Exoplanet Discovery Methods

(1) Direct imaging

Today: Star Wobbles

(2) Astrometry $\rightarrow$ position

(3) Radial velocity $\rightarrow$ velocity

Later:

(4) Transits

(5) Gravitational microlensing

(6) Pulsar timing
Kepler Orbits

Star’s view:

Kepler 1: Planet orbit is an *ellipse with star at one focus* (Newton showed this is due to gravity’s inverse-square law).

Kepler 2: Planet seeps out *equal area in equal time* (angular momentum conservation).

Planet’s view:

Inertial Frame:

Star and planet both orbit around the *centre of mass.*
Kepler Orbits

\[ M = M_\ast + m_p = \text{total mass} \]
\[ a = a_p + a_\ast = \text{semi-major axis} \]
\[ a_p \ m_p = a_\ast M_\ast = a \ M \quad \text{Centre of Mass} \]
\[ P = \text{orbit period} \]
\[ a^3 = G \ M \left( \frac{P}{2\pi} \right)^2 \quad \text{Kepler's 3rd Law} \]
\[ e = \frac{a - b}{a} = \text{eccentricity} \quad 0 = \text{circular} \quad 1 = \text{parabolic} \]
Astrometry

• Look for a periodic “wobble” in the angular position of host star
• Light from the star+planet is dominated by star
• Measure star’s motion in the plane of the sky due to the orbiting planet
• Must correct measurements for parallax and proper motion of star
• Doppler (radial velocity) more sensitive to planets close to the star
• Astrometry more sensitive to planets far from the star

Stellar wobble: Star and planet orbit around centre of mass. Radius of star’s orbit scales with planet’s mass:

\[
\frac{a_*}{a} = \frac{m_p}{M_* + m_p} \quad \frac{a_p}{a} = \frac{M_*}{M_* + m_p}
\]

Angular displacement for a star at distance \(d\):

\[
\Delta \theta = \frac{a_*}{d} \approx \left( \frac{m_p}{M_*} \right) \left( \frac{a}{d} \right)
\]

(Assumes small angles and \(m_p << M_*\))
Scaling to Jupiter and the Sun, this gives:

\[ \Delta \theta \approx 0.5 \left( \frac{m_p}{m_{J}} \right) \left( \frac{M_*}{M_{\text{sun}}} \right)^{-1} \left( \frac{a}{5 \text{AU}} \right) \left( \frac{d}{10 \text{pc}} \right)^{-1} \text{ mas} \]

Note:

• Units are milliarcseconds -> very small effect
• Amplitude increases at \textbf{large orbital separation,} \(a\)
• Amplitude decreases with distance to star \(d\).

• Detecting planets at large orbital radii requires a \textbf{long search time,} comparable to the orbital period.

\[ \frac{P}{\text{yr}} = \left( \frac{M_*}{M_{\text{sun}}} \right)^{-1/2} \left( \frac{a}{\text{AU}} \right)^{2/3} \]
The wobble of the Sun’s projected position due to the influence of all the planets as seen from 10 pc

Epsilon Eridani

Data obtained 1980-2006 to track the orbit

\[ P = 6.9 \text{ yr}, \ m_p = 1.55 \text{ M}_J \]

\[ 1 \text{ mas} \]

\textit{HST Fine Guidance Sensors}
• PRIMA on VLT Interferometer (Paranal, Chile)
• ESA’s GAIA (2011 launch) and NASA’s SIM (not yet funded)
• Planned astrometric errors \(\sim 10 \text{ micro-arcsecond}\)
• May detect planets of a few Earth masses at 1 AU around nearby stars
Astrometry Selection Function

Need to observe (most of) a full orbit of the planet:
No discovery for planets with $P > P_{\text{survey}}$

For $P < P_{\text{survey}}$, planet detection requires a star wobble several times larger than the accuracy of the measurements. $\implies$ minimum detectable planet mass.

Planet mass sensitivity as a function of orbital separation

$$\Delta \theta = \frac{a_*}{d} \approx \left( \frac{m_p}{M_*} \right) \left( \frac{a}{d} \right)$$

$${m_p} \propto a^{-1}$$
Exoplanets: 50+262+8+5=325 (Mar 2009)

**Orbit Period (in Earth Years)**

**Doppler:** 262

**Transits:** 50(+5)

**Microlensing:** 8

**Astrometry:** 0

**Imaging:** 5
Doppler Wobbles: Radial Velocity

Periodic variations in the Radial Velocity of the Host Star

- Most successful method: >300 planets detected
- The first planet around a normal star, 51 Peg, was detected by doppler wobbles in 1995.
- Doppler shift of starlight caused by the star orbiting the center of mass with 1 or more orbiting planets
Consider first a circular orbit.

