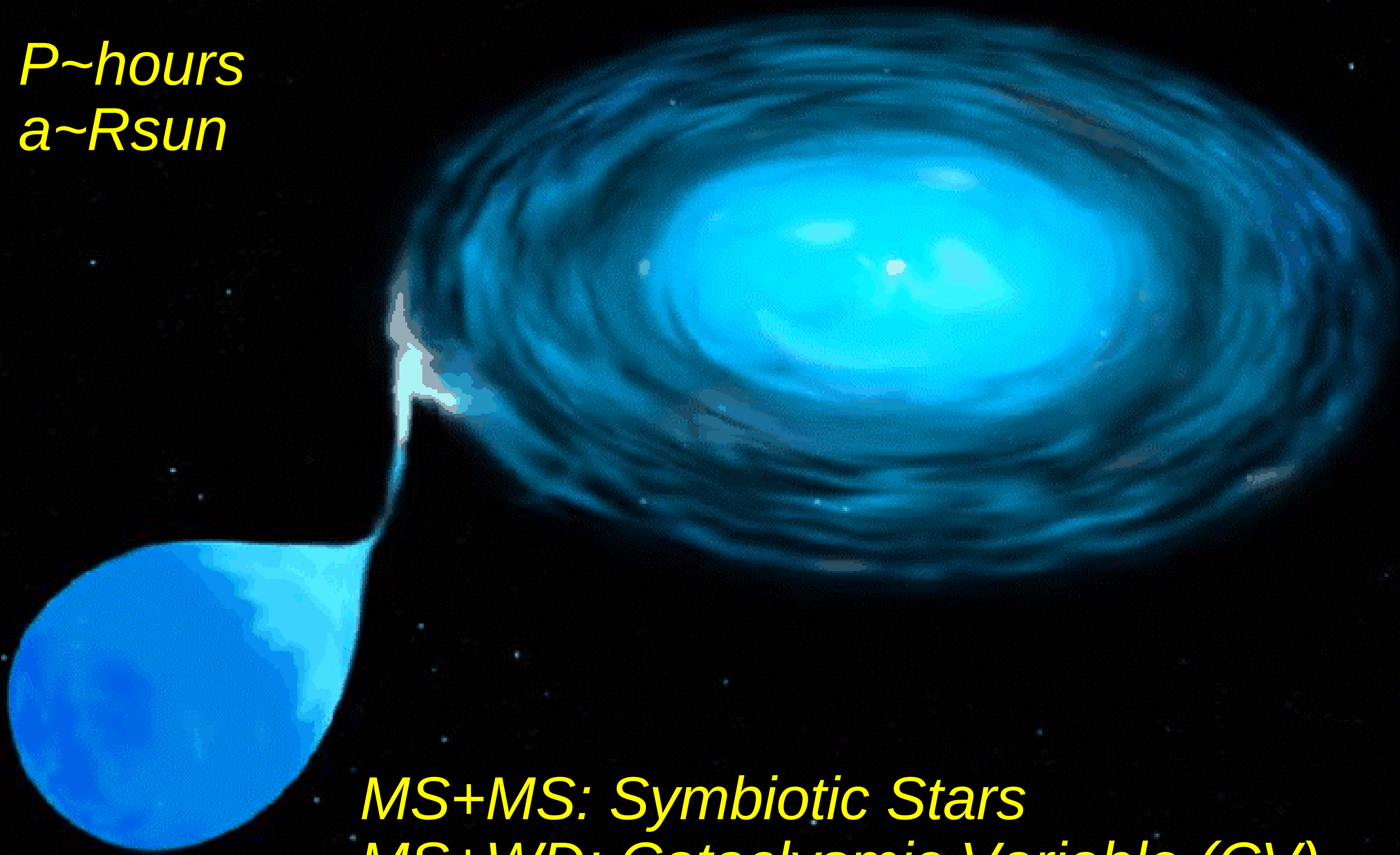
Binary Stars with Accretion Discs

P~hours
a~Rsun



MS+MS: Symbiotic Stars
MS+WD: Cataclysmic Variable (CV)
MS+NS or BH: Low-Mass X-ray Binary (LMXB)

Importance of Accretion Discs

- **Formation of compact objects**
  - **friction moves angular momentum outward**
  - **allows matter to spiral inward**
  - **build up compact object at centre**

- **Generation of light**
  - **gravitational potential energy**
  - **converted by friction to heat**
  - **radiated as light**

