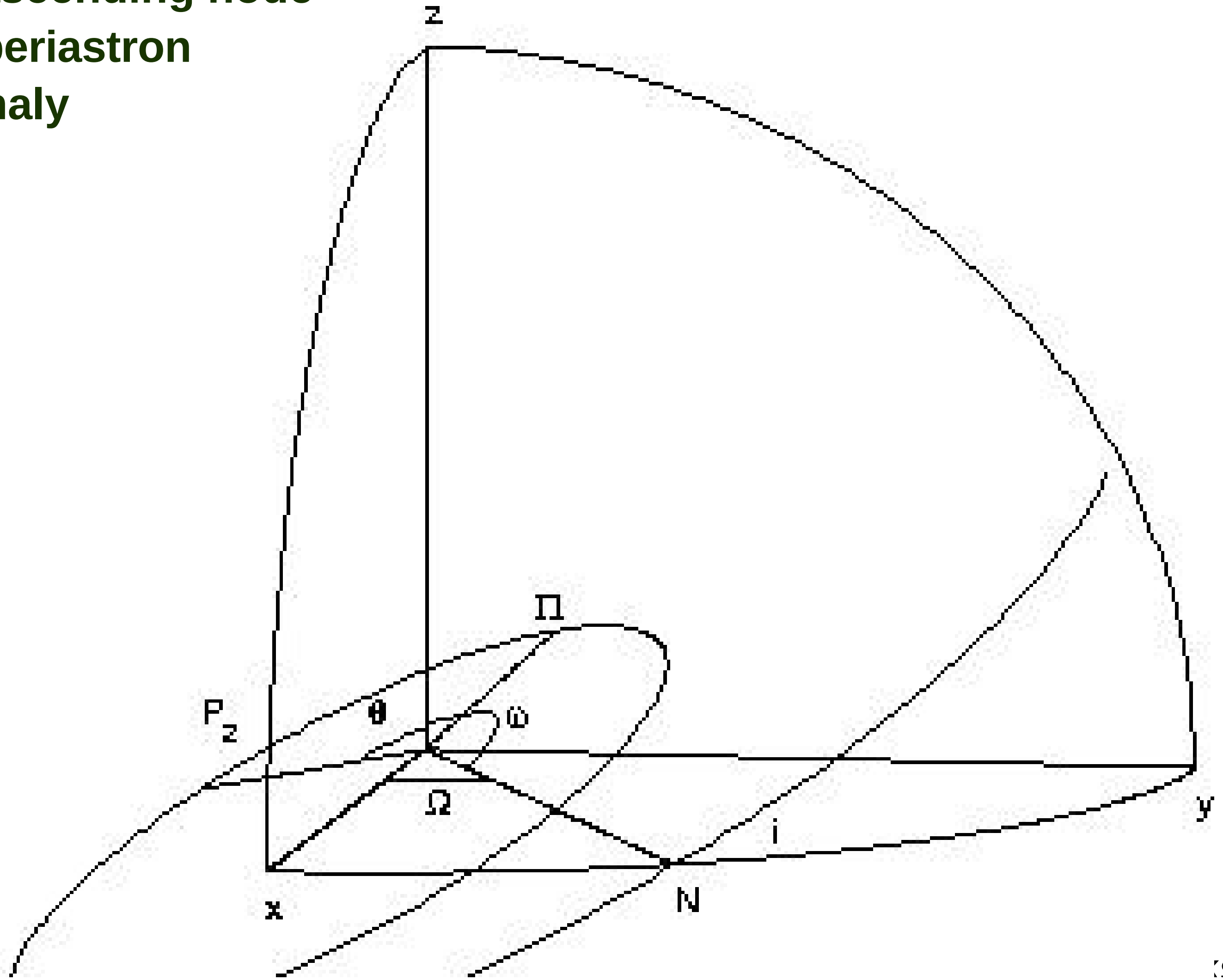
Orbit in Space

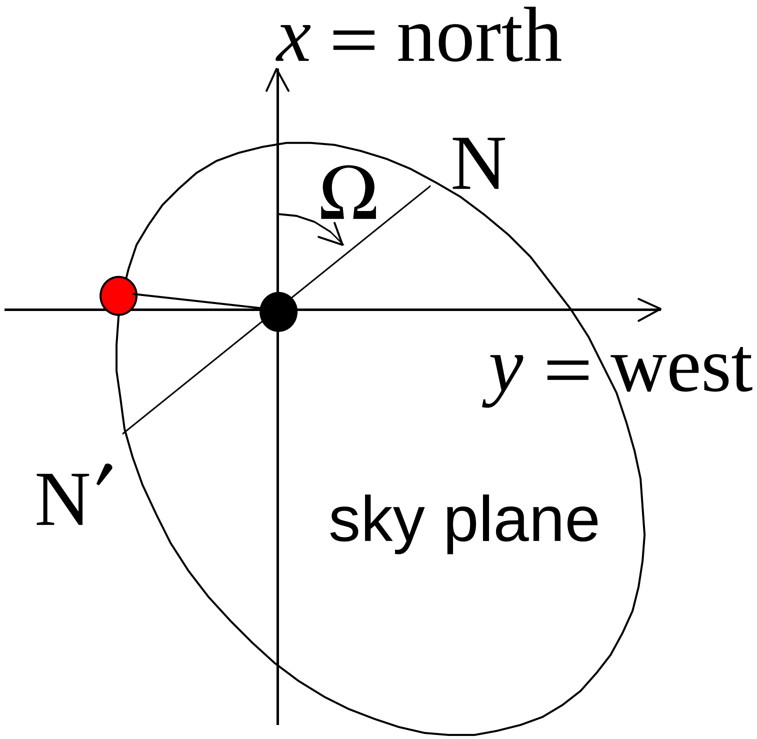
- x, y = north, west on sky plane thru m1 at x,y=0,   observer at -z.
- *i* = inclination between sky and orbit planes
- N = ascending node, m2 crosses x-y plane in +z direction

