MOND & Retarded Gravity

Asher Yahalom
Ariel University, Ariel 40700, Israel

MOND40
5th - 9th June 2023, St Andrews, Scotland, UK

Contents

Bibliography

1. Asher Yahalom "Retardation effects in gravitation and electromagnetics" Proceedings of the 10th International Conference on Materials, Technologies and Modelling (MMT-2018) Ariel, Israel, August 20-24, 2018.
2. Yahalom, A. The effect of Retardation on Galactic Rotation Curves. J. Phys.: Conf. Ser. 1239 (2019) 012006.
3. Yahalom, A. Dark Matter: Reality or a Relativistic Illusion? In Proceedings of Eighteenth Israeli-Russian Bi-National Workshop 2019, The Optimization of Composition, Structure and Properties of Metals, Oxides, Composites, Nano and Amorphous Materials, Ein Bokek, Israel, 17–22 February 2019.

Bibliography

4. Asher Yahalom "Lorentz Symmetry Group, Retardation, Intergalactic Mass Depletion and Mechanisms Leading to Galactic Rotation Curves" Symmetry 2020, 12(10), 1693; https://doi.org/10.3390/sym12101693
5. Yahalom, A. Lensing Effects in Retarded Gravity. Symmetry 2021, 13, 1062. https://doi.org/10.3390/sym13061062. https://arxiv.org/abs/2108.04683.
6. Yahalom A. Effects of Higher Order Retarded Gravity. Universe. 2021; 7(7):207. https://doi.org/10.3390/universe7070207. https://arxiv.org/abs/2108.08246

Bibliography

7. A. Yahalom "Tully - Fisher Relations and Retardation Theory for Galaxies" International Journal of Modern Physics D, (2021), Volume No. 30, Issue No. 14, Article No. 2142008 (8 pages). © World Scientific Publishing Company. https://doi.org/10.1142/S0218271821420086 , https://arxiv.org/abs/2110.05935 .
8. A. Yahalom "The weak field approximation of general relativity, retardation, and the problem of precession of the perihelion for mercury" Proceedings of the International Conference: COSMOLOGY ON SMALL SCALES 2022 Dark Energy and the Local Hubble Expansion Problem, Prague, September 21-24, 2022. Edited by Michal Krizek and Yuri V. Dumin, Institute of Mathematics, Czech Academy of Sciences. .

Bibliography

9. A. Yahalom "Lensing Effects in Galactic Retarded Gravity: Why ”Dark Matter” is the Same for Both Gravitational Lensing and Rotation Curves" IJMPD Vol. 31, No. 14, 2242018 (10 pages), received 23 May 2022, Accepted 31 August 2022, published online 30 September 2022. https://doi.org/10.1142/S0218271822420184
10. Yahalom, A. The Weak Field Approximation of General Relativity and the Problem of Precession of the Perihelion for Mercury. Symmetry 2023, 15, 39. https://doi.org/10.3390/sym15010039

Bibliography

11. Michal Wagman, Lawrence P. Horwitz, and Asher Yahalom "Applying Retardation Theory to Galaxies" 2023 J. Phys.: Conf. Ser. 2482 012005.

12. Gravity Research Foundation - 2023 Honorable Mention. For the Essay "The Virial Theorem for Retarded gravity”.

Preface

Extensive, now 40-year underground and accelerator searches have failed to find any dark matter or establish its existence. The dark matter situation has become even more dire in the last few years as the Large Hadron Collider has failed to find any super symmetric particles, not only of the community’s preferred form of dark matter, but also the form of it that is required in string theory, a theory that attempts to provide a quantized version of Newton–Einstein gravity.

MOND

Is an attempt (maybe the first attempt) to solve the standing problems of gravitation at the large scale not by postulating a new type of matter or particle but by looking more deeply at the phenomena of gravity (and gravity as Einstein taught is nothing but the structure of space-time).

MOND = MOdified Newtonian Dynamics

(or MOND = MOti's New Dynamics)

MOND

A theory of small acceleration defined by a typical acceleration a_0 in which:

$a \gg a_0$ is the Newtonian regime.