3 Classic Papers

- **Black hole accretion discs**

  Shakura, Sunyaev 1973 A&A 24 337

- **Time-dependent discs**

  Pringle, Lynden-Bell 1974 MNRAS 168 603

- **Gas streams**

  Lubow, Shu 1975 ApJ 198 383

Accretion Energy

- **Nuclear energy (H -> He)** **Efficiency:**

$$\frac{\Delta E_{nuc}}{m} \approx 6\times 10^{14}\ \frac{\mathrm{J}}{\mathrm{kg}}$$

$$h = \frac{\Delta E_{nuc}}{m\,c^2} = 0.7\%$$

- **Accretion energy**

$$\frac{\Delta E_{acc}}{m} \approx \frac{G\,M}{R} \approx 1.7\times 10^{11} \left(\frac{M}{M_{sun}}\right)\left(\frac{R}{R_{\mathrm{sun}}}\right)^{-1} \frac{\mathrm{J}}{\mathrm{kg}}$$

$$\approx 1.7\times 10^{16} \left(\frac{M}{1.4\ M_{sun}}\right)\left(\frac{R}{10\ \mathrm{km}}\right)^{-1} \frac{\mathrm{J}}{\mathrm{kg}}$$

- **more efficient for compact objects**

Efficiency

- **Nuclear energy (H -> He)** **Efficiency:**

$$\Delta E_{nuc} = h \; m \; c^2 \qquad h = 0.7\%$$

- **Accretion efficiency**

$$L \approx \frac{G \; M \; \dot{M}}{R} = h \; \dot{M} \; c^2 \qquad h = \frac{G \; M}{R \; c^2} = \frac{R_H}{2 \; R} \qquad R_H \approx 3 \text{ km} \left(\frac{M}{M_{sun}} \right)$$

- **Compactness --- M / R**
- **Black hole - smallest stable orbit**
  - non-rotating

$$R = 6 \; R_H \qquad h = \frac{1}{12} \approx 8\%$$

  - maximally-rotating

$$R = 3 \; R_H \qquad h = \frac{1}{6} \approx 17\%$$

- **White Dwarf**

$$M \approx M_{sun} \qquad R \approx 10^4 \text{ km} \qquad h \sim 10^{-4}$$

Radiation Opposes Gravity

- **Coulomb attraction ties electrons and protons**
- **gravity pulls in**

$$F_{grav} = -\frac{G\,M\,m}{r^2}$$

- **radiation pushes electrons out**

$$F_{rad} = \frac{L\,\sigma_T}{4\pi\,r^2\,c} \qquad L_{acc} = -\frac{G\,M\,\dot{M}}{R}$$

Thompson electron scattering cross-section

$$\sigma_T = 6.7\times10^{-21}\,\mathrm{m}^2$$

- **Notes:**
  - **radiation pressure opposes gravity**
  - **same R scaling**
  - **radiation pressure > gravity at high Mdot**

Eddington Luminosity

- **Photon momentum**

$$p_\gamma = \frac{h\nu}{c}$$

Density of photons

$$n_\gamma = \frac{L / h\nu}{4\pi R^2 c}$$

- **Radiative force (per electron)**

$$F_{rad} = n_\gamma \; p_\gamma \; \sigma_T \; c = \frac{L \, \sigma_T}{4\pi R^2 c}$$

$$\sigma_T = 6.7 \times 10^{-25}\,\mathrm{cm}^2$$

- **Total force (per electron + proton pair)**

$$F_{rad} + F_{grav} = \frac{L \, \sigma_T}{4\pi R^2 c} - \frac{G M \left(m_p + m_e \right)}{R^2}$$

- **Eddington Luminosity**

$$L_{Edd} = \frac{4\pi G M \left(m_p + m_e \right) c}{\sigma_T} = 1.3 \times 10^{38} \left(\frac{M}{M_{sun}} \right) \mathrm{erg\ s}^{-1}$$

Eddington Accretion Rate

- **Eddington Luminosity**

$$L_{Edd} = \frac{4\pi\, G\, M\, m_p\, c}{\sigma_T}$$

- **Accretion Luminosity**

$$L_{acc} = \frac{G\, M\, \dot{M}}{R}$$

- **Eddington Accretion Rate**

$$\frac{L_{acc}}{L_{Edd}} = \frac{\sigma_T\, \dot{M}}{4\pi\, m_p\, c\, R} = \frac{\dot{M}}{\dot{M}_{Edd}}$$

$$\dot{M}_{Edd} = \frac{4\pi\, m_p\, c\, R}{\sigma_T} \approx 10^{-5} \left(\frac{R}{R_{sun}} \right) \frac{M_{sun}}{\text{yr}}$$

	kg/s	M_{sun}/yr
WD	6×10^{19}	10^{-7}
NS	10^{12}	1.4×10^{-12}

Rough Temperatures

- **Optically thick**
  - **Blackbody radiates accretion luminosity**

$$\sigma T_b^4 = \frac{L_{acc}}{4\pi R^2} \qquad T_b = \left(\frac{L_{acc}}{4\pi R^2 \sigma} \right)^{1/4} \approx \left(\frac{G M \dot{M}}{4\pi R^3 \sigma} \right)^{1/4}$$