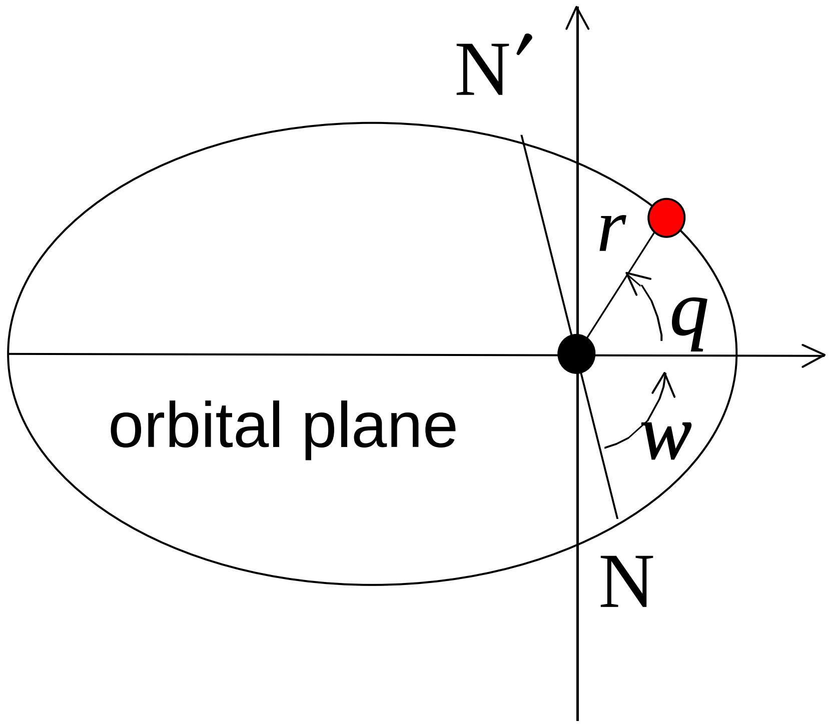
  W = longitude of ascending node

  w = longitude of periastron
- theta = true anomaly



Relative orbit on the Sky





$$x = x' \cos\Omega + y' \sin\Omega \cos i$$

$$y = x' \sin\Omega - y' \cos\Omega \cos i$$

$$z = y' \sin i$$

$$x' = r \cos(q + w)$$

$$y' = r \sin(q + w)$$

Orbital Elements

specific angular momentum :

$$L_x = \quad L \sin i \sin \Omega$$

$$L_y = -L \sin i \cos \Omega$$

$$L_z = -L \cos i$$

7 orbital elements :

size (a), shape (e),

orientation in space (i, w, Ω)

and in time (T, P).

Spectroscopic Binaries

- **m2 at (r, q + w)**
  - **project along the line of nodes: r cos(q + w)**
  - **perpendicular to line of nodes: r sin(q + w)**
    - **project along line of sight: z = r sin(q + w) sin i**
- **radial velocities along line-of sight**

$$V_{rad} = \dot{z} = \sin i \left[\dot{r} \sin(q + w) + r \dot{q} \cos(q + w) \right]$$

use

$$r = \frac{a\,(1-e^2)}{1+e\cos q} \quad \rightarrow \quad \dot{r} = \frac{e \sin(q)\, r\, \dot{q}}{1+e\cos q}$$

and Kepler's 2nd Law

$$r^2 \dot{q} = \frac{2p\, a^2 (1-e^2)^{1/2}}{P}$$

$$\boxed{V_{rad} = \frac{2p\, a \sin i}{P \sqrt{1-e^2}} \left[\cos(q + w) + e\cos(w) \right]}$$

Spectroscopic Orbital Velocities

$$V_{rad} = K\left[\cos(q + w) + e\cos w\right] + g$$

$$K = \frac{2p\ a\sin i}{P\sqrt{1-e^2}}$$

$$\text{ascending node}:\ \ \cos(q + w) = +1$$

$$V_{\max} = K\left[e\cos w + 1\right] + g$$

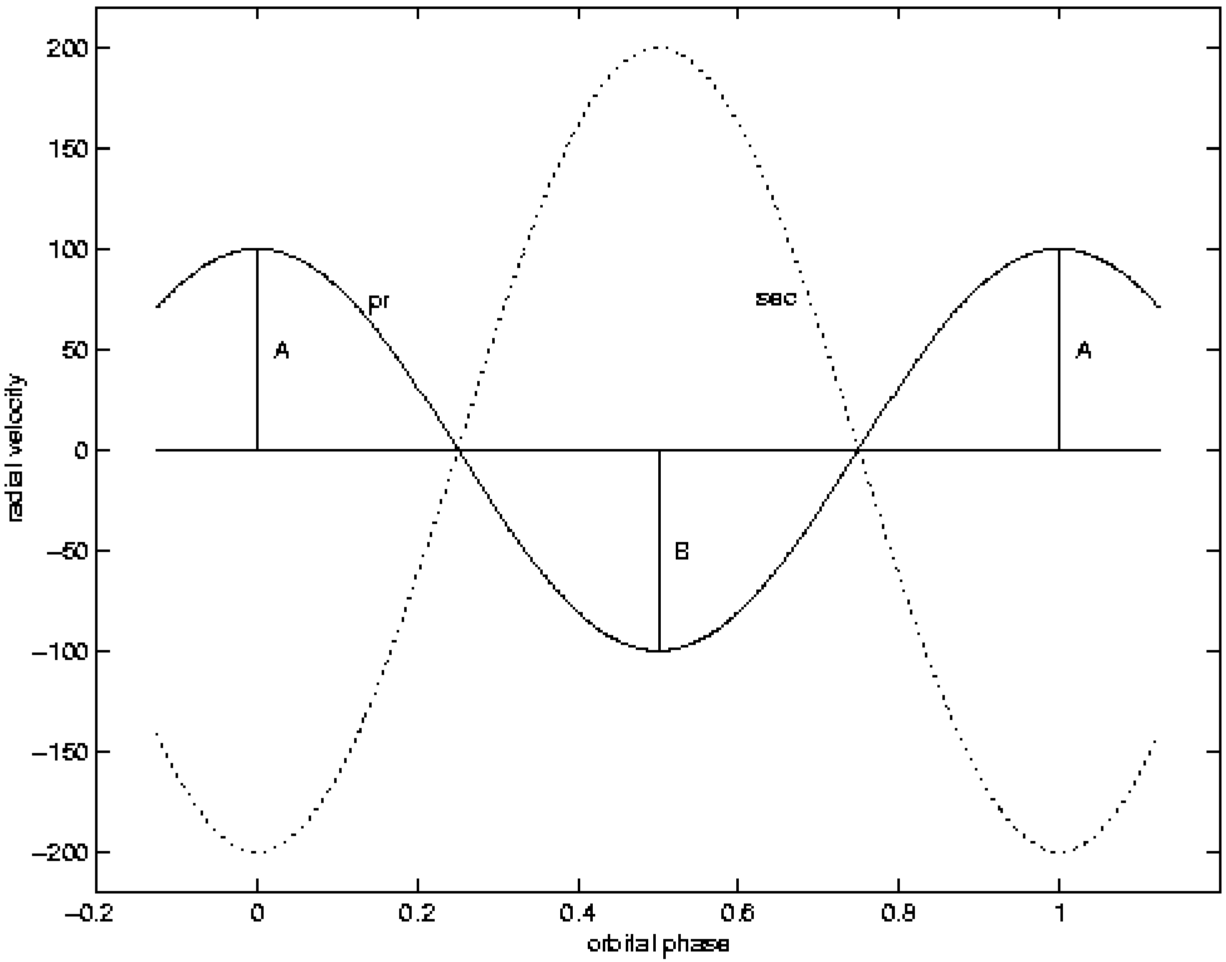
$$\text{descending node}:\ \ \cos(q + w) = -1$$