$a \ll a_0$ is the deep MOND regime.

In between there is an interpolation.

William of Ockham

1285 - 1347

Ockham's Razor:

"Plurality is not to be posited without necessity."

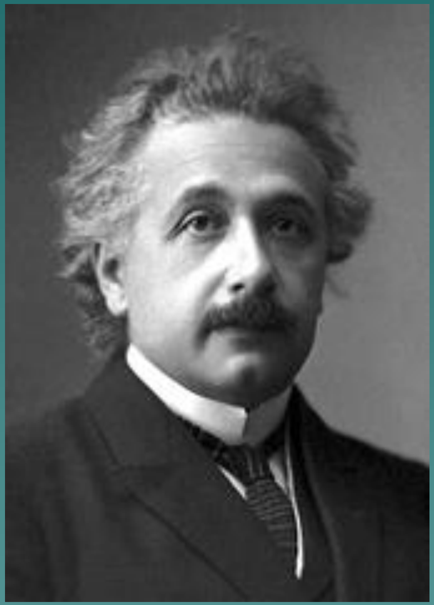
Don't multiply complex causes to explain things when a simple one will do

Law of Parsimony
(lex parsimoniae).

ARIEL UNIVERSITY

Albert Einstein

1879 - 1955



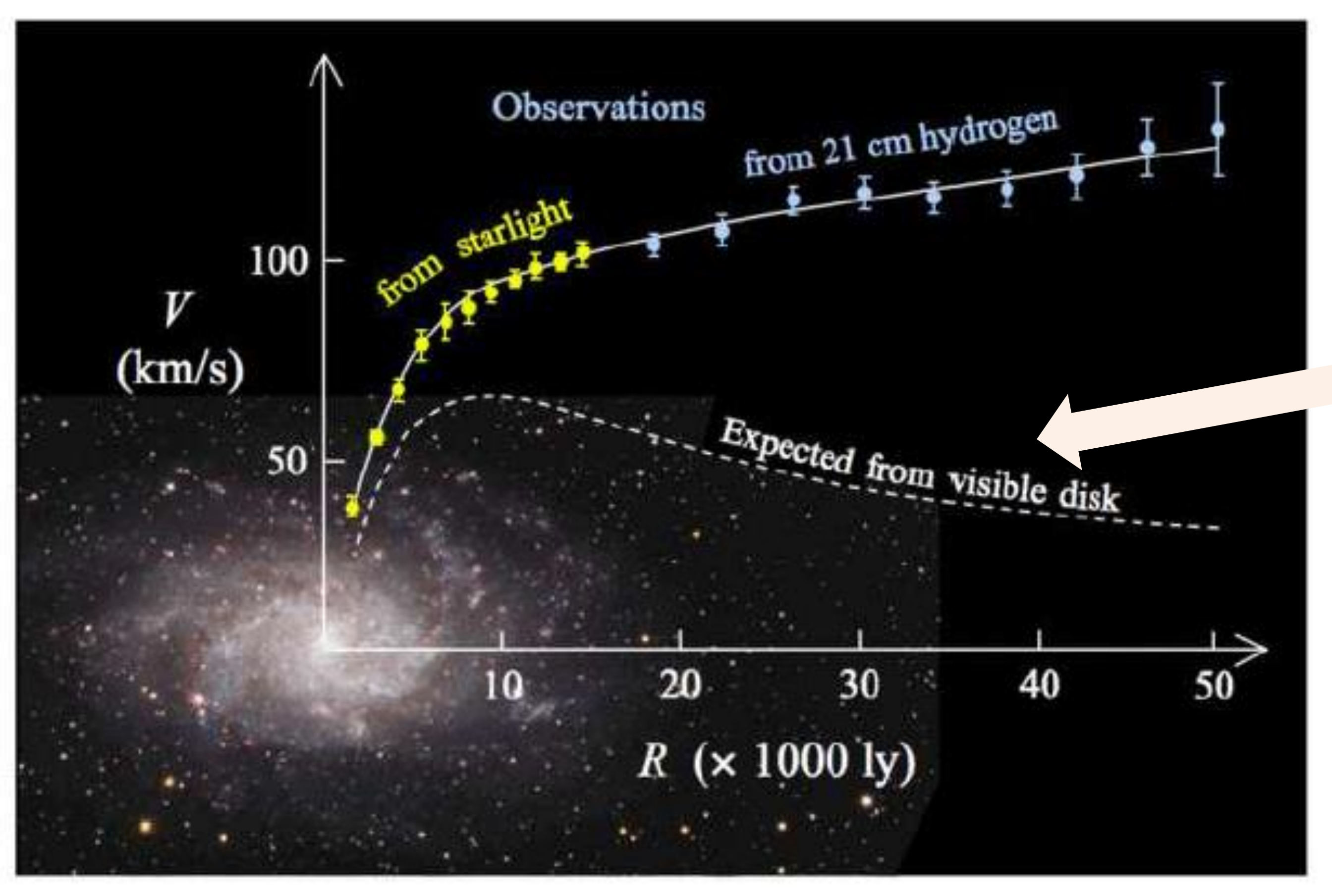
"It can scarcely be denied that the supreme goal of all theory is to make the irreducible basic elements as simple and as few as possible without having to surrender the adequate representation of a single datum of experience."