- **Optically thin**
  - **potential energy released**
  - **= thermal energy of shocked gas**

$$\frac{G M m_p}{R} = 2 \times \frac{3}{2} k T_{th} \qquad T_{th} \approx \frac{G M m_p}{3 k R}$$

- **Radiation temperature and photon energy**

$$T_b < T_{rad} < T_{th} \qquad k T_b < h\nu < k T_{th}$$

Rough Temperatures

- **Neutron Star or Black Hole**

$$L_{acc} \sim L_{Edd} \sim 10^{38}\,\mathrm{erg\,s^{-1}}$$

$$10^{7}\,\mathrm{K} < T_{rad} < 10^{11}\,\mathrm{K}$$

$$1\,\mathrm{keV} < hn < 50\,\mathrm{MeV}$$

  – **mid to hard X-rays**

- **White Dwarf**

$$L_{acc} \sim 10^{33}\,\mathrm{erg\,s^{-1}}$$

$$6\times10^{4}\,\mathrm{K} < T_{rad} < 10^{9}\,\mathrm{K}$$

$$6\,\mathrm{eV} < hn < 100\,\mathrm{keV}$$

  – **optical - uv - soft X-ray**

Roche Lobe Overflow

- **Initial stream velocity**

$$V_{\parallel} \sim c_S \sim 10 \text{ km s}^{-1}$$

- **L1 velocity relative to centre of mass**

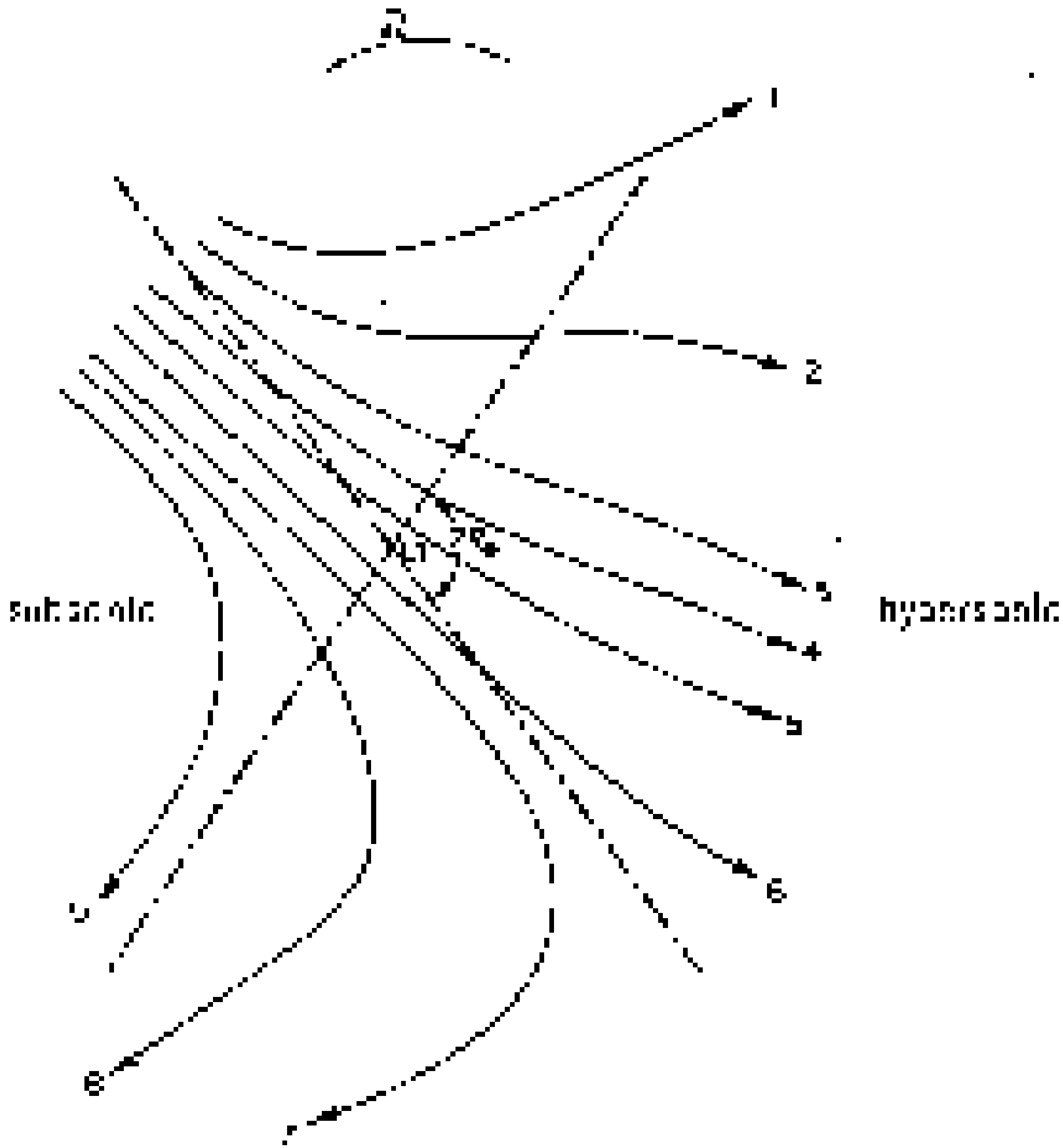
$$V_{\perp} = \frac{2p}{P} R_{L1} \approx \frac{2p\ a}{P} (0.5 - 0.23 \log q)$$

$$\sim 100 \text{ km s}^{-1} \left(\frac{M}{M_{sun}} \right)^{1/3} \left(\frac{P}{\text{day}} \right)^{-1/3}$$