$$V_{\min} = K\left[e\cos w - 1\right] + g$$

$$\text{semi-amplitude:}$$

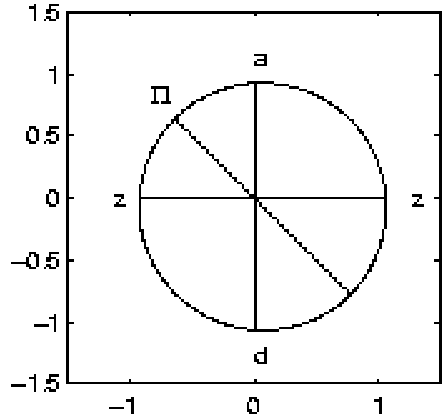
$$K \equiv (V_{\max} - V_{\min})/2$$

Circular Orbit

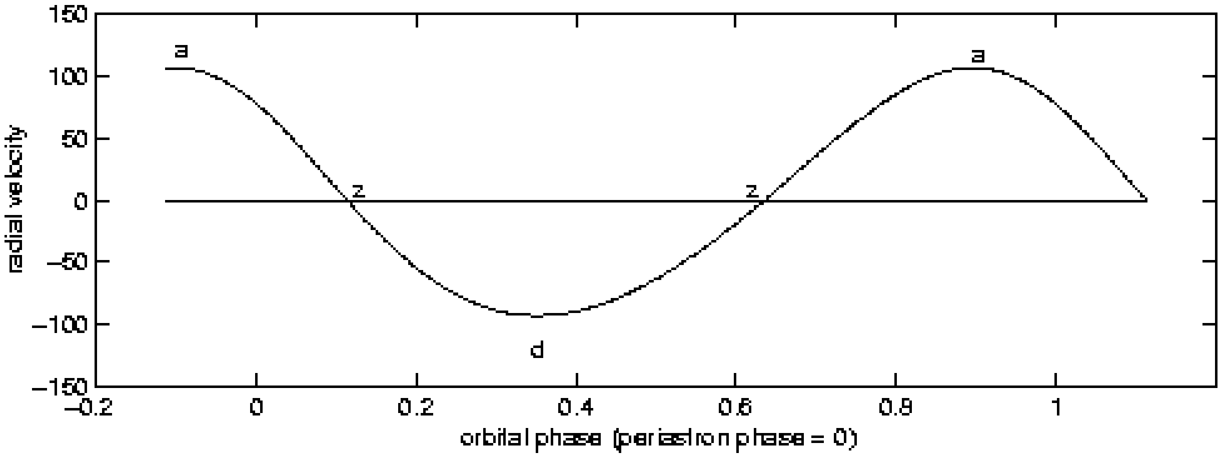


- $K_1 = 100$ **km/s** $K_2 = 200$ **km/s** $q = m_2 / m_1 = 0.5$

Radial Velocities

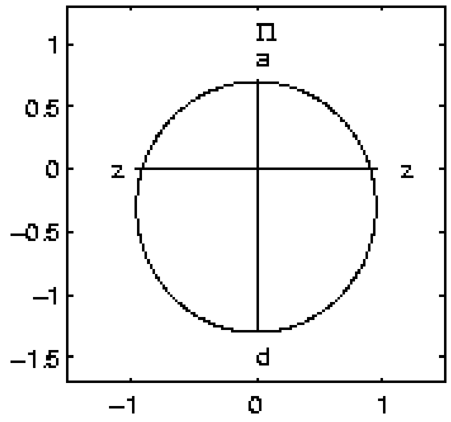


to the observer -->

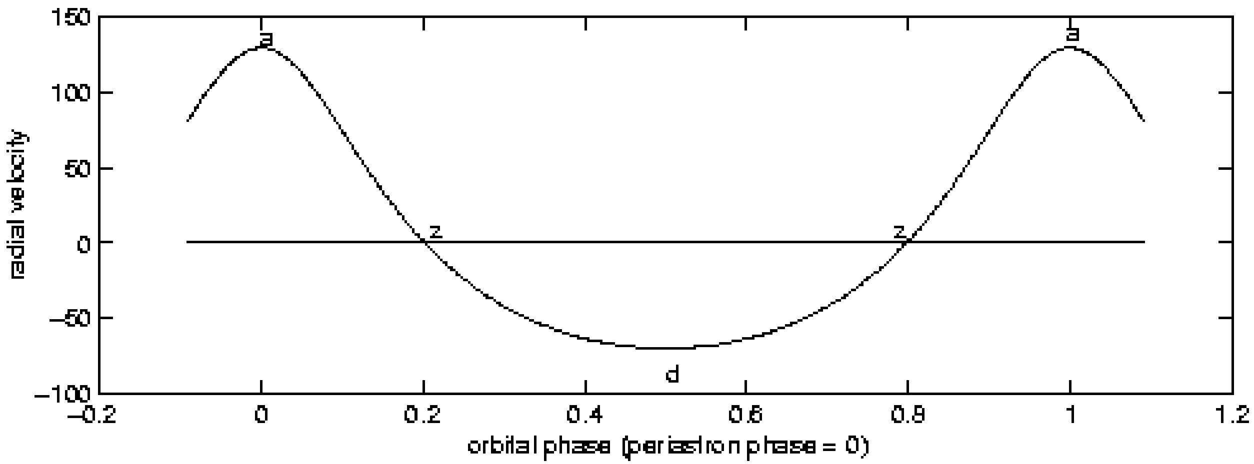


$e = 0.1, \omega = 45^o$

Radial Velocities

