Mass transfer in binary systems

- **Mass transfer occurs when**
  - star expands to fill Roche-lobe
  - due to stellar evolution
  - orbit, and thus Roche-lobe, shrinks till R < R_L
  - due to angular momentum loss
  - e.g. magnetic braking, gravitational radiation

- **Three cases**
  - Case A: mass transfer while donor is on main sequence
  - Case B: donor star is in (or evolving to) Red Giant phase
  - Case C: SuperGiant phase

- **Mass transfer changes mass ratio**
  - changes Roche-lobe sizes
  - can drive further mass transfer

---

**Orbit evolution**

Kepler $a^3 = P^2 M \rightarrow \frac{\dot{a}}{a} = 2 \frac{\dot{P}}{P} \frac{M}{M}$

$M = m_1 + m_2$

orbital angular momentum $J = \frac{m_1 m_2}{M} \left( \frac{2 \pi}{P} \right) \left( 1 - e^2 \right)^{1/2}$

$e = \frac{m_1}{m_2} \left( \frac{1}{2} \right) = 1 \frac{m_1}{m_2} \left( 2 \frac{2 e}{1 - e^2} \right)$

\[
\frac{\dot{a}}{a} = \frac{2}{M} \left( \frac{m_1}{m_2} \right) \frac{\dot{P}}{P} \frac{\dot{e}}{1 - e^2}
\]

Shrinks if $m_1 < m_2$

If $m_1 > m_2$

Period increases

---

**Conservative mass exchange**

\[
\frac{\dot{a}}{a} = \frac{2}{J} \frac{\dot{P}}{P} \left( \frac{m_1}{m_2} \right) \frac{\dot{e}}{1 - e^2}
\]

Circular orbit

Conservative mass exchange:

\[
\frac{\dot{a}}{a} = 0 \quad \frac{\dot{e}}{1 - e^2} = 0 \quad J = 0 \quad m_1 = -m_2 > 0
\]

\[
\frac{\dot{a}}{a} = -2 \left( \frac{m_1}{m_2} \right) \frac{\dot{P}}{P} \left( \frac{m_1}{m_2} \right) > 0
\]

\[
\frac{\dot{P}}{P} \frac{\dot{a}}{a} > 0
\]

Orbit expands

Period increases if $m_1 < m_2$

---

**Roche Lobe size**

Eggleton 1983

\[
R_L = \frac{0.49 q^{1/3}}{a} \left( 0.69 q^{-1/3} + \ln(1 + q^{-1/3}) \right)
\]

Paczynski 0.1 < $q$ < 0.8

\[
R_L = \frac{0.462 q^{1/3}}{1 + q}
\]

Star 2 fills Roche Lobe:

\[
\frac{R}{a} = \frac{2}{3 m_2}
\]

Critical mass ratio:

Lobe shrinks if $q = m_2 / m_1 > 5 / 6$

expands if $q < 5 / 6$

---

**Timescales**

- **Dynamical timescale**
  - time scale for star to establish hydrostatic equilibrium

\[
t_{\text{dyn}} = \frac{R}{G M} \approx 30 \text{ min} \left( \frac{R}{R} \right) \left( \frac{M}{M} \right)^{1/2}
\]

- **Thermal timescale**
  - time scale for star to establish thermal equilibrium

\[
t_{\text{th}} = \frac{G M}{R L} \approx 3 \times 10^7 \text{ yr} \left( \frac{R}{R} \right) \left( \frac{L}{L} \right)
\]

- **Nuclear timescale**
  - time scale of energy source of star
  - i.e. main sequence lifetime

\[
t_{\text{nuc}} = \frac{7 \times 10^9 \text{ yr} \left( \frac{L}{L} \right)}{M}
\]

---

Page 1
Star reacts to mass loss

- Star reacts to mass loss
- Expands/contracts
- Roche-lobe also expands or contracts

Define
- \( \dot{m} \) = \( \frac{d \ln m}{d \ln t} \)
- If \( \dot{m} < \dot{m}_{\text{dyn}} \)
  - Star transfers mass on dynamical timescale
  - Star stripped down too fast to adjust
  - Hydrostatic equilibrium easily maintained
- If \( \dot{m} > \dot{m}_{\text{dyn}} \)
  - Star transfers mass on thermal timescale
  - Stable on thermal timescale
  - Mass transfer due to stellar evolution, nuclear timescale

Mass loss timescales

- For \( M = 0 \), \( \dot{m}_L = 2 \left( \frac{3}{5 - 2q} \right) \)
- Thermal timescale: \( \dot{m}_{\text{dyn}} < \dot{m}_L < \dot{m}_{\text{nuc}} \)
- Nuclear timescale: \( \dot{m}_{\text{nuc}} < \dot{m}_L < \dot{m}_{\text{acc}} \)

\[ R \quad \ln \]
\[ \text{log } M \quad \text{log } t \]

Reaction to mass loss

- Star reacts to mass loss
- Expands/contracts
- Roche-lobe also expands or contracts

Define
- \( \dot{m} = \frac{d \ln m}{d \ln t} \)
- If \( \dot{m} < \dot{m}_{\text{dyn}} \)
  - Star transfers mass on dynamical timescale
  - Star stripped down too fast to adjust
  - Hydrostatic equilibrium easily maintained
- If \( \dot{m} > \dot{m}_{\text{dyn}} \)
  - Star transfers mass on thermal timescale
  - Stable on thermal timescale
  - Mass transfer due to stellar evolution, nuclear timescale

Angular momentum loss

- Magnetic braking
- Gravitational radiation
- Stable q < 5/6
- Donor stays on main-seq
- R - M

\[ \dot{J} < 0 \]
\[ \frac{\dot{m}}{m} = \left( \frac{3 - q}{2q - 3} \right) \]

Cataclysmic variables

- White dwarf primary
- Late K or M donor

- Case A,B,C mass transfer
- Many interacting binaries of this type

\[ t = \frac{R_L}{R_i} = \frac{m_i \left( \frac{5}{3} - 2q \right)}{m} \]

- Orbital expands
- Period increases on nuclear timescale

q > 5/6 -- unstable mass transfer

- Unstable (runaway) mass transfer
- q > 5/6
- Roche lobe shrinks down around the star, stripping it down.
- Rapid (dynamical)
- Violent
- Rare because very fast
- Must occur (more massive stars evolve first)
- Possible example
  - Super-soft x-ray binaries:
    - WD primary accreting from main sequence secondary with q > 5/6
    - Orbit and Roche lobe shrink from orbital angular momentum lost
    - High accretion rate (e.g., 10^{-4} Msun/yr)

q < 5/6 -- stable mass transfer

- q < 5/6
- Conservative mass transfer makes Roche lobe expand.
- Cuts off mass transfer
- Mass transfer if
- 1) Star expands
- 2) Angular momentum lost
  - Winds
  - Gravitational radiation
- Donor star fills Roche lobe
- Case A,B,C mass transfer
- Many interacting binaries of this type

\[ \frac{\dot{J}}{J} = \frac{\dot{m}}{\dot{J}} = \left( \frac{3 - q}{2q - 3} \right) \]