

Tutorial Questions for AS4022 [2012 version]

Q1. Argue about  $10^5$  photons fit in a 10cm x 10cm x 10cm microwave oven. [Hint:  $3kT = h c / \lambda$ ] Show the approximate solutions  $R(t)$  of the Friedman equation  $(dR/dt/R)^2 = A R^{-n}$  where  $n=4$  for radiation,  $n=3$  for matter, and  $n=2$  (negative) curvature, and  $n=0$  for vacuum energy. Show that Einstein's static universe has formally infinite Omegas. [due 28Feb]

Q2. A baby universe is initially at time  $t_i=10^{-40}$  sec curved with an Omega = 1.10 (i.e., there is 10% more energy density than the critical energy density for a flat universe). If this toy universe expands first under certain energy density  $\rho \sim R^{-1}$  from time  $t_i$  to time  $t_f=1$ sec, and then expands normally under radiation with  $\rho \sim R^{-4}$  from  $t_f=1$ sec until its 1-year birthday  $t_b = 1$  year. Show at its birthday, the baby universe is no longer curvature-dominated. Prove that in the earliest era (with  $n=1$ ) this universe satisfies the thermo-dynamical law  $PdV = -dE$ , where  $V=R^3$ ,  $E=V \rho c^2$ ,  $P = -2/3 \rho c^2$ . [due 28Feb]

Q3. Adopt the standard concordance division of energy (0.7:0:3:0.0001:-0.0001 being the Omegas of vacuum:matter:radiation:curvature) and a cosmic age of 13Gyrs. Estimate the fraction of the time of the universe that the radiation dominates or the vacuum dominates. [due 28Feb]

Estimate the fraction of the time the CMB temperature is high enough to ionise hydrogen. [Hint: - 13.6eV is the energy for the ground state of hydrogen]. [due 6Mar]

Q4. Consider a micro-cosmos of  $N$ -ants inhabiting an expanding spherical surface of radius  $R=R_0 (t/t_0)^a$ , where presently we are at  $t=t_0 = 1$ min,  $R=R_0 = 1$ lightsecond. Let  $a=1/2$ ,  $N=100$ . What is the present rate of expansion  $dR/dt/R =$  in units of 1/min? How does the ant surface density change as function of cosmic time? [due 6Mar]

Light emitted by ant-B travels a half circle and reaches ant-A now, what redshift was the light emitted? What is the angular diameter distance to the emission redshift? [due 10Apr]

Let each ant conserve its random angular momentum per unit mass  $J=(1\text{lightsecond})*(1\text{m/min})$  with respect to the centre of the sphere. Estimate the age of universe when the ants were moving relativistic. How far has ant B travelled since the emission and since the beginning of the universe? [due 24Apr]

Q5. Derive the time-redshift relation; Do a Taylor expansion of the angular diameter distance and luminosity distance at low  $z$ . Use the Friedman equation to argue that in a universe made purely of normal matter has a negative  $d^2R/dt^2$ . [due 10Apr]

Q6. For a coupled radiation-matter fluid, the sound speed  $C_s^2 = c^2/3/(1+Q)$ ,  $Q = (3 \rho_m) / (4 \rho_r)$ , show the sound speed  $C_s$  drops from  $c/\text{sqrt}(3)$  at radiation-dominated era to  $c/\text{sqrt}(5.25)$  at matter-radiation equality. Estimate the sound horizon up to the time of CMB. [due 10Apr]

Q7. The edge of void is lined up by galaxies. What direction is their peculiar gravity and peculiar motion? A patch of void is presently cooler in CMB by 3micro Kelvin than average. How much was it cooler than average at the last scattering ( $z=1000$ )? Argue that a void in universe now originates from an under-dense perturbation at  $z=10^{10}$  with  $\delta$  about  $10^{-17}$ . [due 24Apr]

Q8. Summarize the evidences for concordance cosmology. Show that a light ray grazing a spherical galaxy cluster of  $10^{14}$  Msun at typical impact parameter  $b=100$  kpc scale will be bent  $\sim 4GM/b/c^2$  radian  $\sim 100$  arcsec. Show the angular diameter distance to a lensing arc of size 100 kpc and angular size 100 arcsec is about  $10^9$  light years. [due 24Apr]