Velocities: \( V_* = \left( \frac{2\pi a_*}{P} \right) \)
\( V_p = \left( \frac{2\pi a_p}{P} \right) \)

Conservation of momentum: \( M_* V_* = m_p V_p \)
thus \( M_* a_* = m_p a_p \)

Kepler’s 3rd Law: \( a^3 = G M \left( \frac{P}{2\pi} \right)^2 \)
\( M = M_* + m_p \)

\[ V_* = \frac{2\pi a_*}{P} = \frac{2\pi m_p}{P M} a = \frac{2\pi m_p}{P M} \left( G M \left( \frac{P}{2\pi} \right)^2 \right)^{1/3} = m_p \left( \frac{2\pi G}{P M^2} \right)^{1/3} \]
Star’s Orbit Velocity

Centre of mass

Kepler’s law applies for \( V = \) relative velocity, \( M = \) total mass

\[
\frac{V^2}{a} = \frac{G M}{a^2} \quad \Rightarrow \quad V = \left( \frac{G M}{a} \right)^{1/2} = \frac{2\pi a}{P} \quad M \equiv M* + m_p
\]

\[
V_* = \frac{a_*}{a} V = \frac{m_p}{M} \left( \frac{G M}{a} \right)^{1/2} = m_p \left( \frac{G}{a M} \right)^{1/2}
\]

Star’s centrifugal acceleration due to planet’s gravity:

\[
\frac{V_*^2}{a_*} = \frac{G m_p}{a^2} \quad \Rightarrow \quad V_* = \left( \frac{G m_p}{a^2 a_*} \right)^{1/2} = \left( \frac{G m_p}{a^2 M} \right)^{1/2} = m_p \left( \frac{G}{a M} \right)^{1/2}
\]
Star’s Orbit Velocity

From Kepler’s Law and $a_* M_* = a_p m_p$ (center of mass), the star’s velocity is:

$$V_* \approx \left( \frac{m_p}{M} \right)^{1/2} \sqrt{\frac{GM}{a}}$$

where $M \equiv M_* + m_p \approx M_*$

Star’s velocity scales with planet’s mass.
Hot Jupiter ($P = 5$ days) orbiting a 1 $M_{\text{sun}}$ star: 125 m/s
Jupiter orbiting the Sun: 12.5 m/s
Sun due to Earth: 0.1 m/s

Thermal velocity width of spectral lines $\sim 10$ km/s $(T/10^4 \text{K})^{1/2}$

Special techniques and spectrographs needed to measure such tiny radial velocity shifts stably over many years.
**Key Technology:**

Iodine Gas Cell

Pass the starlight through an Iodine Cell and then into a Spectrograph
High sensitivity to small radial velocity shifts:
• Achieved by comparing high S/N \(\sim 200\) spectra with template stellar spectra
• Large number of lines in spectrum allows shifts of much less than one pixel to be determined

Absolute wavelength calibration and stability over long timescales:
• Achieved by passing stellar light through a cell containing iodine, imprinting large number of additional lines of known wavelength into the spectrum.
• Calibration suffers identical instrumental distortions as the data
Examples of radial velocity data

51 Pegasi

Mass = 0.46 M_{Jup} / \sin i
P = 4.230 day
K = 55.9 m s^{-1}
e = 0.00

RMS = 5.23 m s^{-1}

Orbital Phase

51 Peg b, the first known exoplanet, with a 4.2 day circular orbit.
Orbital inclination => lower limits

The *observed* velocity is component along the line of sight, thus reduced by the sine of the orbit’s inclination angle:

\[ V_{\text{obs}} = V_* \sin(i) \]

With

\[ V_* \approx \left( \frac{m_p}{M_*} \right) \sqrt{ \frac{G M_*}{a} } \]

The measured quantity is: \( m_p \sin(i) \)

(assuming \( M_* \) is well determined e.g. from spectral type)

