Eddington-N: a proposal to improve Eddington's planet hunting and asteroseismology capabilities by making N clones of a smaller wide-field telescope.

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Summary

The proposal is to implement Eddington as a set of N identical but smaller wide-field telescopes. A simplified cost-benefit analysis suggests that this may be realized within a fixed cost envelope. Potential benefits for both asteroseismology and planet hunting are discussed, as well as several risk factors. For N telescopes of aperture D and angular field of view θ, the survey volume for planet hunting scales as $N^{3/2}D^3\theta^2$ for N co-aligned telescopes, and as $ND^3\theta^2$ for N telescopes pointing at different fields. We argue that these expressions, which assume uniform density of stars and neglect galactic absorption, will be reasonable approximations for the volumes surveyed for small planets that are the primary aim of the mission. At fixed F ratio and detector size ℓ, and with $D \propto 1/\sqrt{N}$ to maintain the count rate for asteroseismology, the planet catch scales as N for co-aligned telescopes and as $\sqrt{N}$ non-overlapping fields. A more comprehensive cost-benefit analysis, utilizing realistic cost functions, can identify optimal design parameters that maximize the survey volume for planet detection at each total cost.

1 Improved Capabilities at Fixed Cost

To see how we may build a system of N telescopes with improved capabilities for both asteroseismology and planet hunting, within the confines of a fixed budget, consider a specific example. Assume that the cost C per telescope scales with telescope aperture D, focal ratio F, and linear size ℓ of the CCD mosaic as

$$C(D,F,\ell) \propto D^\alpha F^\beta \ell^\gamma \ . \tag{1}$$

We expect $\alpha > 2$ and $\beta < 0$, and $\gamma \approx 1/2$. If C_0 is the cost of building a single telescope with aperture D_0, focal ratio F_0, and CCD mosaic size ℓ_0, then building N smaller telescopes with aperture reduced from D_0 to $D = D_0/\sqrt{N}$, but keeping $F = F_0$ and $\ell = \ell_0$, will cost:

$$C(N,D,F,\ell) = N \cdot C_0 \cdot \left(\frac{D}{D_0}\right)^\alpha \left(\frac{F}{F_0}\right)^\beta \left(\frac{\ell}{\ell_0}\right)^\gamma = N \cdot C_0 \cdot N^{-\alpha/2} \ . \tag{2}$$

If $\alpha > 2$, the cost is reduced as $N^{1-(\alpha/2)}$. While the power-law cost model adopted here is only a crude approximation, the purpose of this specific example is to motivate a more realistic study by showing that it may be possible to deliver better mission capabilities at fixed cost.

2 Better Asteroseismology

For asteroseismology, the N telescopes may be co-aligned to point at the same field of view, thereby delivering N independent lightcurves of each star. With $D = D_0/\sqrt{N}$, summing the N lightcurves will yield a lightcurve with the same photon count rate as that achieved by the single larger telescope. There will thus be no loss in detection capabilities for stars of a given apparent magnitude so long as the detection S/N is dominated by Poisson noise on detected star photons rather than on background photons or crowding effects.

With N co-aligned telescopes, the N independent measurements of each star should permit improved internal consistency checks, rejection of bad data, and self-calibration of systematic errors.

Bandpass filters may also be optionally inserted into the beams on 1 or more of the N telescopes to acquire colour information whenever this is desirable. This permits greater flexibility to optimize the science return of the observations. Each telescope can deploy a different filter to obtain simultaneous lightcurves in a number of bandpasses. Alternatively, the total count rate can be kept higher by acquiring unfiltered data on most of the telescopes and introducing filters on 1 or 2 telescopes. With a single telescope the same information can be acquired by moving filters in and out of the beam rapidly enough to sample the timescale of the phenomenon under study. But this is undesirable because it delivers dithered rather than simultaneous coverage in different filters, and it requires moving the filters more often, which may increase the chances of failure. With N telescopes the impact is also diminished of a failure in one of the filter mechanisms during the mission because those on other telescopes remain operational.

3 Better Planet Hunting

For planet hunting, the larger field of view of the smaller telescopes increases the number of potential targets. The angular field of view is

$$\theta = \theta_0 \left(\frac{\ell}{\ell_0}\right) \left(\frac{F}{F_0}\right)^{-1} \left(\frac{D}{D_0}\right)^{-1} . \tag{3}$$

For our example with $D = D_0/\sqrt{N}$, $F = F_0$, and $\ell = \ell_0$, the number of potential targets at given apparent magnitude increases as $\theta^2 \propto N$, assuming the targets are uniformly distributed across the field.

For N co-aligned telescopes, the photon count n for a star of given type at distance d scales as $n \propto N(D/d)^2$. The detection signal-to-noise ratio then scales as $S/N \propto \sqrt{n} \propto \sqrt{N}D/d$. Thus at fixed S/N, the limiting distance is proportional to the telescope aperture, $d \propto \sqrt{N}D$. The survey volume is then scales as $\theta^2 d^3 \propto \theta^2 D^3 N^{3/2}$. In terms of D, F and ℓ, this is

$$\theta^2 d^3 = \theta_0^2 d_0^3 \cdot \left(\frac{\ell}{\ell_0}\right)^2 \left(\frac{F}{F_0}\right)^{-2} \left(\frac{D}{D_0}\right)^{+1} \cdot N^{3/2} . \tag{4}$$

For our example with $D = D_0/\sqrt{N}$, $F = F_0$ and $\ell = \ell_0$ the survey volume increases as N.

