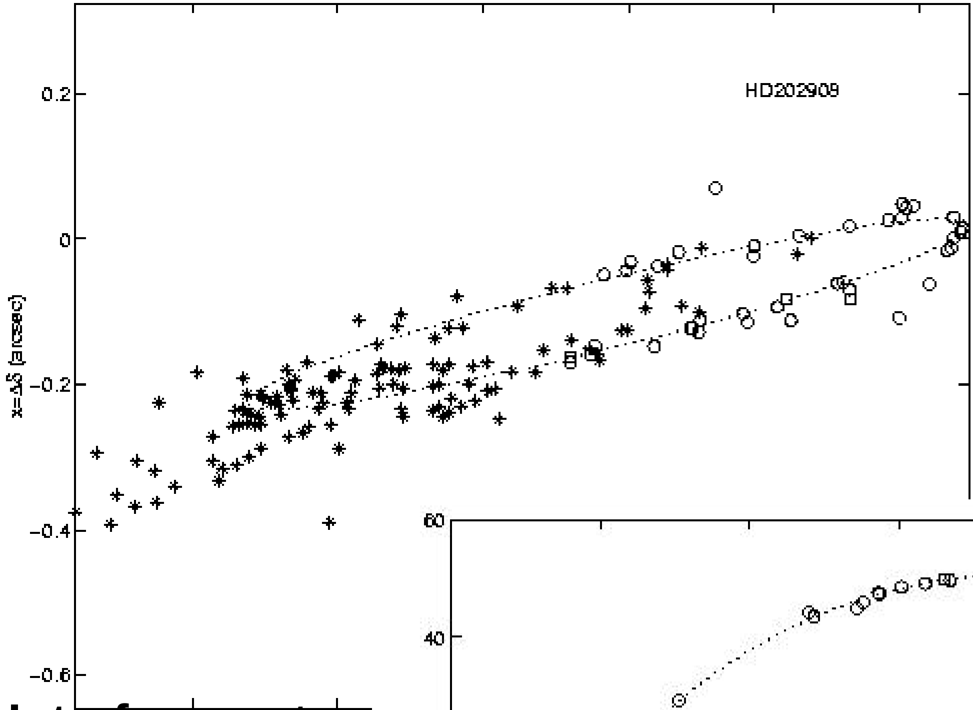
Inclinations

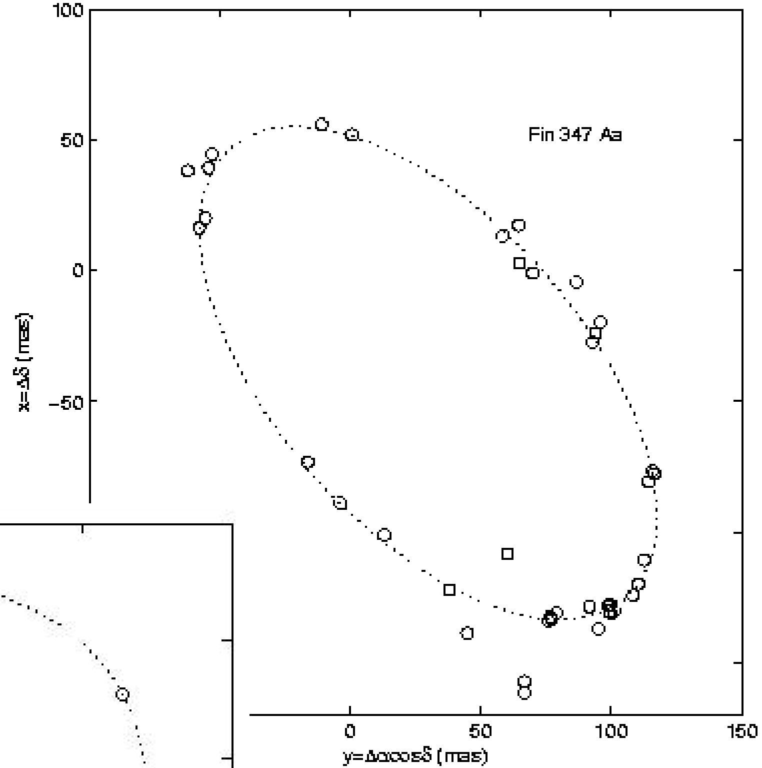
- **Radial velocities give minimum masses**

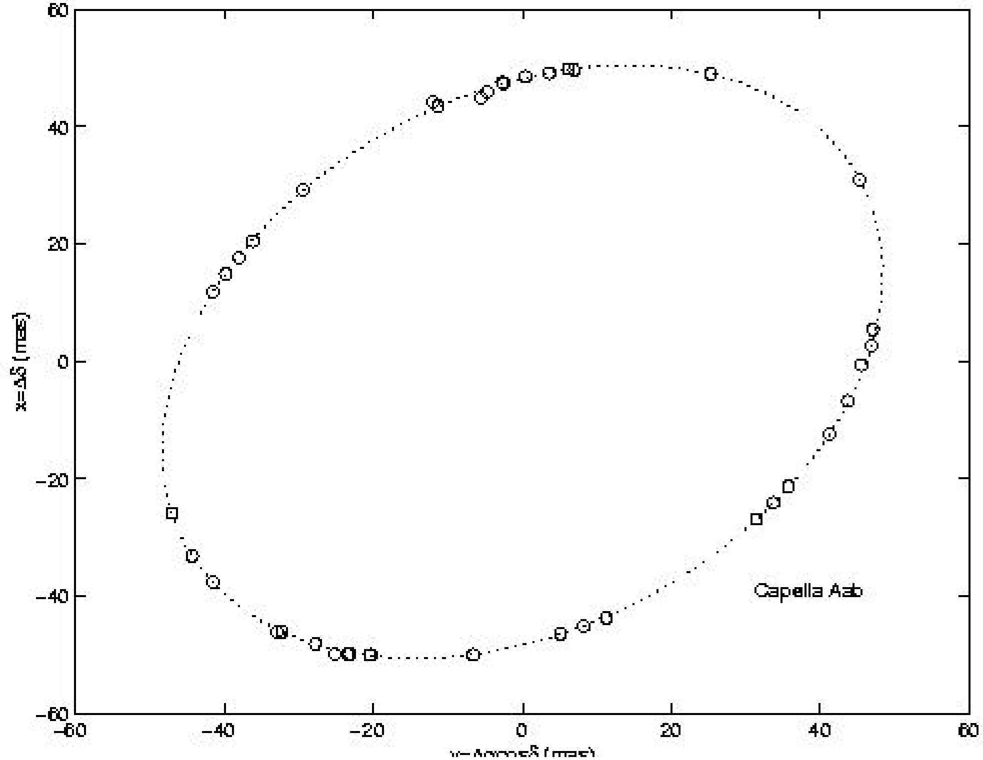
$$\text{SB2:} \quad m \sin^3 i \qquad \text{SB1:} \quad \frac{m^3 \sin^3 i}{M^2}$$