- **subsonic --> supersonic transition at nozzle**
- **ballistic trajectory in Roche potential**
  - **(neglect pressure forces)**

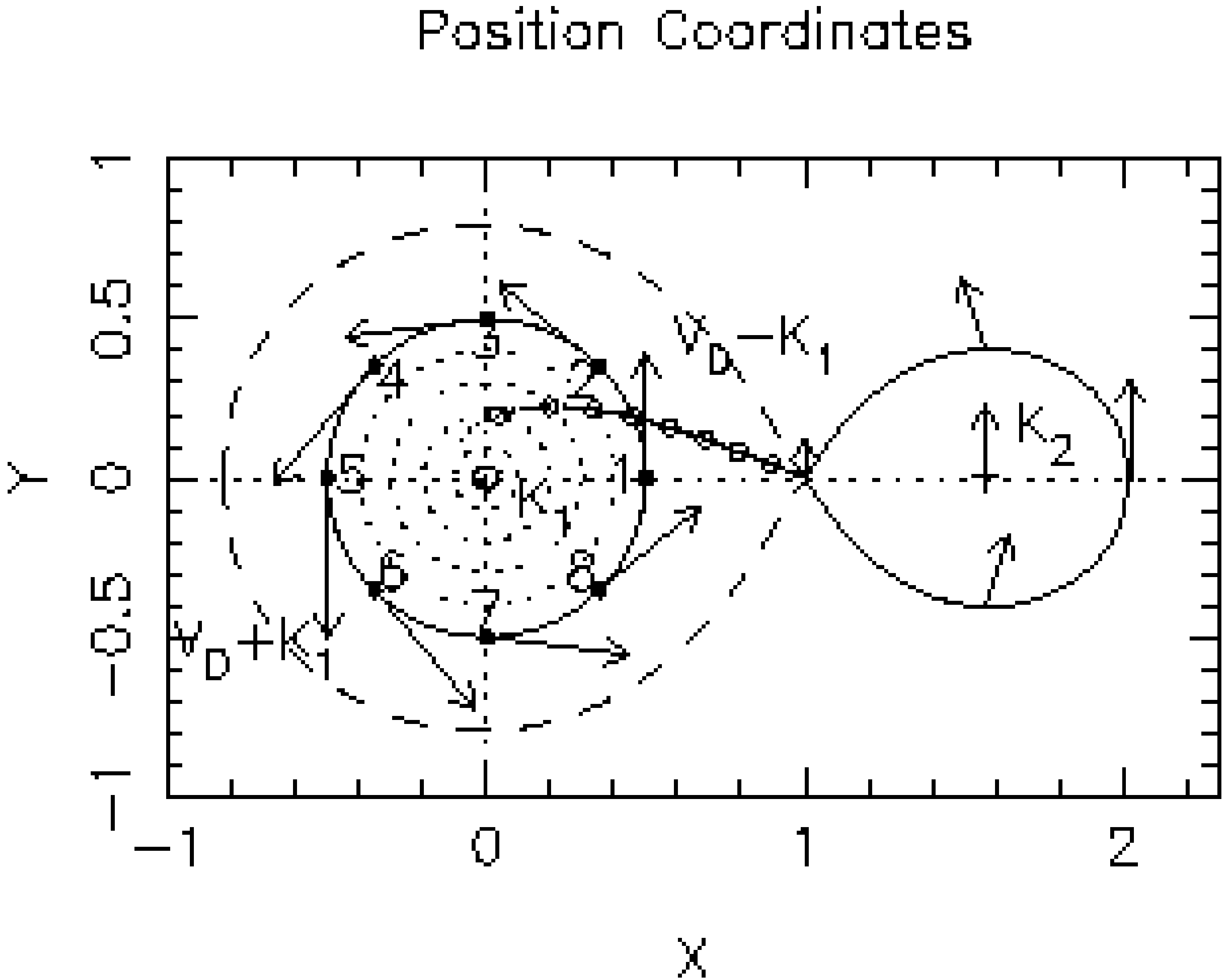
