Inclinations

• Radial velocities give minimum masses

\[
\begin{align*}
SB2: \quad m \sin^3 i \\
SB1: \quad \frac{m^3 \sin^3 i}{M^2}
\end{align*}
\]

• need inclinations
  – astrometric orbits
  – eclipsing binaries
  – polarimetry
  – apsidal motion
Astrometric Orbits

Interferrometry measures the relative orbit (milli-arcsec accuracy).

Fit elliptical orbit model to $x(t)$ and $y(t)$.

Get inclination, hence masses
Polarimetry

Scattered light is polarised (e.g. our sky)

Get inclination, hence masses

AS 4024  Binary Stars and Accretion Disks
Eclipsing Binaries

- **Binary properties from eclipses**
  - sizes, shapes of stars, inclination,
  - temperatures, limb darkening, apsidal motion
  - 4 contact phases:
    - 1
    - 2
    - 3
    - 4
    - primary eclipse
    - secondary eclipse

Diagram showing phases of an eclipse with graphs showing magnitudes vs. orbital phase.
Partial Eclipse
Eclipses

\[ R_1 > R_2 \quad \theta \equiv 2\pi \phi \]

edge-on \ (i = 90^\circ)\ : 
4 contact phases:
\[ R_1 \pm R_2 = a \sin \theta \]

\[
\frac{R_2}{a} = \frac{\sin \theta_1 - \sin \theta_2}{2} \\
\frac{R_1}{a} = \frac{\sin \theta_1 + \sin \theta_2}{2}
\]

\[ a \sin \Theta = R_1 + R_2 \]
Eclipses

4 contact phases:

\[
R_1 \pm R_2 = a \sqrt{\sin^2 \theta + \cos^2 \theta \cos^2 i}
\]

\[
= a \sqrt{1 + \cos^2 \theta \sin^2 i}
\]

4 measurements: \( \phi_1, \phi_2, \phi_3, \phi_4 \)

4 parameters: \( \frac{R_1}{a}, \frac{R_2}{a}, i, \phi_0 \)

mid eclipse: \( \theta = 0 \)

total eclipse: \( a \cos i < R_1 - R_2 \)

partial eclipse: \( R_1 - R_2 < a \cos i < R_1 + R_2 \)
Colour changes $\rightarrow T, L$
Application to Binary Pulsars

- **binary system where one star is a pulsar**
  - emits ‘pulses’ of radiation
  - accurate timing possible (accurate clocks)
  - need narrow pulses
    - radio signals from neutron stars

- **solitary pulsar**
  - if at 0 velocity relative to us
    - time between pulses, $dt = \text{constant}$ (unless being spun up/down)
  - if at $V_{\text{rel}}$ relative velocity
    - $dt = \text{constant} \times \text{pulse number}$
  - if pulsar spins up, $dt$ decreases with pulse number
    - concave curve
  - if pulsar spins down, $dt$ increases with pulse number
    - convex curve
Timing Residuals

\[ O-C = \text{observed time minus calculated time} \]

- Period too long
- Correct period
- Period too short

- Period increasing
- Period constant
- Period decreasing

Cycle number
Binary Pulsars

- In binary system, time between pulses affected by orbital motions
  - due to light travel time (distance) changing along orbit

![Graph showing pulse arrival time vs. pulse number for a binary system](image)
Light travel time

pulsar orbit: \[ r_p = \frac{a_p (1+e^2)}{1+e \cos \theta} \]

distance along line of sight:
\[ z_p = r_p \sin i \sin(\theta + \omega) \]

light travel time:
\[ z_p = \frac{a_p \sin i}{c} \left( \frac{1+e^2}{1+e \cos \theta} \right) \sin(\theta + \omega) \]

circular orbit
\[ z_p = \frac{a_p \sin i}{c} \sin \left( \frac{2\pi}{P} (t - T_0) \right) \]
light travel time
Binary Pulsar timing residuals

- Time difference between predicted, $j_n \tau$, and actual (binary) pulse arrival times, $t_n$, is

$$\Delta t = t_n - j_n \tau = a \, t + b \sin \left[ \frac{2\pi (t - T_0)}{P} \right]$$

- $P$ is the orbital period, $T_0$ is a reference time
- $a, b$ are determined by the velocity of the pulsar
  - $a$: from systematic velocity
  - $b$: from Keplerian velocity
- for circular orbits: $b = \left( \frac{a_p}{c} \right) \sin i$
Binary Pulsar Orbits

radial velocity

timing

AS 4024

Binary Stars and Accretion Disks
Mass determinations

- **visible companion star**
  - O-B star in High-Mass X-ray Binaries (HMXB)
  - A-K star in Low-Mass X-ray Binaries (LMXB)

\[
a_c \sin i = \frac{(1-e^2)^{1/2} K_c P}{2\pi}
\]

mass function

\[
f(m_p) = \frac{m_p^3 \sin^3 i}{M^2} = \frac{(1-e^2)^{3/2} K_c^3 P}{2\pi G}
\]

mass ratio, q,

\[
q = \frac{m_p}{m_c} = \frac{a_c \sin i}{a_p \sin i}
\]

- If inclination, \(i\), can be found, then masses follow
Frequency shifts

- **Binary orbit also affects pulsar frequency**
  - radio pulsars, very narrow pulse widths
  - pulse frequency affected by orbital velocity
  - Doppler shift:

\[
\Delta f = f \frac{V_{rad}}{c} = f \frac{\dot{z}}{c}
\]

- gives a phase lag of:

\[
\Delta \phi = \int_{T_0}^{t} \Delta f \, dt \approx f_0 \frac{\dot{z}}{c} (t - T_0)
\]

\[
= f_0 \frac{\dot{z}}{c} \left[ \frac{z}{c} \right]
\]
Pulsar Phase lag

- **Combined phase lag is**
  - from light travel time due to orbit
    \[ \Delta \phi_L = -f \frac{Z}{c} \]
  - and from Doppler shift
    \[ \Delta \phi_D = f_0 \frac{\dot{z}}{c} \left[ \frac{Z}{c} \right] \]

- hence
  \[ \Delta \phi = \Delta \phi_D + \Delta \phi_L \approx \left[ \frac{Z}{c} \right] \left[ f_0 \frac{\dot{z}}{c} - f_0 - \dot{f}_0 (t - T_0) \right] \]

- generally
  \[ \Delta \phi_D \approx 0.001 \Delta \phi_L \]

- but measurable in radio pulsars