- **need inclinations**
  - **astrometric orbits**
  - **eclipsing binaries**
  - **polarimetry**
  - **apsidal motion**

Astrometric Orbits







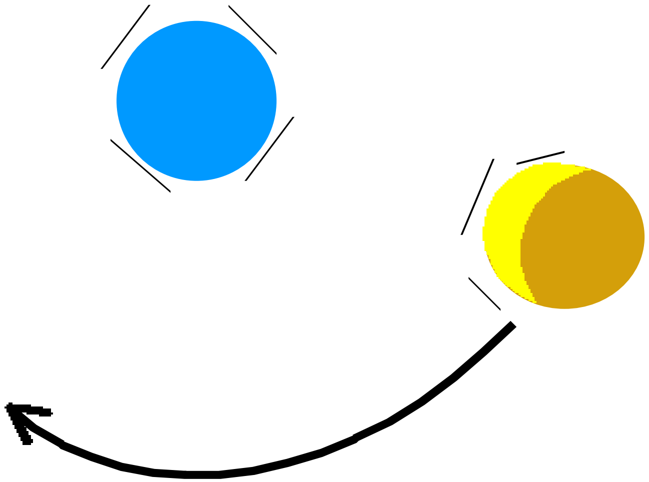
Interferrometry measures the relative orbit (milli-arcsec accuracy).

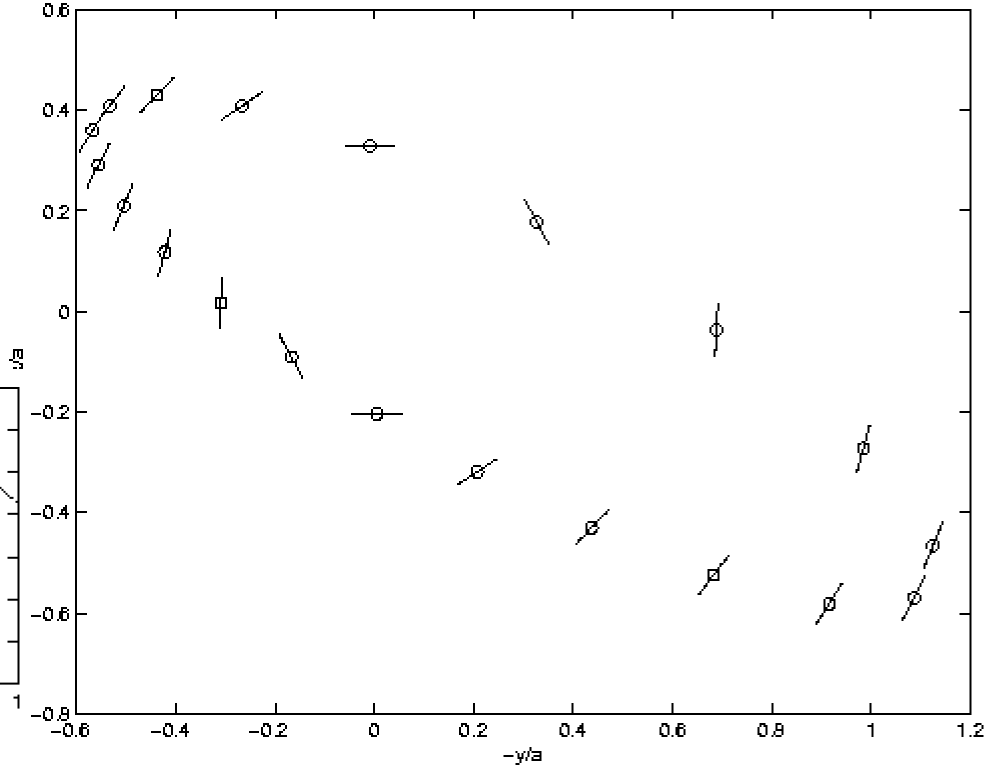
Fit elliptical orbit model to x(t) and y(t).

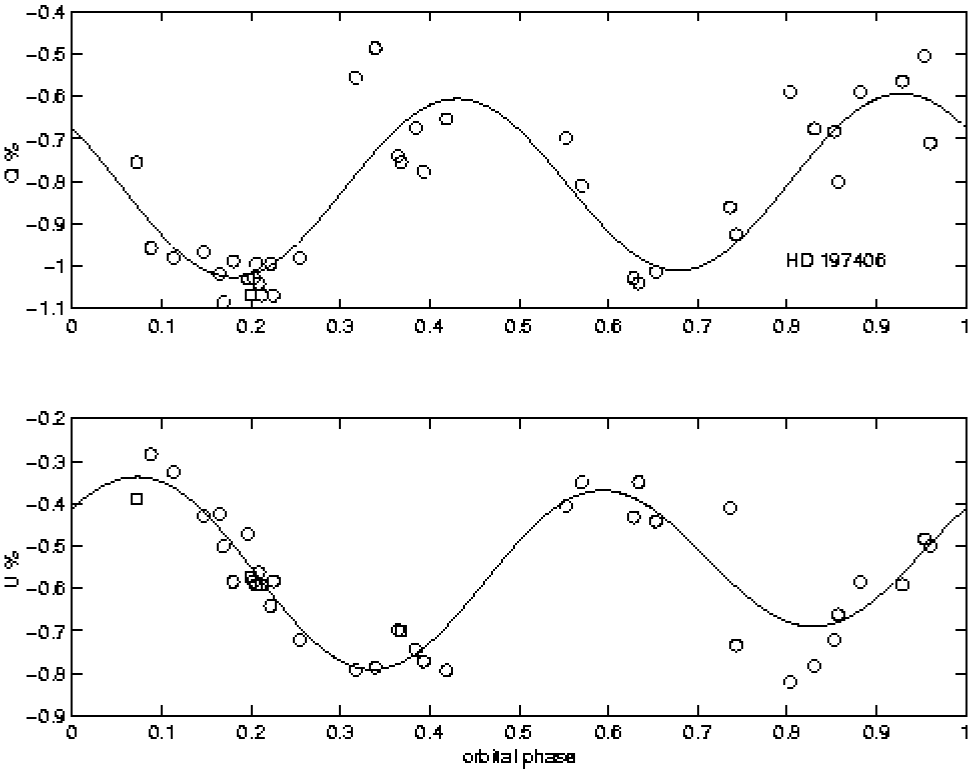
Get inclination, hence masses

Polarimetry

Scattered light is polarised (e.g. our sky)