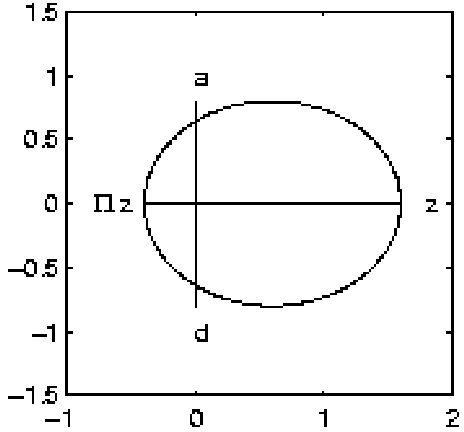


Less time near periastron

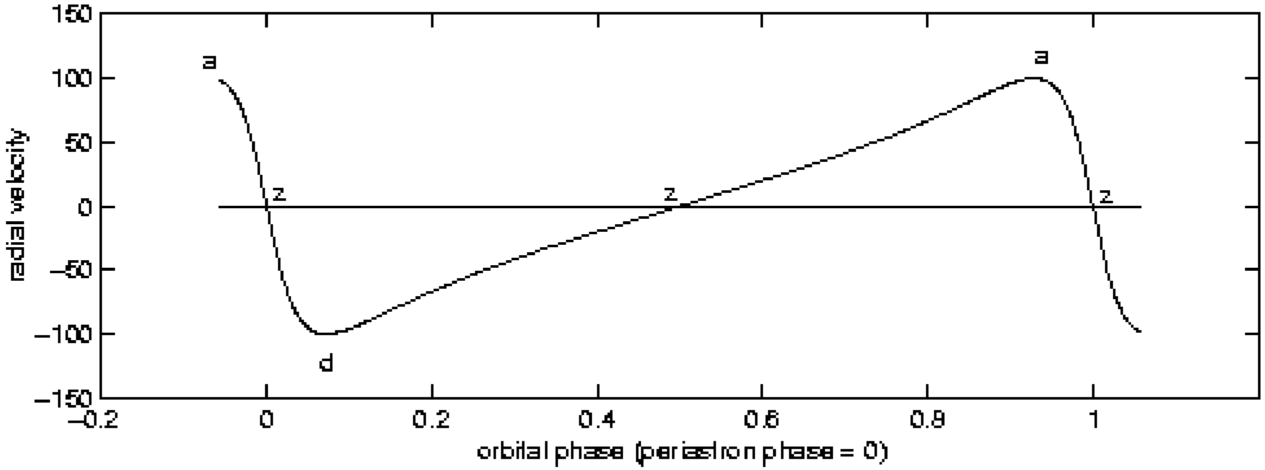


e = 0.3, w= 0º

Radial Velocities

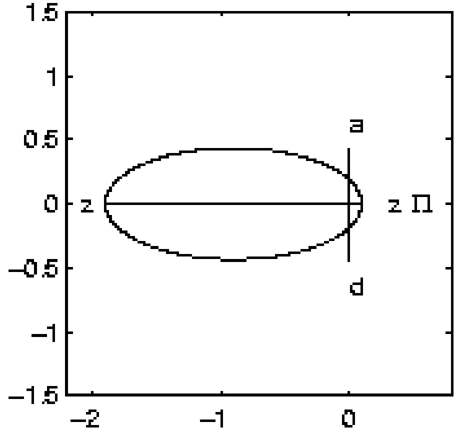


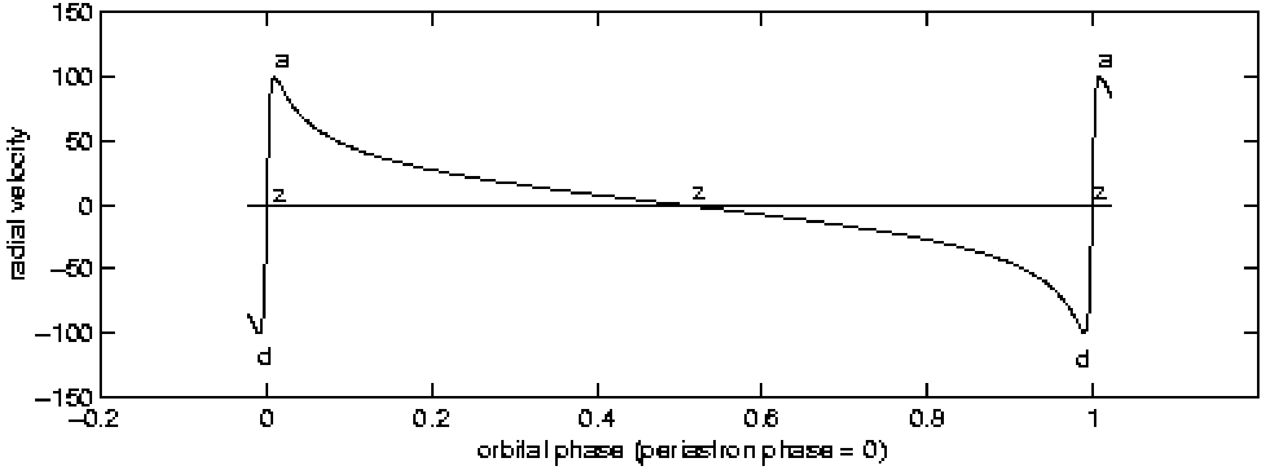
Max /Min velocity at ascending/descending nodes



- ***e=0.6 , w= 90^o***

Radial Velocities





- ***e=0.9 , w= 270º***

Orbits from radial velocities

- **Observations *n* radial velocity measurements:**
  - $V_r(i)$ at times $t(i)$ $i = 1 \ldots n$