"On the Method of Theoretical Physics" the Herbert Spencer Lecture, Oxford, June 10,1933.

Fact I - Retardation

Galaxies are huge physical systems having dimensions of many tens of thousands of light years. Thus, any change at the galactic center will be noticed at the rim only tens of thousands of years later. Those retardation effects seems to be neglected in naïve galactic modelling used to calculate rotational velocities of matter in the rims of the galaxy and surrounding gas. (And more so in galaxy clusters)

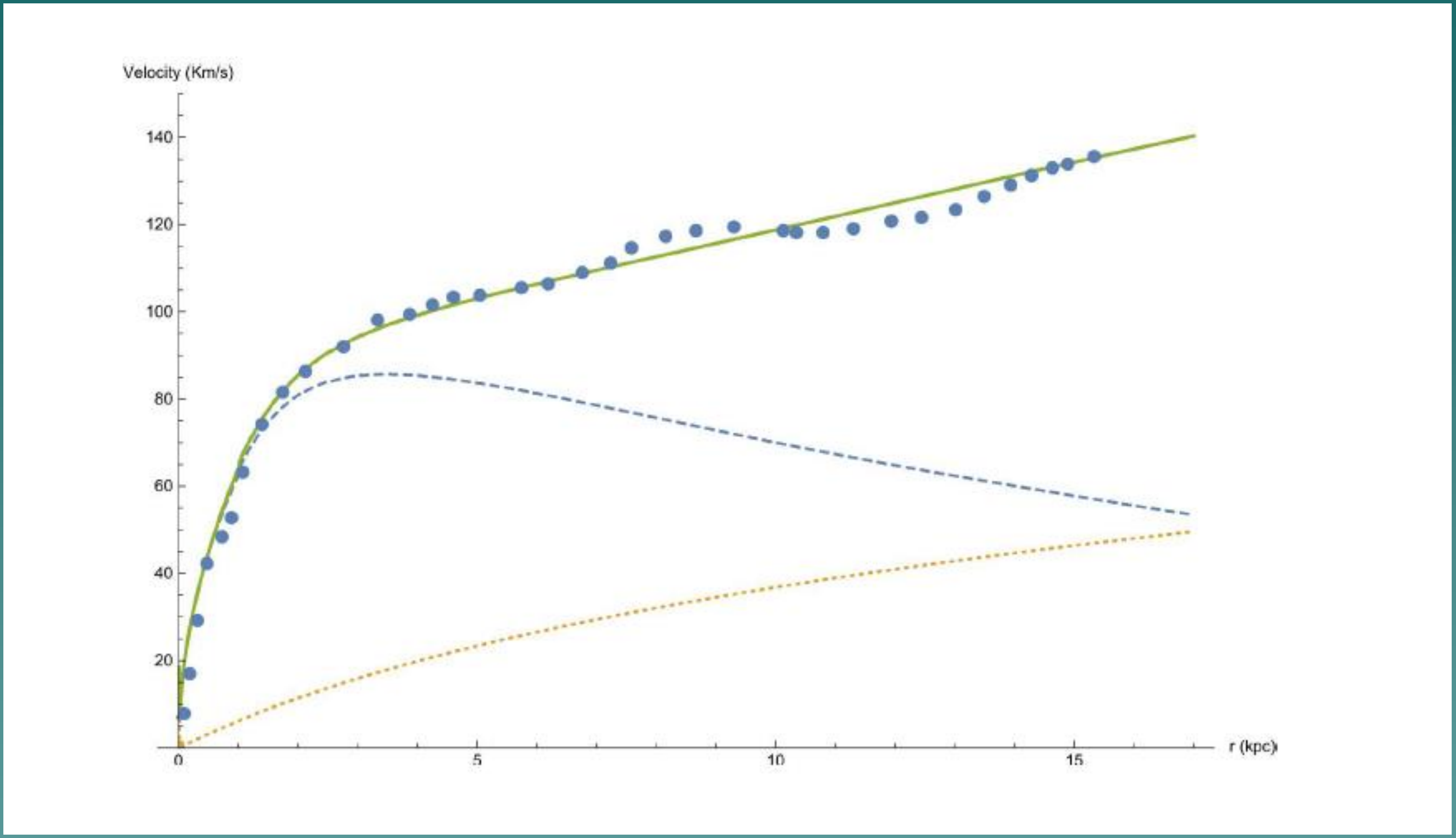
Fact II – Strange Rotation Curves



If you forget about retardation

Are those two facts connected?

M33



Rotation curve for M33. The observational points were supplied by Dr. Michal Wagman, a former PhD student at Ariel University, under my supervision. The full line describes the complete rotation curve, which is the sum of the dotted line, describing the retardation contribution, and the dashed line, which is the Newtonian contribution.

General Relativity