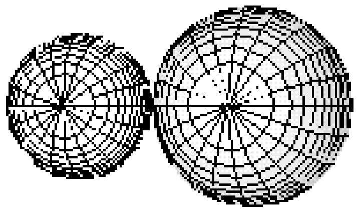
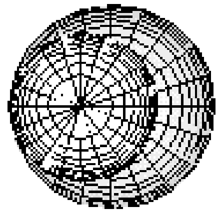
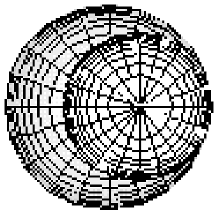
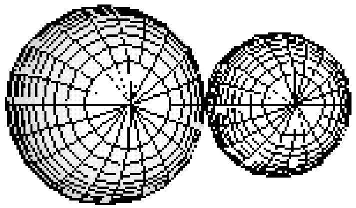


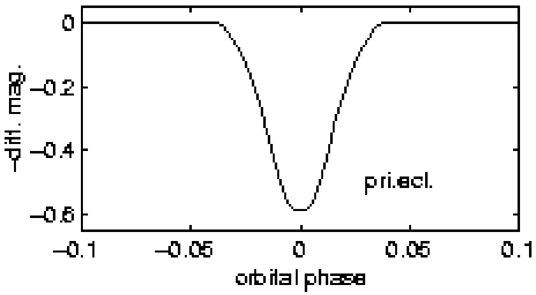


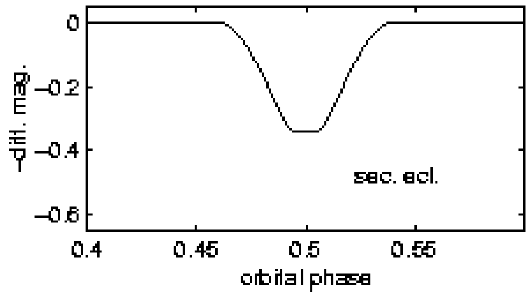


Get inclination, hence masses

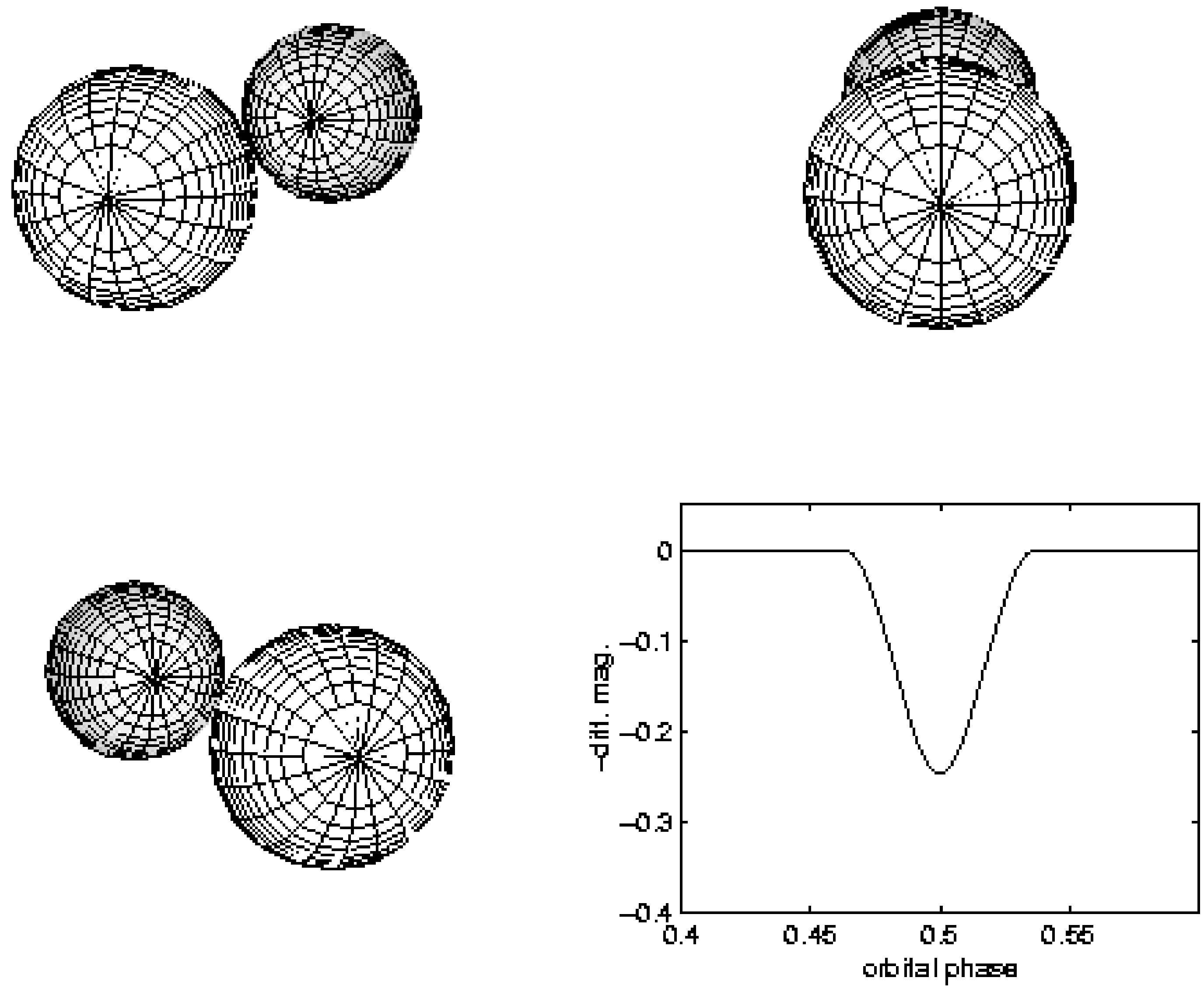
Eclipsing Binaries

- **Binary properties from eclipses**
  - **sizes, shapes of stars, inclination,**
  - **temperatures, limb darkening, apsidal motion**
  - **4 contact phases:**
  - **1**  **2** 
  - **3**  **4** 
  - **primary eclipse** **secondary eclipse**