Flow thru the L1 nozzle

subsonic -> hypersonic

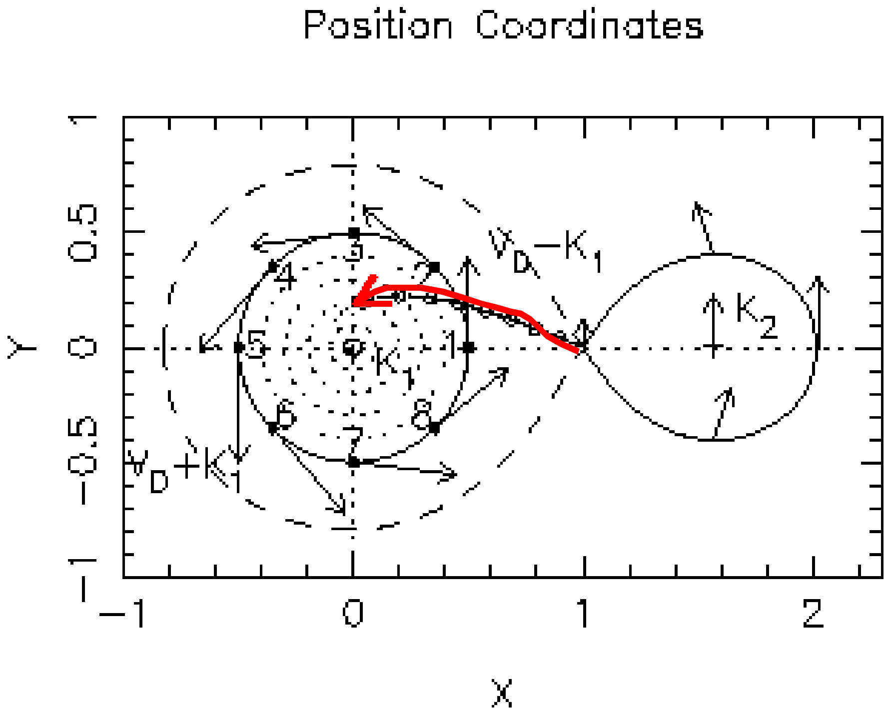


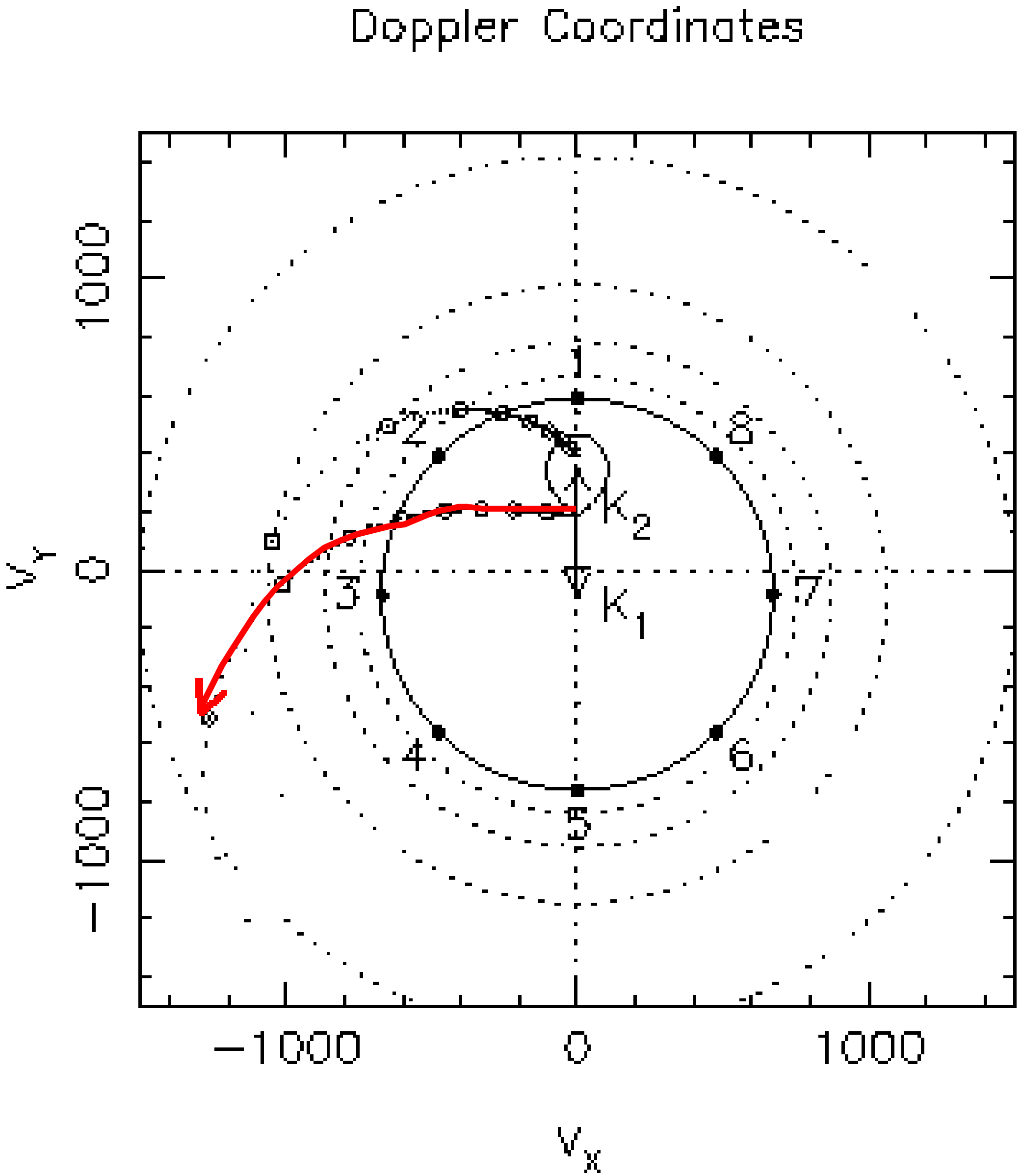
Lubow, Shu 1975 ApJ 198 383

Balistic Stream Trajectory

