AS4022 — Cosmology — Tutorial Sheet 2

1. 10^4 galaxies with redshifts z between 0.3 and 0.4 are counted in a 1 square degree field of view. Calculate the co-moving volume of this survey, and hence the number density of galaxies per *co-moving* cubic Megaparsec. (The co-moving volume is the survey volume expanded to the current epoch t_0.) Assume a flat universe with $h = 0.7$ and $\Omega_M = 1.0$. For a challenge, do the same using the Concordance model $(h, \Omega_M, \Omega_\Lambda) = (0.7, 0.3, 0.7)$.

2. A radio jet emerging from the nucleus of a quasar at redshift $z = 0.2$ is observed to be 3 arcseconds long. Assuming that the jet is perpendicular to the line of sight, calculate its physical length in pc for two cosmological models with parameters $(h, \Omega_M, \Omega_\Lambda) = (0.7, 0.3, 0.7)$ and $(0.7, 0.3, 0.0)$.

3. Analyze Olber's paradox in the context of modern cosmology, using the Robertson-Walker metric and the dimensionless Friedmann equation

$$H^2 = H_0^2 \left(\Omega_M \, x^3 + \Omega_\Lambda + (1 - \Omega_M - \Omega_\Lambda) x^2 \right) \ .$$

   Assume a Universe filled with stars of luminosity $L_\star$, uniformly distributed with volume density n_0 today. Write an expression in terms of an integral over $x = (1+z)$ for the surface brightness $\mu(z) \equiv F(z)/\Omega$ where $F(z)$ is the observed flux from all stars closer than redshift z, and Ω is the solid angle of the entire sky. From your result, what constraints on Ω_M and Ω_Λ are needed for the sky to have a finite surface brightness? What is the main effect, redshift, finite age, or curvature, that keeps our sky dark at night?

4. Write a formula for the "sound horizon", $t_H(z)$, which is the distance travelled by a sound wave starting at time $t = 0$ and finishing at a particular redshift z. Assume a sound speed $c_S = c/\sqrt{3}$ in the photon-baryon plasma prior to recombination. Evaluate the size in Mpc of the sound horizon at $z_R = 1000$. To what size does this region expand at the present epoch $(z = 0)$?

5. After recombination at $z \approx 1000$, the absence of free electrons removes the electron scattering opacity, releasing photons to flow freely. Matter, released from the photons, is now able to fall into dark matter potential wells to form compact structures. Some time later, the first stars appeared, ending the Dark Ages. UV radiation from the first high mass stars re-ionised the intergalactic gas, producing free electrons once again, which scattered some of the background photons, producing polarisation. WMAP detected this polarisation at a strength indiating that $\approx 15\%$ of the background photons were scattered.

   Write a formula for the optical depth to electron scattering along the path of a photon that was emitted at redshift z and observed today. Assuming $h = 0.7$, $\Omega_b = 0.04$, $\Omega_M + \Omega_\Lambda = 1$, and $\Omega_M = 0.3$, at what redshift does the optical depth become 0.15? This is the redshift of re-ionisation, when the first stars formed.