3.1 co-aligned telescopes vs independent fields

It may also be considered to reconfigure the N telescopes to point at N different fields, probably but not necessarily adjacent fields on the sky. This increases the solid angle of the survey from θ^2 to $\Omega = N\theta^2 \propto N^2$. In this splayed-out configuration, each star is observed by only 1 of the N telescopes. The effective aperture is thus reduced from D_0 to $D = D_0/\sqrt{N}$, and the limiting distance of the survey d is reduced accordingly. The photon count n for a star of given type at distance d scales as $n \propto (D/d)^2$. The detection signal-to-noise ratio then scales as $S/N \propto \sqrt{n} \propto D/d$. Thus at fixed S/N, the limiting distance is proportional to the telescope aperture, $d \propto D$. In the splayed-out configuration, then, the limiting distance of the survey is reduced from d_0 to $d = d_0(D/D_0) = d_0/\sqrt{N}$. The survey volume is then $N\theta^2 d^3 \propto N\theta^2 D^3$, or

$$N\theta^2 d^3 = N \cdot \theta_0^2 d_0^3 \cdot \left(\frac{\ell}{\ell_0}\right)^2 \left(\frac{F}{F_0}\right)^{-2} \left(\frac{D}{D_0}\right)^{+1} . \tag{5}$$

For our example with $D = D_0/\sqrt{N}$, $F = F_0$ and $\ell = \ell_0$ the survey volume increases as $\sqrt{N}$. In this example, the limiting distance of the survey decreases, but the larger solid angle more than compensates. The planet catch is larger compared with the single-telescope option, but less than with N co-aligned telescopes.

The planets discovered in a splayed-out configuration will be transiting brighter stars that are closer by, making follow-up observations easier.

3.2 background noise and de-focus

The above considerations assume that signal detections are limited by Poisson noise on the detected star photons, rather than by background noise. For small N this assumption is justified for both the asteroseismology and planet detection components of the Eddington mission. The assumption of negligible background noise may be violated for sufficiently large N, since the solid angle subtended by each CCD pixel increases as θ^2 and θ increases with N.

A noise model incorporating the degradation of S/N due to background photons coincident with the de-focussed star image is straightforward to implement given knowledge of the wavelength-dependent telescope and camera throughputs, CCD quantum efficience, angular size of star images, sky background spectrum, and target star spectra. This detailed study is beyond the scope of this report.

An additional design parameter that may need to be adjusted is the degree to which star images are de-focussed in order to distribute the starlight among more than a few CCD pixels. Eddington uses de-focussed star images to spread the light over more CCD pixels. This permits a larger number of detected star photons to be accumulated in each exposure without saturating the CCD, and also reduces systematic errors related to flat-fielding and image motion.

Note that with N telescopes we effectively divide the photon counts among N independent CCD detectors, relaxing the saturation limit so that saturation may still be avoided while using exposure times longer by a factor N, or while using de-focussed star images that are smaller in area by a factor $1/N$.

By using smaller de-focussed star images, we could keep the number of background photons coincident on the star image at the same level, independent of N, until the star image size shrinks to 1 pixel. This sets an upper limit on N, beyond which the background counts must increase relative to star counts.

In practice the benefits of improved flat-fielding are progressively lost as star images are focussed onto a small number of CCD pixels. There is thus a trade-off here that needs to be optimized.

3.3 uniform star density and neglect of absorption

The focus above on survey volume as the measure of Eddington's planet detection performance neglects effects due to non-uniform density of stars and galactic absorption.

Absorption reduces the apparent brightness of target stars near the limit of the survey volume. We argue that while this reduces significantly the planet catch for large planets, the effect is less important for smaller ones which are arguably of more interest.

For small planets the Monte-Carlo simulations (Fig 1) indicate that Eddington's survey volume extends to $m_V \sim 15$ and $d \sim 700$ pc, neglecting absorption. For small habitable planets the volume is even smaller since the typical host star has smaller mass and radius. We will presumably choose a field of view that is relatively dust-free, in order to minimize absorption effects over such a distance. Absorption will therefore not have a large effect on the catch of small planets, and these will scale as $N^{3/2}\theta^2 D^3$ for N co-aligned telescopes.

Absorption will of course cut off the larger planets, which can be detected on fainter stars at larger distances. If small planets are the criterion of success, however, then the maximizing the survey volume seems justified.

The Eddington field of view is small enough that the conical survey volume will remain inside Galactic Disk scale heights unless we point well out of the Galactic plane. Assumption of a uniform star density within the survey volume should therefore be a reasonably good approximation for purposes of designing the telescope capabilities.

Of course the exact positioning of the field to optimize the planet detection capabilities will require a detailed study based on catalogs of stars with known radius, distance, and apparent magnitude. Chosing dust-free directions with high star densities will make a big difference.