Partial Eclipse



Eclipses

$$R_1 > R_2 \quad q \equiv 2p\ f$$

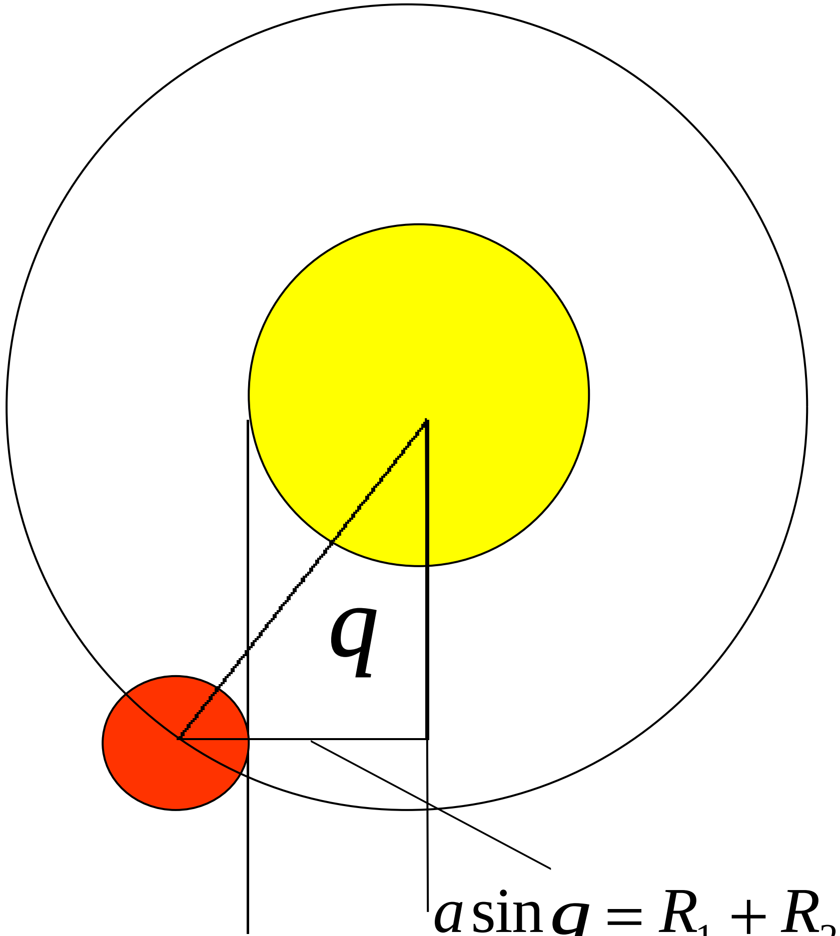
edge-on $(\ i = 90^{\circ}\ )$:

4 contact phases:

$$R_1 \pm R_2 = a \sin q$$

$$\frac{R_2}{a} = \frac{\sin q_1 - \sin q_2}{2}$$

$$\frac{R_1}{a} = \frac{\sin q_1 + \sin q_2}{2}$$

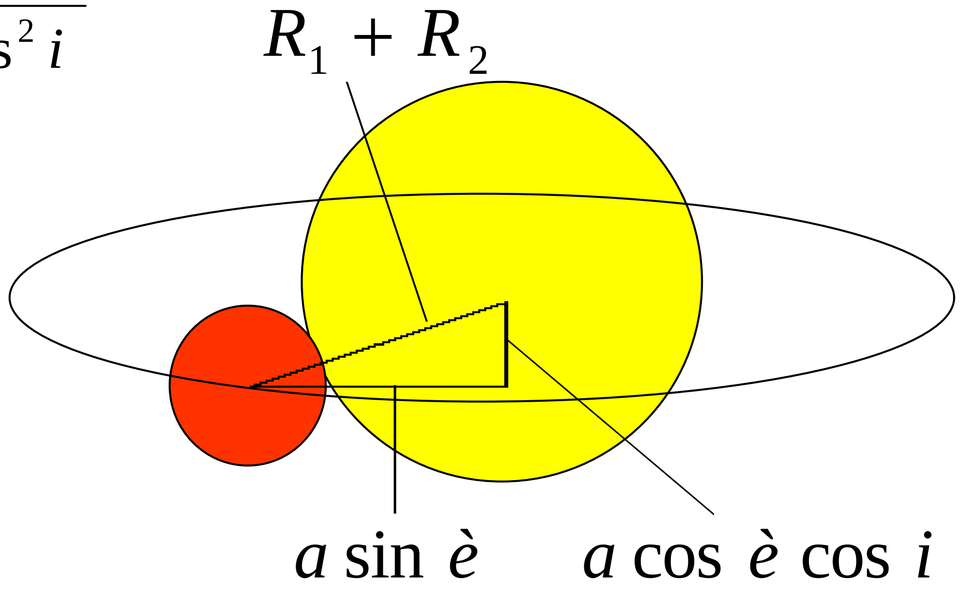


Eclipses

4 contact phases :