\( V_{\text{obs}} \) gives us \( m_p \sin(i) \), a **lower limit** on the planetary mass, if there are no other constraints on the inclination angle.
Error sources

(1) Theoretical: photon noise limit
   • flux in a pixel that receives N photons uncertain by $\sim N^{1/2}$
   • implies absolute limit to measurement of radial velocity
   • depends on spectral type - more lines improve signal
   • $< 1$ m/s for a G-type main sequence star with spectrum recorded at S/N=200
   • practically, S/N=200 can be achieved for V=8 stars on a 3m class telescope in survey mode

(2) Practical:
   • stellar activity - young or otherwise active stars are not stable at the m/s level
   • remaining systematic errors in the observations
Currently, best observations achieve:

Best RV precision ~ 1 m/s

...in a single measurement. Allowing for the detection of low mass planets with peak Vobs amplitudes of ~ 3 m/s

HD 40307, with a radial velocity amplitude of ~ 2 m/s, has the smallest amplitude wobble so far attributed to a planet.

Radial velocity monitoring detects massive planets (gas giants, especially those at small a. It is now also detecting super-Earth mass planets (< 10 ME)
Selection Function

Need to observe (most of) a full orbit of the planet:
No discovery for planets with $P > P_{\text{survey}}$

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Planet mass sensitivity as a function of orbital separation

$$V_* \approx \left( \frac{m_p}{M_*} \right) \sqrt{\frac{GM_*}{a}}$$

$$m_p \sin(i) \propto a^{1/2}$$
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Orbit Radius in Astronomical Units (AU)
Eccentric Orbits

Circular orbit: velocity curve is a sine wave.
Elliptical orbit: velocity curve more complicated, but still varies periodically.

Eccentric orbit:

\[ V_{rad} = \frac{2\pi a \sin(i)}{P(1 - e^2)^{1/2}} \left[ \cos(\theta - \omega) + e \cos(\omega) \right] \]

Circular orbit: \( e \rightarrow 0, \omega \rightarrow 0 \)
Example of a planet with an eccentric orbit: $e=0.67$

**HD 89744**

- Mass = $7.56 \, M_{\text{Jup}}$ $/ \sin i$
- $P = 256.3 \, \text{day}$
- $K = 262, \, \text{m s}^{-1}$
- $e = 0.67$

**RMS = 13.3 \, \text{m s}^{-1}$**

**Orbital Phase**
Eccentric (non-circular) Orbits

Not yet well understood.

Early star-star encounters?
Planet-planet interactions?
Eccentricity pumping.
Small planets ejected?
Tidal circularisation.
A planetary system with 3 gas giants

Planet b: 0.059 AU, 0.72 M\(_J\), e=0.04
Planet c: 0.83 AU, 1.98 M\(_J\), e=0.23
Planet d: 2.5 AU, 4.1 M\(_J\), e=0.36

Upsilon And: F8V star
M\(_*\)=1.28 Msun
Teff=6100 K
A system of “super Earths”

- 22 planets discovered with $m_p < 30 \text{ M}_E$
- 9 super-Earths ($2 \text{ M}_E < m_p < 10 \text{ M}_E$)
- Found at a range of orbital separations:
  - Microlensing detection of $5.5 \text{ M}_E$ at 2.9 AU
  - RV detections at $P \sim$ few days to few hundred days
- 80% are found in multi-planet systems

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<thead>
<tr>
<th>System</th>
<th>Mass</th>
<th>Orbit</th>
<th>Temperature</th>
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| GJ 581 | M3V    | 0.31  | 581b: 5.3d 15.7 M\(_E\)  
|        |        |       | 581c: 12.9d 5.0 M\(_E\)  
|        |        |       | 581d: 83.6d 7.7 M\(_E\)  

Bonfils, et al. 2005  
Udry, et al. 2007
Summary

Observables:
(1) Planet mass, up to an uncertainty from the normally unknown inclination of the orbit. Measure $m_p \sin(i)$
(2) Orbital period -> radius of the orbit given the stellar mass
(3) Eccentricity of the orbit

Current limits:
- Maximum ~ 6 AU (ie orbital period ~ 15 years)
- Minimum mass set by activity level of the star:
  - ~ 0.5 $M_J$ at 1 AU for a typical star
  - 4 $M_E$ for short period planet around low-activity star
- No strong selection bias in favour / against detecting planets with different eccentricities