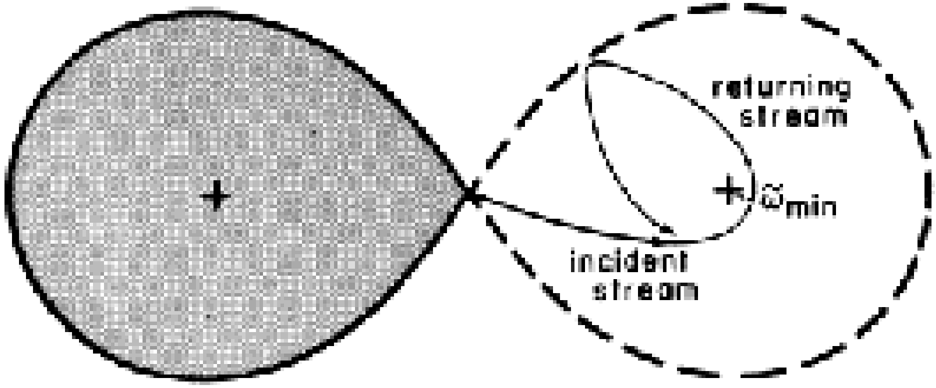


Balistic Stream Trajectory

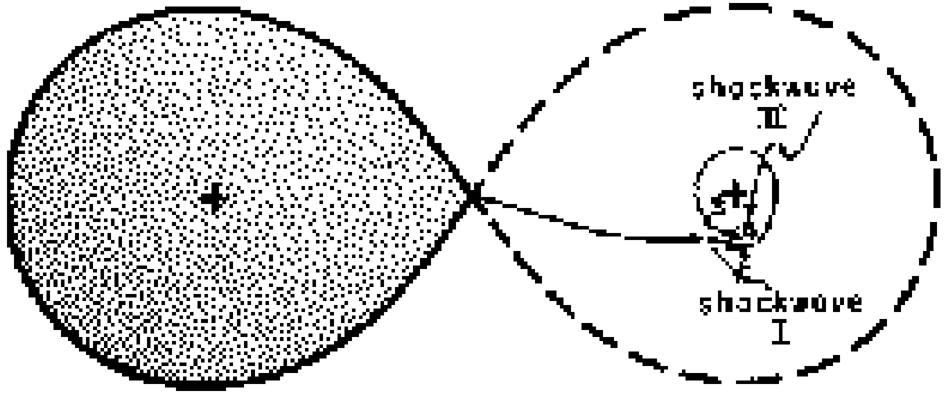




Formation of a Ring



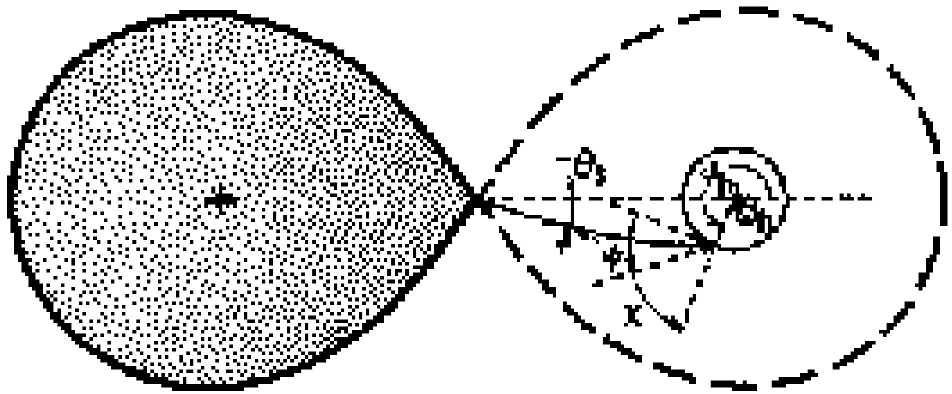
(a)