$$G_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu}$$

$$T_{\mu\nu} = (p + \rho c^2) u_\mu u_\nu - p \, g_{\mu\nu}$$

$$\frac{d^2 x^\alpha}{ds^2} + \Gamma^\alpha_{\mu\nu} \frac{dx^\mu}{ds} \frac{dx^\nu}{ds} = \frac{du^\alpha}{ds} + \Gamma^\alpha_{\mu\nu} u^\mu u^\nu = 0$$

General Relativity

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R.$$

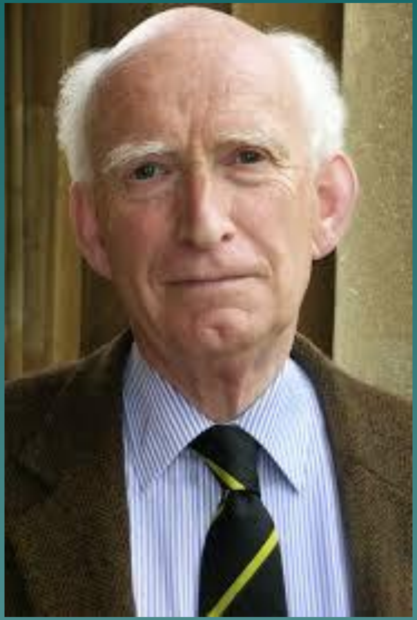
$$R^{\mu}_{\nu\alpha\beta} = \Gamma^{\mu}_{\nu\alpha,\beta} - \Gamma^{\mu}_{\nu\beta,\alpha} + \Gamma^{\sigma}_{\nu\alpha}\Gamma^{\mu}_{\sigma\beta} - \Gamma^{\sigma}_{\nu\beta}\Gamma^{\mu}_{\sigma\alpha}, \quad R_{\alpha\beta} = R^{\mu}_{\alpha\beta\mu}, \quad R = g^{\alpha\beta} R_{\alpha\beta}$$

$$\Gamma^{\alpha}_{\mu\nu} = \frac{1}{2} g^{\alpha\beta} \left(g_{\beta\mu,\nu} + g_{\beta\nu,\mu} - g_{\mu\nu,\beta} \right), \qquad g_{\beta\mu,\nu} \equiv \frac{\partial g_{\beta\mu}}{\partial x^{\nu}}$$

How can one solve those complex, tensor, non-linear partial differential equations for the case of galaxies?