- **Elliptical orbit model:**

$$E - e\sin E = \frac{2p}{P}(t - T)$$

$$\tan\left(\frac{q}{2}\right) = \left(\frac{1+e}{1-e}\right)^{1/2} \tan\left(\frac{E}{2}\right)$$

$$V_r = K\left(\cos(q + w) + e\cos w\right) + l$$

- **Best fit determines (*K, e, w, g, T, P*)**
- **e.g. using a least squares procedure**

Minimum masses

$$\text{SB2 : measure } P,\ K_1,\ K_2,\ e$$

$$K_i = \frac{2p\ a_i \sin i}{P\sqrt{1-e^2}} \quad \rightarrow \quad a_i \sin i = \frac{\sqrt{1-e^2}}{2p} K_i\ P$$

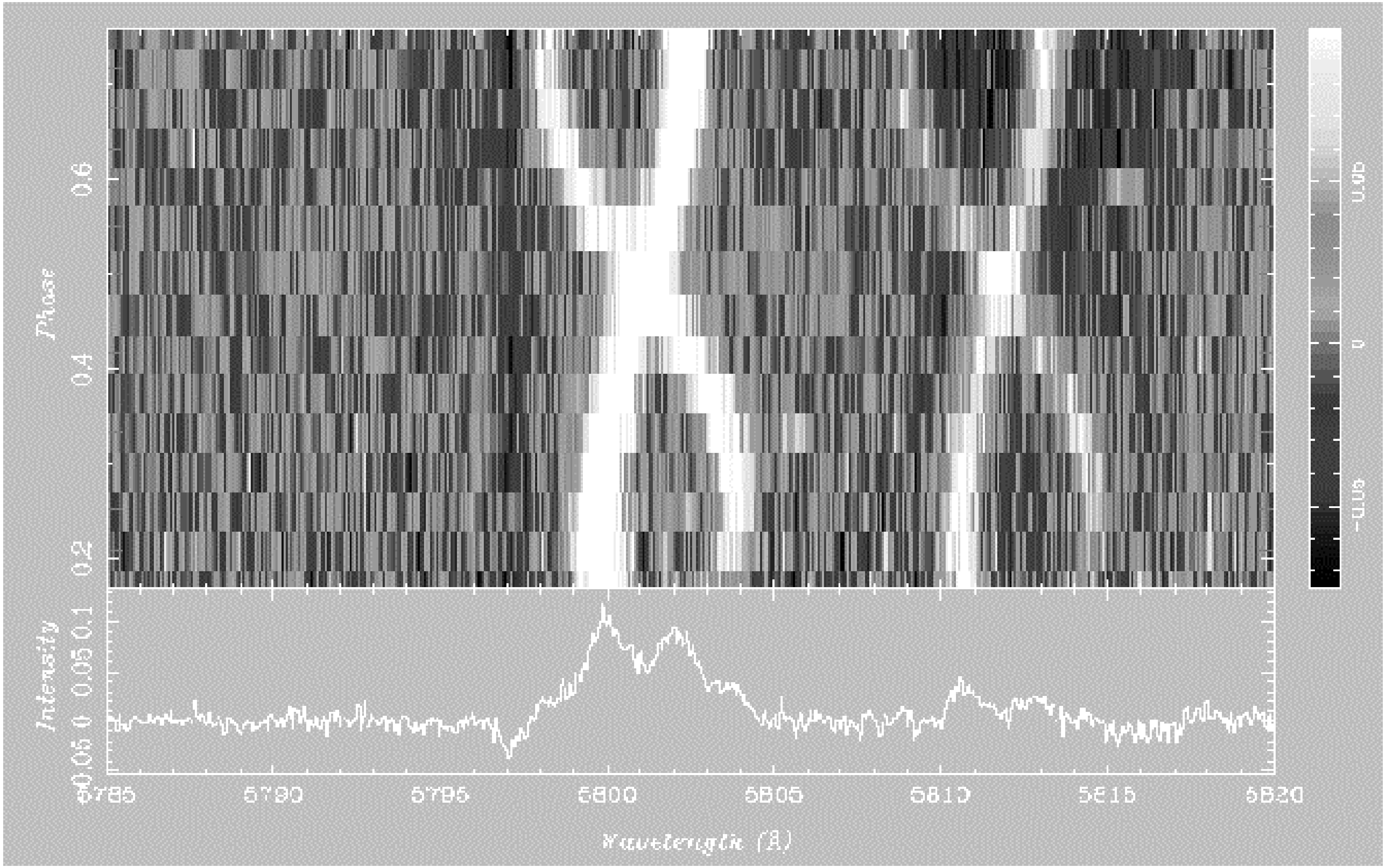
$$m_1 = M\ \frac{a_2}{a} = \frac{4p^2\ a^2\ a_2}{G\ P^2}$$

$$m_1 \sin^3 i = \frac{4p^2 a^2\ a_2\ \sin^3 i}{G\ P^2} = \frac{\left(1-e^2\right)^{3/2} \left(K_1 + K_2\right)^2 K_2\ P}{2p\ G}$$

$$\text{SB1 : measure } P,\ K_1,\ e,\ \text{ calculate the } \underline{\text{mass function}}\ :$$

$$f\,(m_2) = \frac{m_2{}^3 \sin^3 i}{M^2} = \frac{\left(1-e^2\right)^{3/2} K_1{}^3 P}{2p\ G}$$