(b)

Circularisation Radius

same angular momentum as L1



(c)

$$\left(G\ M\ \ R_{circ} \right)^{1/2} = \frac{2p}{P}\, R_{L1}{}^{2}$$

$$\frac{R_{circ}}{a} = \left(1+q\right)\left(\frac{R_{L1}}{a}\right)^{4} \approx \left(1+q\right)\left(0.5-0.23\log q\right)^{4}$$

Friction -> Spreading

- **Kepler Velocity**

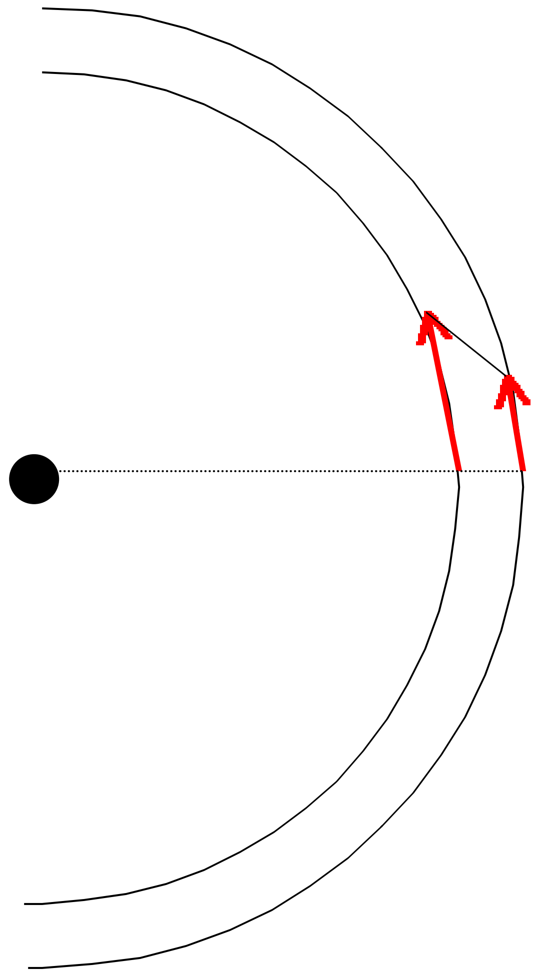
$$V = \sqrt{\frac{G\,M}{R}}$$

- **Differential rotation (Shear)**

$$\Delta V = \frac{d}{dR}\left(\sqrt{\frac{G\,M}{R}}\right)\Delta R$$

$$= \frac{V}{2}\,\frac{\Delta R}{R}$$

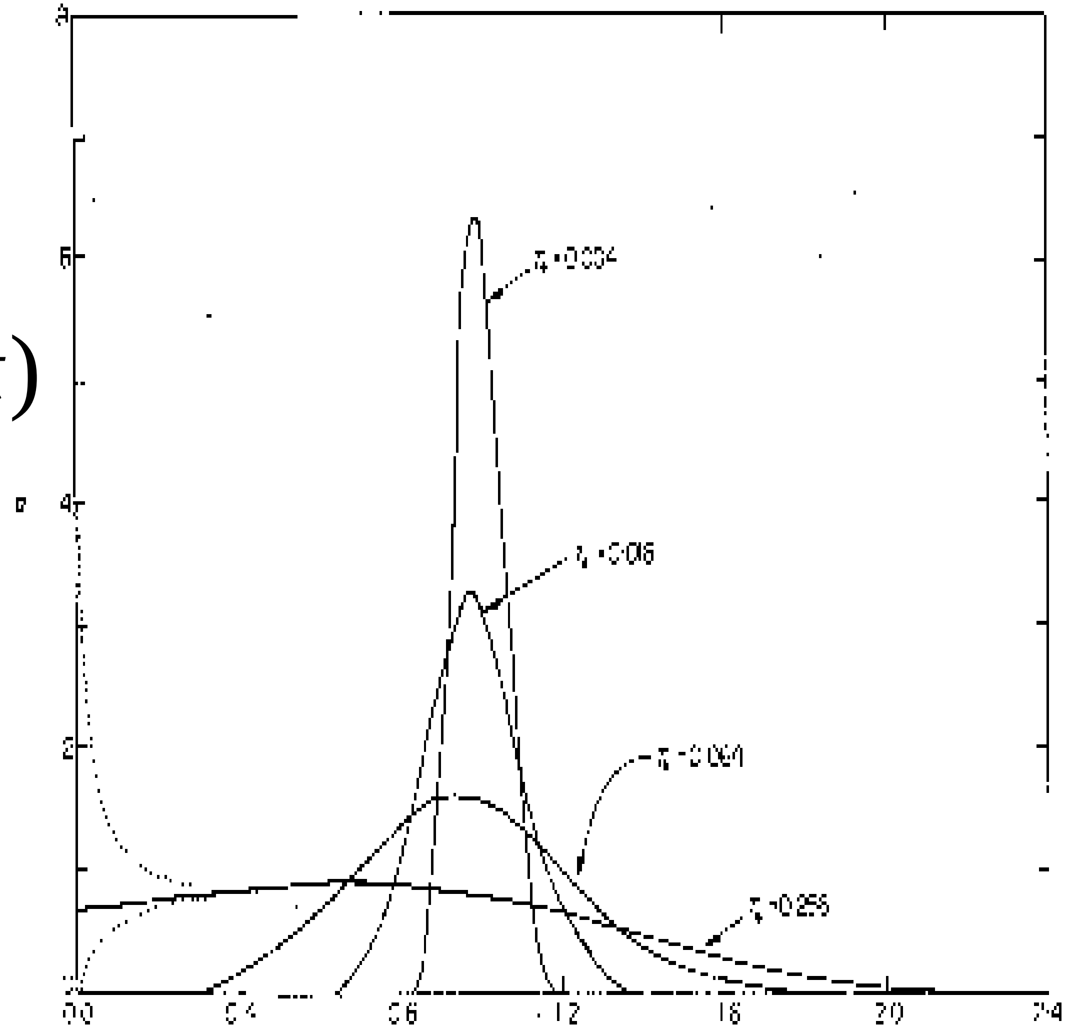
- **Friction**
  - opposes shear
  - causes ring to spread
  - inward + outward
  - diffusion