$$R_1 \pm R_2 = a \sqrt{\sin^2 q + \cos^2 q \, \cos^2 i}$$

$$= a \sqrt{1 + \cos^2 q \, \sin^2 i}$$



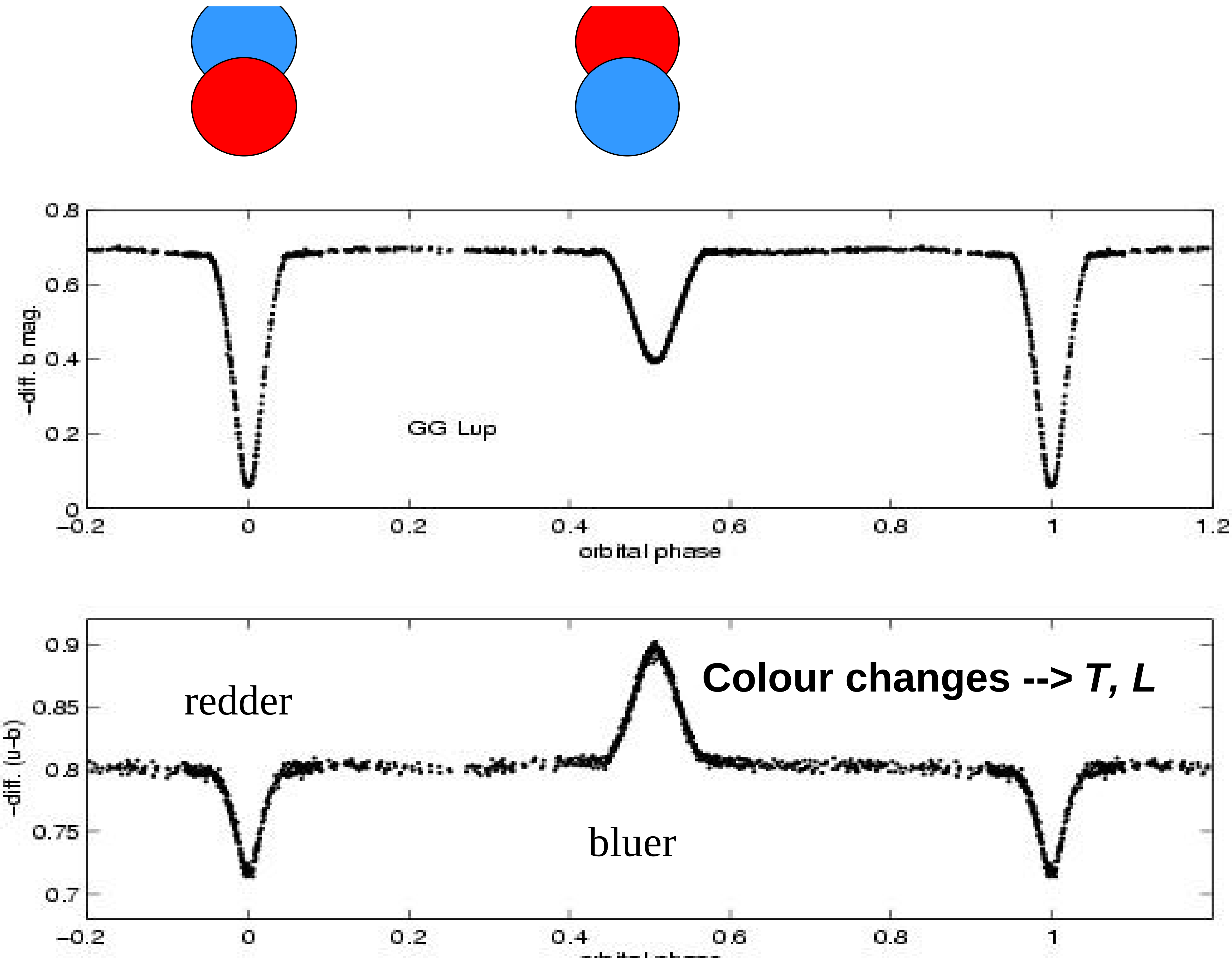
4 measurements : $f_1 \quad f_2 \quad f_3 \quad f_4$

4 parameters : $\frac{R_1}{a} \quad \frac{R_2}{a} \quad i \quad f_0$

mid eclipse : $q = 0$

total eclipse : $a \cos i < R_1 - R_2$

partial eclipse : $R_1 - R_2 < a \cos i < R_1 + R_2$



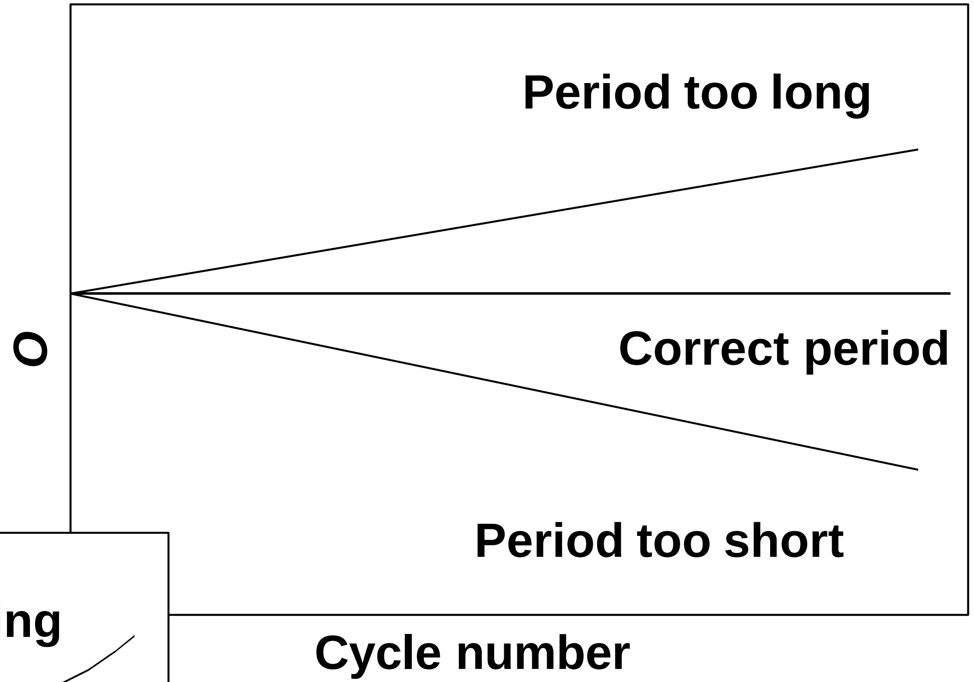
-diff. b mag.
GG Lup
orbital phase
redder
Colour changes --> *T, L*
bluer
-diff. (u-b)