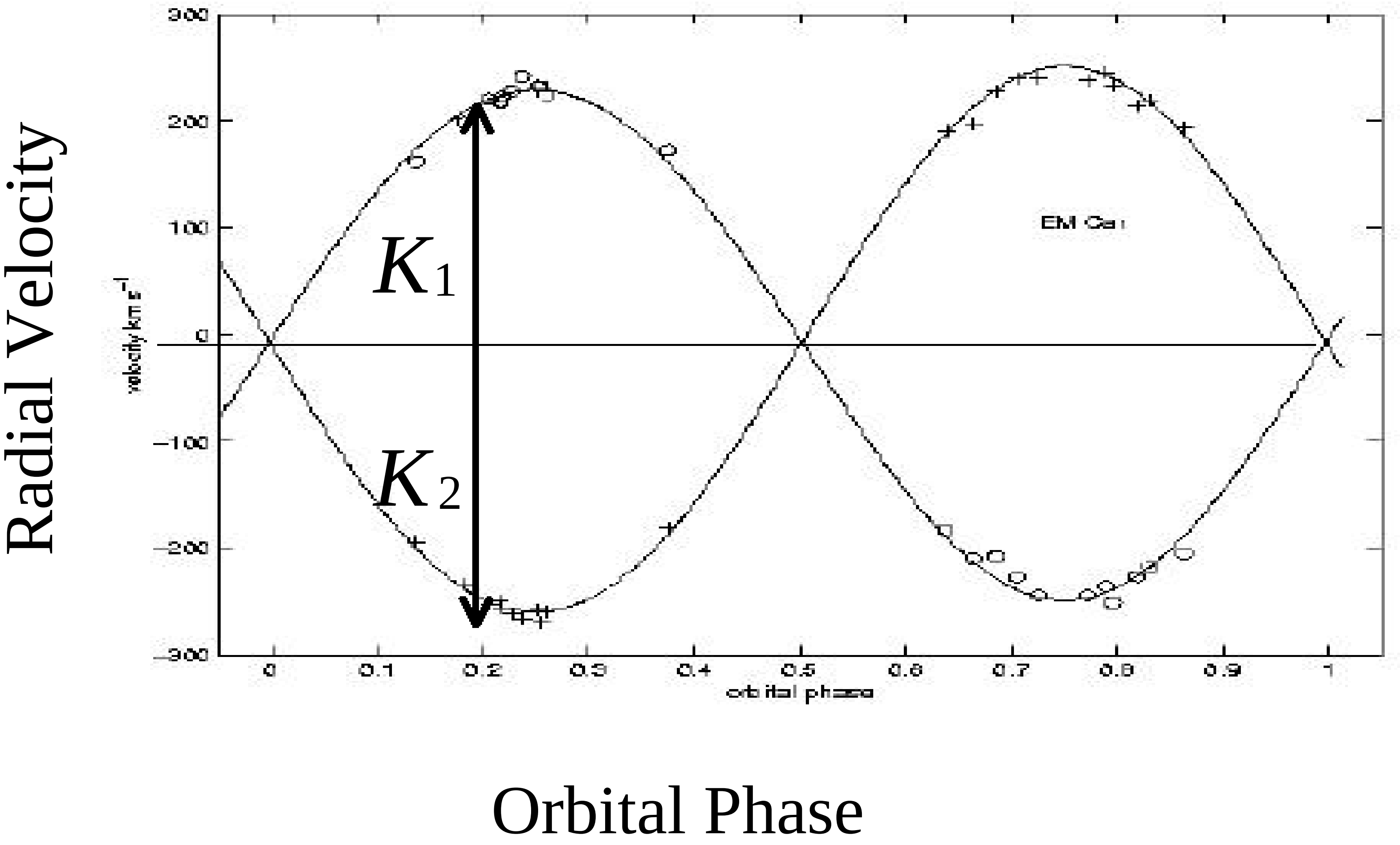
KV Vel (sdO+M)



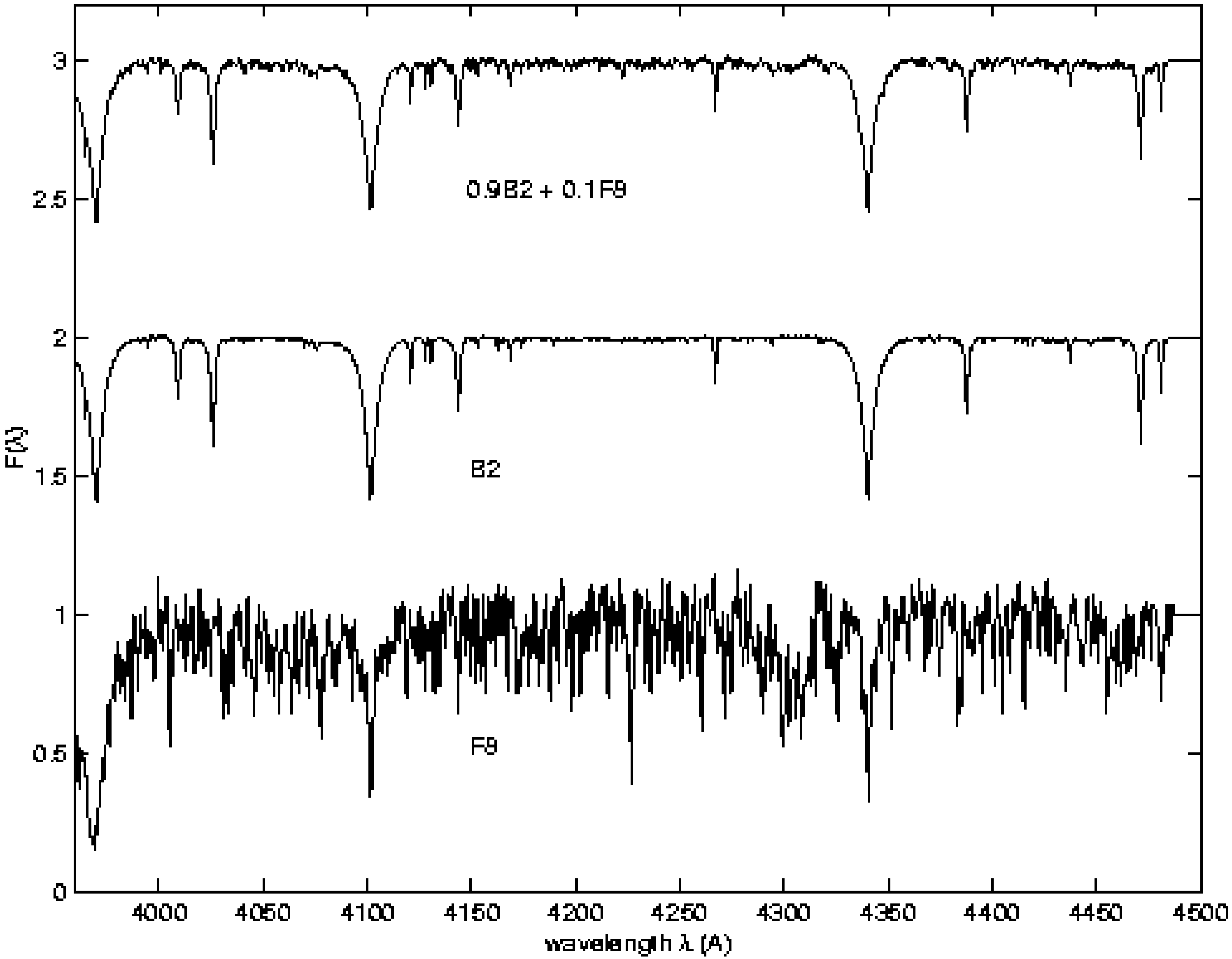
Phase
0.6
0.4
0.2
Intensity
0.1
0.05
0
-0.05
5785
5790
5795
5800
5805
5810
5815
5820
Wavelength (Å)
0.05
0
-0.05