Answer:

- For most cases (galaxies included) it is not necessary to solve the full Einstein equation but only a linear approximation to them as only weak gravitational fields are involved.
- For some cases such as compact objects (black holes) and the very early universe (big bang) strong gravitational fields are involved, and one needs to use the exact Einstein equations.



5 April 1935 – 6 February 2018

I would like to thank the late Professor Donald Lynden-Bell for this very important observation.

Linear Approximation (weak gravity)

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad \eta_{\mu\nu} \equiv \text{ diag } (1, -1, -1, -1), \quad |h_{\mu\nu}| \ll 1$$

$$\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h, \quad h = \eta^{\mu\nu} h_{\mu\nu}.$$

$$\Box \bar{h}_{\mu\nu} \equiv \bar{h}_{\mu\nu,\alpha}{}^{\alpha} = -\frac{16\pi G}{c^4} T_{\mu\nu}, \qquad \bar{h}_{\mu\alpha,}{}^{\alpha} = 0.$$

Solution:

$$\bar{h}_{\mu\nu}(\vec{x},t) = -\frac{4G}{c^4}\int \frac{T_{\mu\nu}(\vec{x}',t-\frac{R}{c})}{R} d^3x',$$

$$t \equiv \frac{x^0}{c}, \quad \vec{x} \equiv x^a \quad a,b \in [1,2,3], \quad \vec{R} \equiv \vec{x}-\vec{x}', \quad R = |\vec{R}|.$$

$$h_{\mu\nu} = \bar{h}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\bar{h}.$$

$$\frac{4G}{c^4} \simeq 3.3 \ 10^{-44}$$

$$\Gamma^{\alpha}_{\mu\nu} = \frac{1}{2}\eta^{\alpha\beta}\left(h_{\beta\mu,\nu} + h_{\beta\nu,\mu} - h_{\mu\nu,\beta}\right).$$

$$\frac{d^2x^\alpha}{ds^2} + \Gamma^\alpha_{\mu\nu}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds} = \frac{du^\alpha}{ds} + \Gamma^\alpha_{\mu\nu}u^\mu u^\nu = 0$$

→ **Affine connection is first order.**

$u^\mu u^\nu$ **zeroth order**

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu,$$

$$u^0 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}, \qquad u^a = \vec{u} = \frac{\frac{\vec{v}}{c}}{\sqrt{1-\frac{v^2}{c^2}}}, \qquad \vec{v} \equiv \frac{d\vec{x}}{dt}, \quad v = |\vec{v}|.$$

$$u^0 \simeq 1, \qquad \vec{u} \simeq \frac{\vec{v}}{c}, \qquad u^a \ll u^0 \qquad \text{for} \quad v \ll c.$$

The resulting geodesic:

$$\frac{dv^a}{dt} \simeq -c^2 \Gamma^a_{00} = -c^2 \left(h^a_{0,0} - \frac{1}{2} h_{00,}{}^a \right)$$

$$\rho c^2 \gg p \quad \Rightarrow$$

$$\frac{dv^a}{dt} \simeq \frac{c^2}{4} \bar{h}_{00,}{}^a \Rightarrow \frac{d\vec{v}}{dt} = -\vec{\nabla}\phi = \vec{F}, \qquad \phi \equiv \frac{c^2}{4} \bar{h}_{00}$$

Back to Newton?

$$\phi = \frac{c^2}{4}\bar{h}_{00} = -\frac{G}{c^2}\int \frac{T_{00}(\vec{x}', t - \frac{R}{c})}{R} d^3x' = -G\int \frac{\rho(\vec{x}', t - \frac{R}{c})}{R} d^3x'$$

If retardation is neglected or the density is static:

$$\phi = \phi_N = -G\int \frac{\rho(\vec{x}')}{R} d^3x'$$

However, we suggest that retardation cannot be neglected on galactic scales and the density is not static. A possible cause for this is mass accreted from the intergalactic medium.

