Inclinations

- Radial velocities give minimum masses
  \[ m \sin^2 i \]
  \[ m' \sin^3 i / M^2 \]

- Need inclinations
  - Astrometric orbits
  - Eclipsing binaries
  - Polarimetry
  - Apsidal motion

Astrometric Orbits

- Interferometry measures the relative orbit (milli-arcsec accuracy).
- Fit elliptical orbit model to \( x(t) \) and \( y(t) \).

Polarimetry

- Scattered light is polarised (e.g., our sky)

Eclipsing Binaries

- Binary properties from eclipses
  - Sizes, shapes of stars, inclination,
  - Temperatures, limb darkening, apsidal motion
  - 4 contact phases:
  - 1
  - 2
  - 3
  - 4
  - Primary eclipse
  - Secondary eclipse

Partial Eclipse

- \( R_1 > R_2 \), \( \theta = 2 \phi \)

- Edge-on \( (i = 90^\circ) \):
  - 4 contact phases:
  - \( R_1 \pm R_2 = a \sin \theta \)
  - \( R_1 / a = [\sin \theta_1 - \sin \theta_2] / 2 \)
  - \( R_2 / a = [\sin \theta_1 + \sin \theta_2] / 2 \)
**Eclipses**

- **4 contact phases**
  - $R_i - R_c = a \sqrt{\sin \theta + \cos \theta \cos i}$
  - 4 measurements: $\theta, \psi, \phi, \phi_i$
  - 4 parameters: $a, R_i, i, \phi_i$

- Mideclipse: $\theta = 0$
- Total eclipse: $a \cos \theta = R_i - R_c$
- Partial eclipse: $R_i - R_c < a \cos \theta < R_i + R_c$

**Application to Binary Pulsars**

- **Binary system where one star is a pulsar**
  - Emits ‘pulses’ of radiation
  - Accurate timing possible (accurate clocks)
  - Need narrow pulses
  - Radio signals from neutron stars

- **Solitary pulsar**
  - If at 0 velocity relative to us
  - Time between pulses, $dt = \text{constant}$ (unless being spun up/down)
  - If at $V_{rel}$ relative velocity
  - $dt = \text{constant} \times \text{pulse number}$
  - If pulsar spins up, $dt$ decreases with pulse number
  - Concave curve
  - If pulsar spins down, $dt$ increases with pulse number
  - Convex curve

**Timing Residuals**

- **O-C** = observed time minus calculated time

- Period too long
- Correct period
- Period too short

**Binary Pulsars**

- **In binary system, time between pulses affected by orbital motions**
  - Due to light travel time (distance) changing along orbit

- Pulsar orbit: $r_p = a_p \left( \frac{1 + e^2}{1 - e \cos \theta} \right)$
- Distance along line of sight: $z_p = r_p \sin i \sin (\theta + \omega)$
- Light travel time:
  - Circular orbit: $z_p = a_p \sin i \left( \frac{2\pi}{P} (t - T_p) \right)$

**Light travel time**

- Observer:
  - $z_p = a_p \sin i \left( \frac{2\pi}{P} (t - T_p) \right)$

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**Redder and Bluer Colour Changes**

- Colour changes $\rightarrow T, L$
Binary Pulsar timing residuals

- Time difference between predicted, \( t_a \), and actual (binary) pulse arrival times, \( t_n \), is

\[
\Delta t = t_a - t_n = a t + b \sin \left( \frac{2\pi}{P} (t - T_0) \right)
\]

- \( P \) is the orbital period, \( T_0 \) is a reference time
- \( a, b \) are determined by the velocity of the pulsar
- \( a \): from systematic velocity
- \( b \): from Keplerian velocity
- for circular orbits: \( b = \left( \frac{a_p}{c} \right) \sin i \)

Mass determinations

- visible companion star
  - O-B star in High-Mass X-ray Binaries (HMXB)
  - A-K star in Low-Mass X-ray Binaries (LMXB)

\[
f(m_i) = \frac{m_i \sin i}{M^2} \sin \left( \frac{1 - \cos^2 i}{\cos^2 i} \frac{P}{K} \right) \frac{2\pi}{2K} G
\]

\[
mass ratio, q, \quad q = \frac{m_2}{m_1} = \frac{a_2 \sin i}{a_1 \sin i}
\]

- If inclination, \( i \), can be found, then masses follow

Frequency shifts

- Binary orbit also affects pulsar frequency
  - radio pulsars, very narrow pulse widths
  - pulse frequency affected by orbital velocity
  - Doppler shift:

\[
\Delta f = f \frac{V_{\text{rad}}}{c} = f \frac{\dot{z}}{c}
\]

- gives a phase lag of:

\[
\Delta \Phi = \int_{t_0}^{t} \Delta f \, dt = f_0 \frac{\dot{z}}{c} (t - T_0)
\]

Pulsar Phase lag

- Combined phase lag is
  - from light travel time due to orbit

\[
\Delta \phi_0 = -f_0 \frac{\dot{z}}{c}
\]
  - and from Doppler shift

\[
\Delta \phi_0 = f_0 \frac{\dot{z}}{c}
\]

- hence

\[
\Delta \phi = \Delta \phi_0 + \Delta \phi_1 = \left[ f_0 \frac{\dot{z}}{c} - f_0 (t - T_0) \right]
\]

- generally

\[
\Delta \phi_0 = 0.001 \Delta \phi_1
\]

- but measurable in radio pulsars