4 Performance Optimization at Fixed Cost

The adopted assumption of N telescopes with smaller aperture $D = D_0/\sqrt{N}$ but identical focal ratio $F = F_0$ and CCD detector mosaic size $\ell = \ell_0$ should be regarded as illustrative only. It shows that with certain assumptions about cost scalings with various telescope parameters D, F, ℓ, a better mission may be accomplished by building a larger number N of smaller telescopes than by building a single telescope. In practice a performance optimization study is needed to establish the best values to choose for N, D, F, and ℓ.

Eddington's planet detection capabilities can be optimized by maximizing the volume of the survey. If this is to be done subject to a fixed total cost, the design parameters can be optimized by minimizing the cost function $C(N, D, F, \ell)$ at each fixed survey volume. The survey volume is either $N^{3/2}D^3\theta^2$ for co-aligned telescopes, or $ND^3\theta^2$ for independent fields. The optimization needs to be done for both options, and at each volume the cheapest option can then be selected. This minimization will deliver an optimum design parameters that minimize the mission cost at each survey volume. The minimum cost should increase, of course, with the survey volume. Parameters for a range of optimized missions will therefore emerge from the study, bracketing the eventual planned cost available to fund the mission.

A more realistic cost function than the simple power-laws adopted above will of course need to account separate for costs that are independent of N (telescope and camera design costs) and costs that scale with N (hardware). Each component (telescope, CCDs, cameras, ...) will presumably scale differently with the design parameters D, F, ℓ. Information necessary to model the costs is not available to the author, and the optimization is beyond the scope of the current report, but needs to be done if Eddington is to realize its full potential.

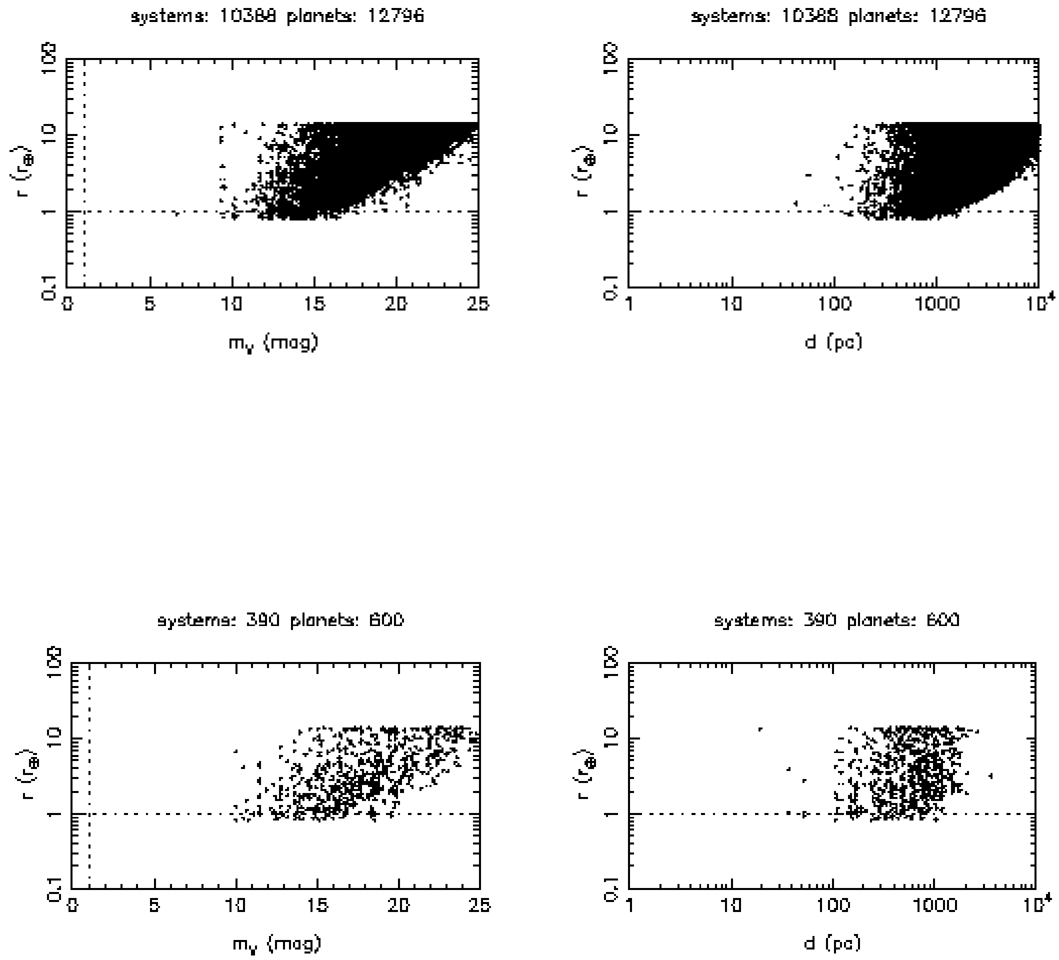

Figure 1: Monte-Carlo simulation of the Eddington planet catch for fields centred at galactic latitudes $b = 10°$ and $b = 90°$.