Ring spreads to form a disc

Pringle, Lynden-Bell 1974
MNRAS 168 603

$$\Sigma(R,t)$$



$$R$$

Surface density evolution

diffusion in radius

Accretion Discs

- **Keplerian Orbits**

$$V_f = V_K = \left(\frac{G\ M}{R} \right)^{1/2} \qquad \Omega \equiv \frac{V_f}{R} = \sqrt{G\ M\ R^3}$$

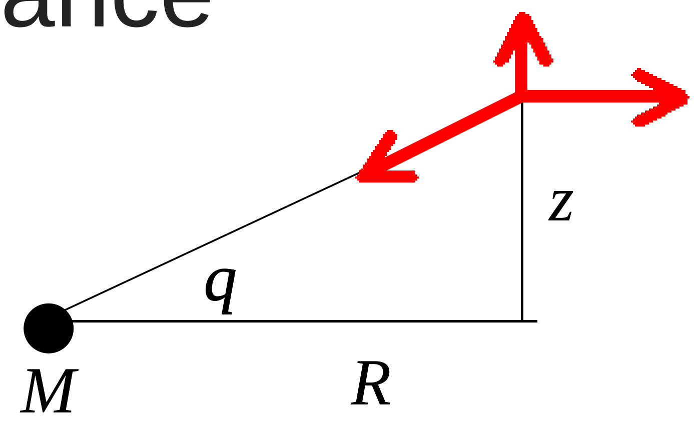
- **Vertical thickness: thin <--> supersonic**

$$\frac{H}{R} \approx \frac{c_S}{V_f}$$

- **sound speed for perfect gas**

$$c_S{}^2 \equiv \frac{dP}{dr} = g\ \frac{P}{r} = \frac{g\ k\ T}{m\ m_H}$$

Force Balance



- **horizontal:**
  - gravity in
  - centrifugal out

$$\frac{V^2}{R} = \frac{G\,M}{R^2 + z^2}\cos q = \frac{G\,M\,R}{\left(R^2 + z^2\right)^{3/2}} \approx \frac{G\,M}{R^2} \qquad \left(z << R \right)$$

- **vertical:**
  - gravity down
  - pressure gradient up

$$\frac{dP}{dz} = -r\; g_z$$

$$g_z = \frac{G\,M}{R^2 + z^2}\sin q = \frac{G\,M\,z}{\left(R^2 + z^2\right)^{3/2}} \approx \frac{G\,M\,z}{R^3}$$

Vertical Hydrostatic Equilibrium

- **Assume vertically isothermal**
- **vertical structure (Gaussian if iso-thermal)**

$$\frac{dP}{dz} = -r \; g_z = -\frac{g \; P}{{c_S}^2} \frac{G \; M \; z}{R^3} = -\frac{P \; z}{H^2}$$

$$\frac{dP}{P} = -\frac{z \; dz}{H^2}$$

$$\ln P = \ln P_0 - \frac{1}{2}\left(\frac{z}{H}\right)^2$$

$$P = P_0 \exp\left\{-\frac{1}{2}\left(\frac{z}{H}\right)^2\right\}$$

$$H^2 = \frac{{c_S}^2 \; R^3}{g \; G \; M} = \frac{k \; T \; r^3}{m \; m_H \; G \; M}$$

$$\frac{H^2}{R^2} = \frac{1}{g} \frac{{c_S}^2}{{V_f}^2}$$