EM Car (O8V+O8V)

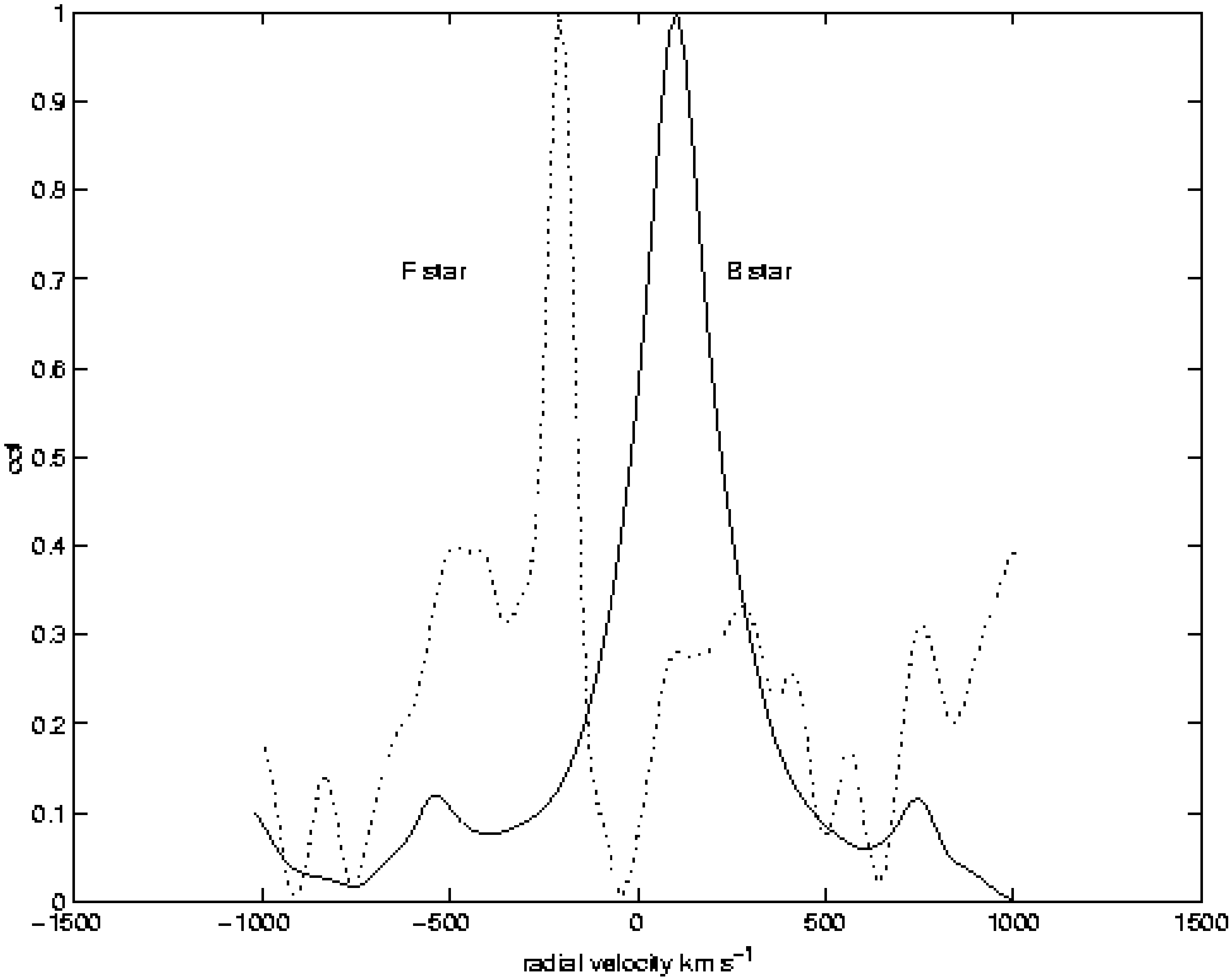
$$\text{observe}: \quad K = V \sin i$$



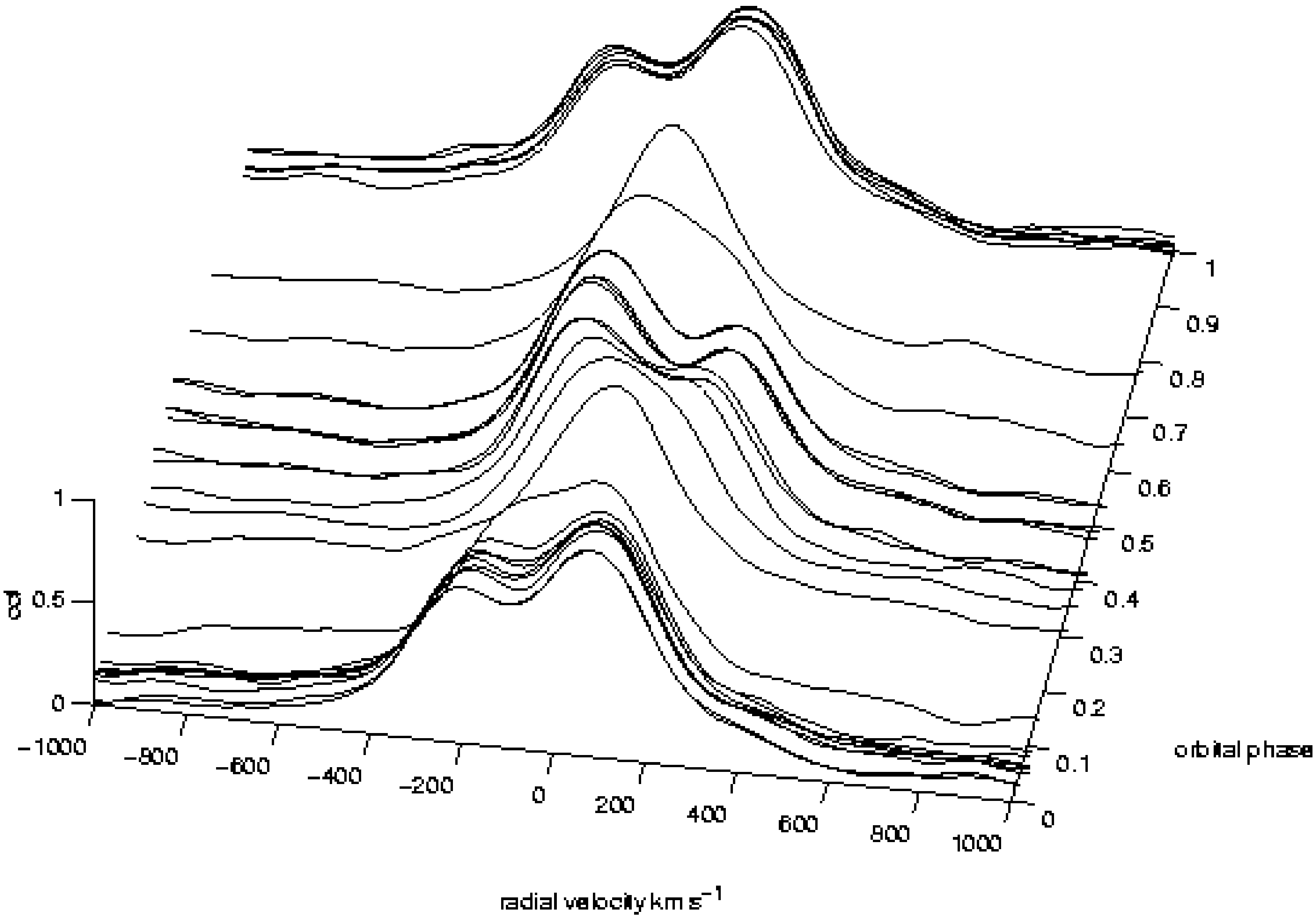
Composite spectrum
(90% B2 + 10% F8)