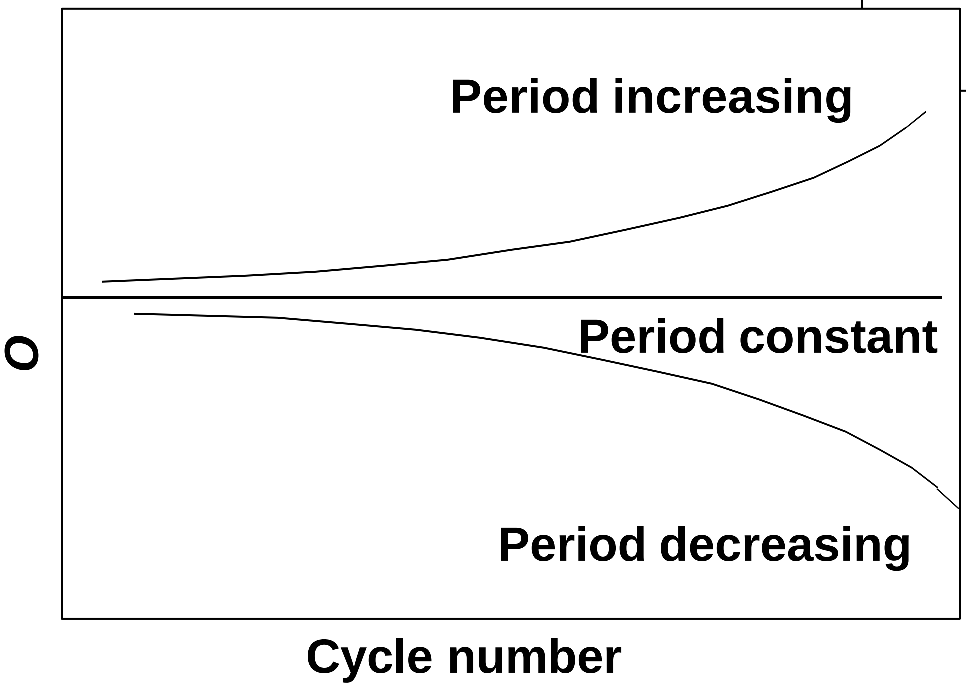
Application to Binary Pulsars

- **binary system where one star is a pulsar**
  - emits ‘pulses’ of radiation
  - accurate timing possible (accurate clocks)
  - need narrow pulses
    - radio signals from neutron stars
- **solitary pulsar**
  - if at 0 velocity relative to us
    - time between pulses, dt = constant (unless being spun up/down)
  - if at V_{rel} relative velocity
    - dt = constant x pulse number
  - if pulsar spins up, dt decreases with pulse number
    - concave curve
  - if pulsar spins down, dt increases with pulse number
    - convex curve

Timing Residuals

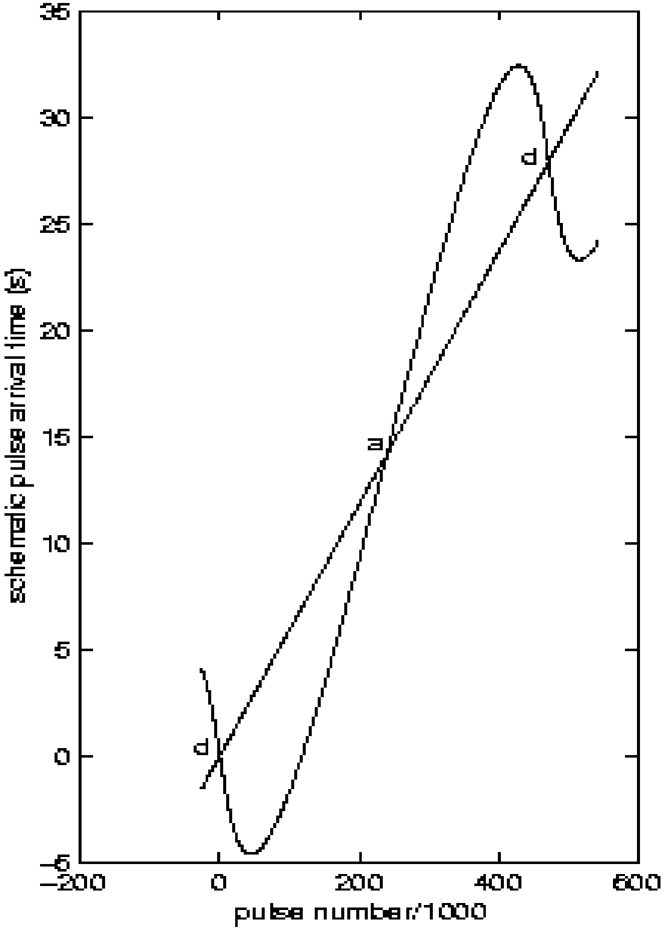
O-C = observed time minus calculated time

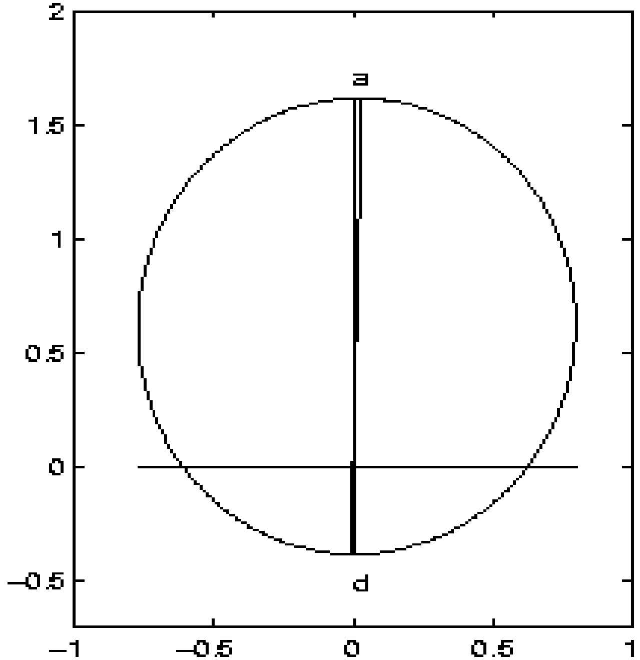




Binary Pulsars

- **In binary system, time between pulses affected by orbital motions**
  - **due to light travel time (distance) changing along orbit**





Observer-->

Light travel time

$$\text{pulsar orbit}: \quad r_p = \frac{a_p\,(1+e^2)}{1+e\cos q}$$

distance along line of sight:

$$z_p = r_p \sin i \sin\left(q + w \right)$$

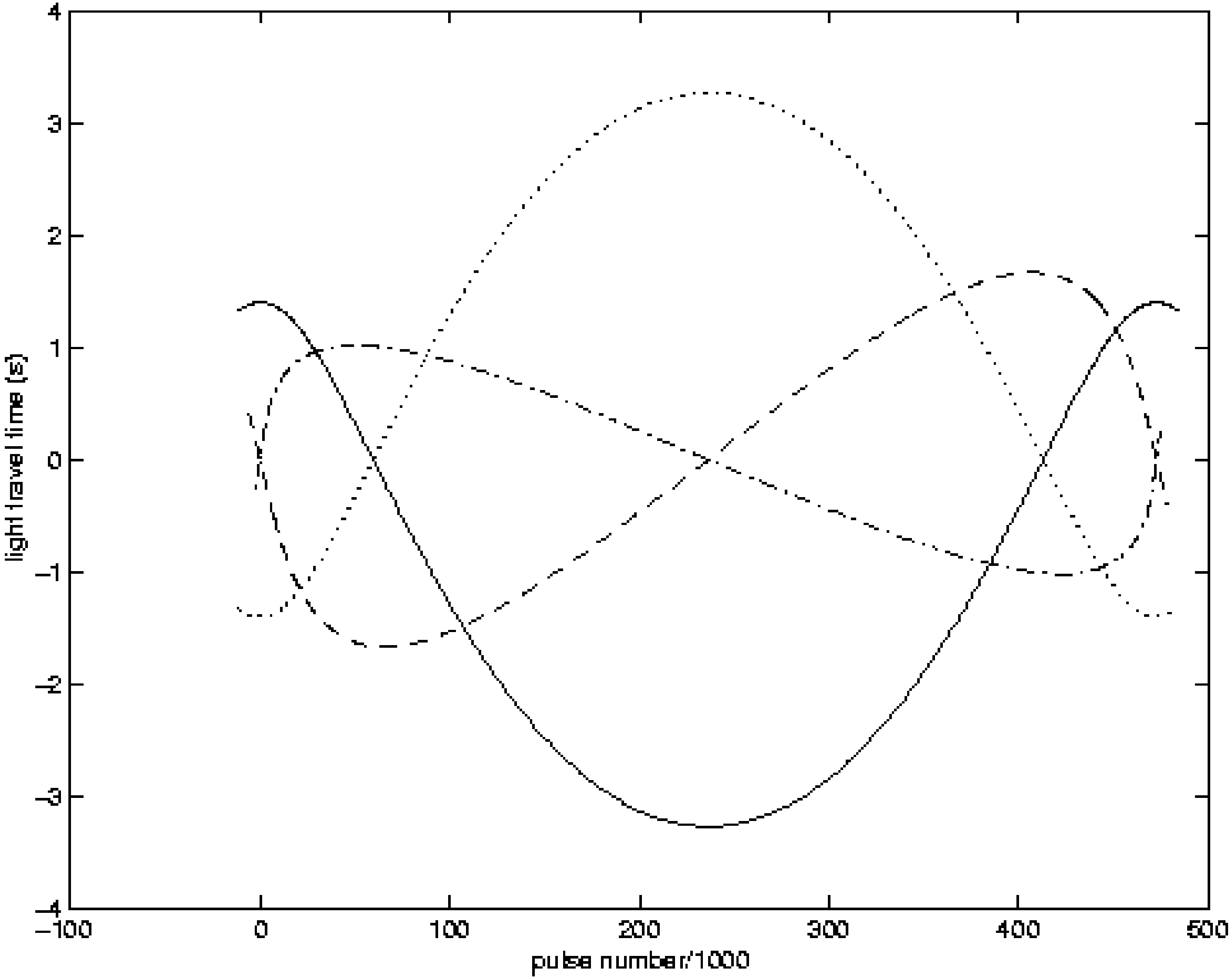
light travel time:

$$\frac{z_p}{c} = \frac{a_p \sin i}{c}\,(1+e^2)\,\frac{\sin(q + w)}{1+e\cos q}$$

circular orbit

$$\frac{z_p}{c} = \frac{a_p \sin i}{c} \sin\left(\frac{2p}{P}(t - T_0) \right)$$

light travel time



Binary Pulsar timing residuals

- **Time difference between predicted , j_n t, and actual (binary) pulse arrival times, t_n is**

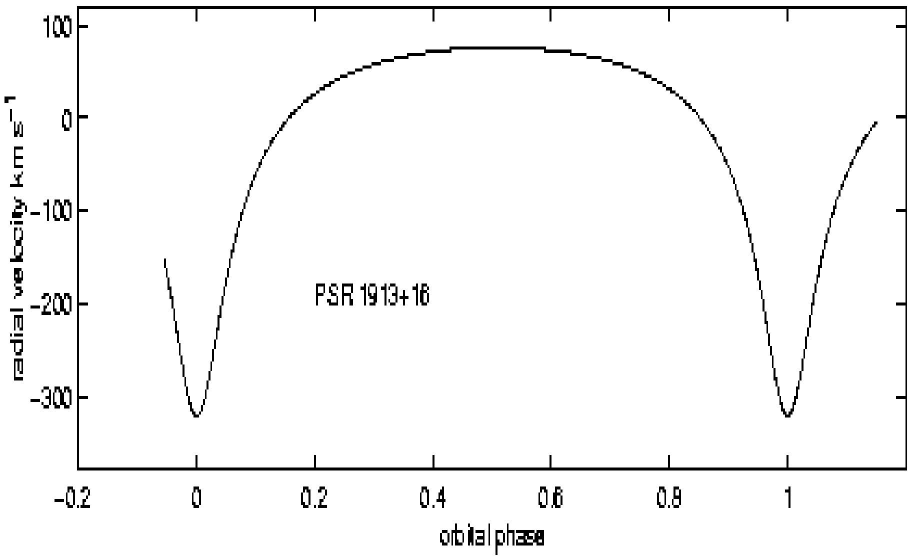
$$\Delta t = t_n - j_n t = a\,t + b \sin\left[\frac{2p\,(t - T_0)}{P}\right]$$

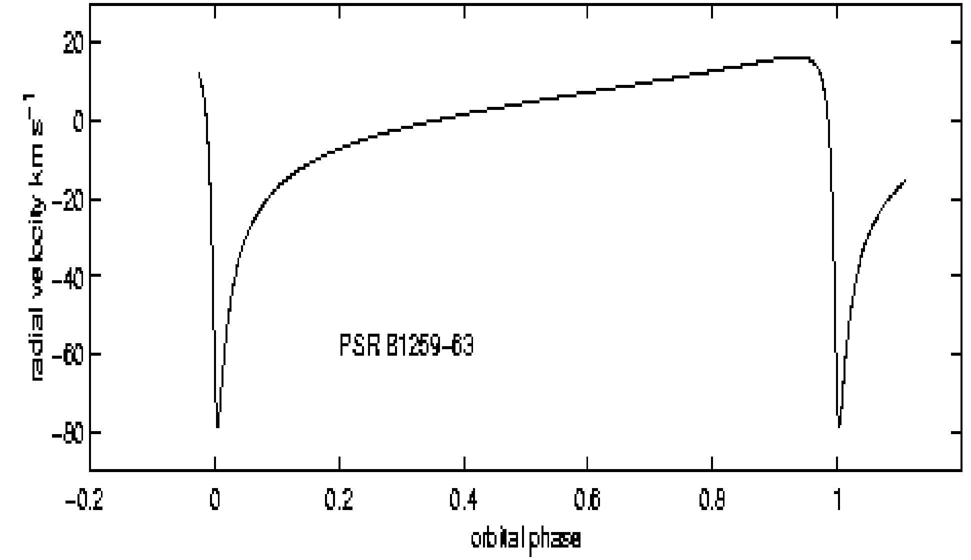
  - **P is the orbital period, T_0 is a reference time**
  - **a,b are determined by the velocity of the pulsar**
    - **a: from systematic velocity**
    - **b: from Keplerian velocity**

