec}}{\mathrm{radian}} = 1.2\ \mathrm{arcsec}$$

The Centre of the Milky Way (detail)
(VLT YEPUN + NACO)

Star Motions at the Galactic Centre



The Motion of a Star around the Central Black Hole in the Milky Way

Black Hole at the Galactic Centre

- From the proper motions, measure sizes and periods of the star orbits.
- Kepler's law :

$$\frac{M}{M_{\mathrm{sun}}} = \left(\frac{a}{\mathrm{AU}}\right)^3 \left(\frac{P}{\mathrm{yr}}\right)^{-2}$$

- the black hole mass

$$M \approx 3\times10^6\, M_{\mathrm{sun}}$$

THE END