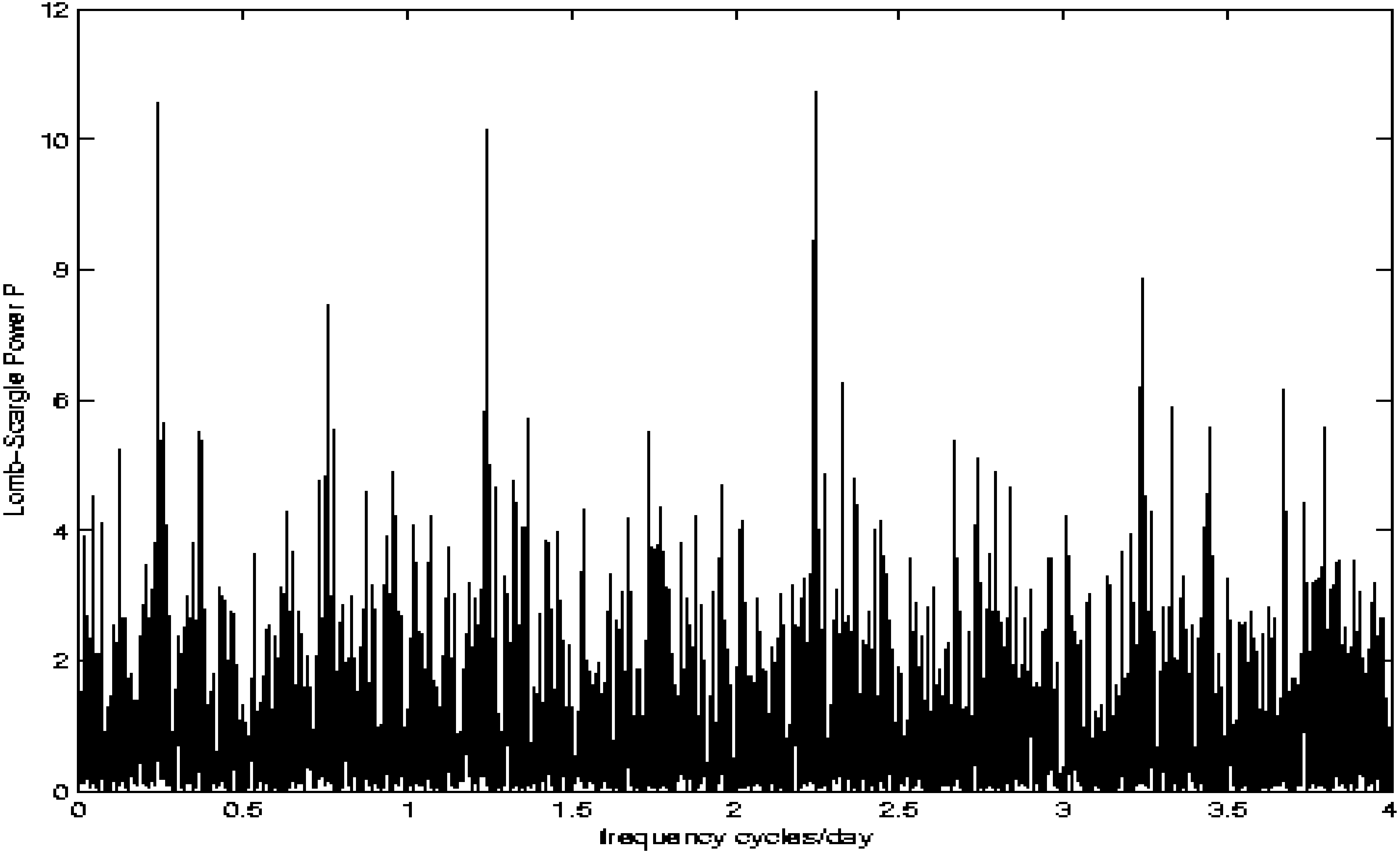
Cross-correlate with B2 and F8 template spectra



LZ Cep (O9.5V + O9.5V)



HR 1165 Periodogram



V1425 Cyg (B5V+B9V)

HR 7000 (F4V+ F6V)