  - **for circular orbits: b = (a_p/c) sini**

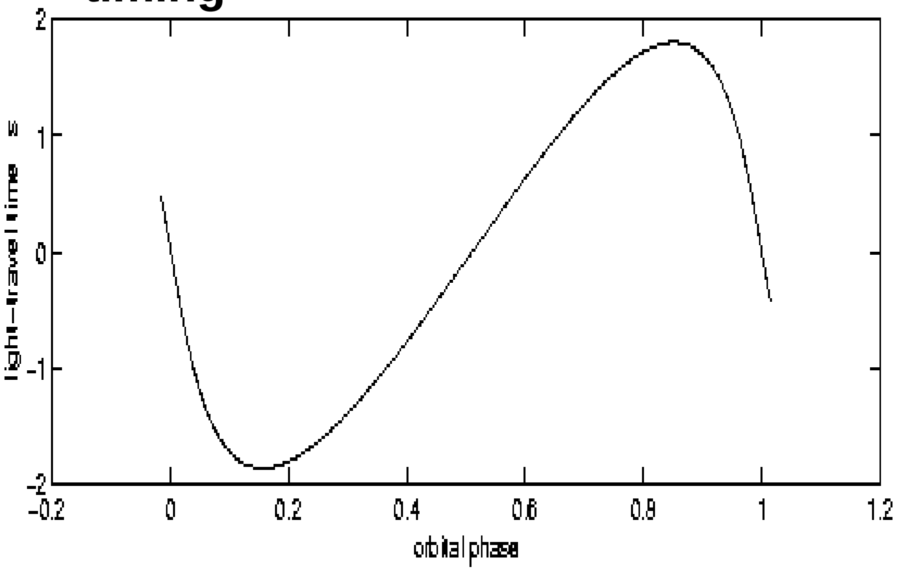
Binary Pulsar Orbits

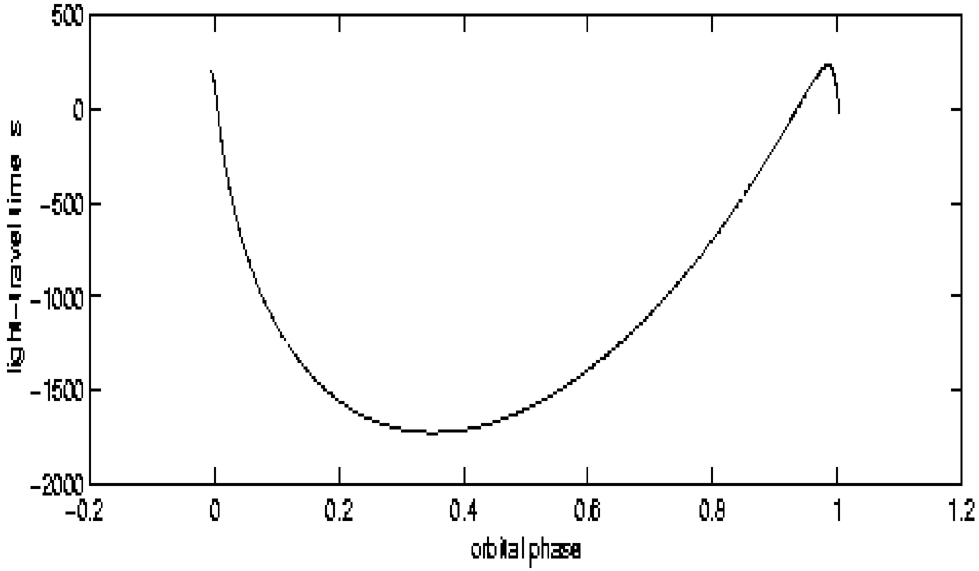
radial velocity





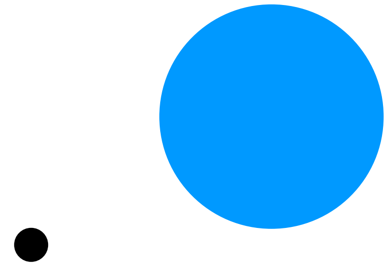
timing





Mass determinations

- **visible companion star**
  - **O-B star in High-Mass X-ray Binaries (HMXB)**
  - **A-K star in Low-Mass X-ray Binaries (LMXB)**



$$a_c \sin i = \frac{(1-e^2)^{1/2} K_c P}{2p}$$

mass function

$$f(m_p) = \frac{m_p^3 \sin^3 i}{M^2} = \frac{(1-e^2)^{3/2} K_c^3 P}{2p G}$$

mass ratio, q,

$$q = \frac{m_p}{m_c} = \frac{a_c \sin i}{a_p \sin i}$$

- **If inclination, *i*, can be found, then masses follow**

Frequency shifts

- **Binary orbit also affects pulsar frequency**
  - **radio pulsars , very narrow pulse widths**

  - **pulse frequency affected by orbital velocity**
  - **Doppler shift:**

$$\Delta f = f \frac{V_{rad}}{c} = f \frac{\dot{z}}{c}$$

  - **gives a phase lag of:**

$$\Delta f = \int_{T_0}^{t} \Delta f \, dt \approx f_0 \frac{\dot{z}}{c} (t - T_0)$$

$$= f_0 \frac{\dot{z}}{c} \left[\frac{z}{c} \right]$$

Pulsar Phase lag

- **Combined phase lag is**
  - **from light travel time due to orbit**

$$\Delta f_L = -f\frac{z}{c}$$

  - **and from Doppler shift**

$$\Delta f_D = f_0\frac{\dot{z}}{c}\left[\frac{z}{c}\right]$$

  - **hence**

$$\Delta f = \Delta f_D + \Delta f_L \approx \left[\frac{z}{c}\right]\left[f_0\frac{\dot{z}}{c} - f_0 - \dot{f}_0(t - T_0)\right]$$

  - **generally** $\Delta f_D \cong 0.001\Delta f_L$
  - **but measurable in radio pulsars**