Beyond the Newtonian Approximation

$$\rho(\vec{x}',t-\frac{R}{c})=\sum_{n=0}^{\infty}\frac{1}{n!}\rho^{(n)}(\vec{x}',t)(-\frac{R}{c})^{n}, \qquad \rho^{(n)}\equiv\frac{\partial^{n}\rho}{\partial t^{n}}.$$

$$\rho(\vec{x}',t-\frac{R}{c})\simeq\rho(\vec{x}',t)-\rho^{(1)}(\vec{x}',t)\frac{R}{c}+\frac{1}{2}\rho^{(2)}(\vec{x}',t)(\frac{R}{c})^{2}.$$

A Taylor series expansion has a limited range hence the current approach is "near field" that is of limited distance Validity from the galaxy.

$$\phi = -G\int \frac{\rho(\vec{x}', t)}{R} d^3x' + \frac{G}{c}\int \rho^{(1)}(\vec{x}', t) d^3x' - \frac{G}{2c^2}\int R\rho^{(2)}(\vec{x}', t) d^3x'$$

$$\phi_r = -\frac{G}{2c^2}\int R\rho^{(2)}(\vec{x}', t) d^3x'$$

$$\begin{aligned}
\vec{F} &= \vec{F}_N + \vec{F}_r \\
\vec{F}_N &= -\vec{\nabla}\phi_N = -G\int \frac{\rho(\vec{x}', t)}{R^2}\hat{R} d^3x', \qquad \hat{R} \equiv \frac{\vec{R}}{R} \\
\vec{F}_r &\equiv -\vec{\nabla}\phi_r = \frac{G}{2c^2}\int \rho^{(2)}(\vec{x}', t)\hat{R} d^3x'
\end{aligned}$$

Obviously for small distances Newtonian forces dominate over Retardation forces but what happens for large distances were Newtonian forces decline and Retardation forces are not reduced?

Δt is the typical duration in which the density ρ changes.

$$R_r \equiv c\Delta t$$

Retardation distance

$$R \ll R_r$$

Newtonian regime

$$R \gg R_r$$

Retardation regime

For large distances:

$$\hat{R} \simeq \frac{\vec{x}}{|\vec{x}|} \equiv \hat{r}$$

$$\vec{F}_r = \frac{G}{2c^2}\hat{r}\int \rho^{(2)}(\vec{x}', t)d^3x' = \frac{G}{2c^2}\hat{r}\ddot{M}, \qquad \ddot{M} \equiv \frac{d^2M}{dt^2}.$$

Retardation forces can be repulsive or attractive.

$$\dot{M} > 0$$

Mass is accreted from the intergalactic gas.

$$\ddot{M} < 0$$

The intergalactic gas is depleted.

$$\vec{F}_r = -\frac{G}{2c^2}|\ddot{M}|\hat{r}$$

The retardation force is attractive in the galactic scenario.

When is retardation theory important?

Defining "dark matter"

$$-\frac{v_c^2}{r}\hat{r} = \vec{F}_d = -\frac{GM_d(r)}{r^2}\hat{r}$$

However, F_d is really F_r

$$\vec{F}_r = -\frac{G}{2c^2}|\ddot{M}|\hat{r}$$

→

$$M_d(r) = \frac{r^2|\ddot{M}|}{2c^2}$$

$$M_d(r) = 4\pi \int_0^r r'^2 \rho_d(r') dr', \qquad \frac{dM_d(r)}{dr} = 4\pi r^2 \rho_d(r)$$

$$\rho_d(r) = \frac{|\ddot{M}|}{4\pi c^2 r}$$

"dark matter" density decreases as $r^{-1.3}$ for the M33 galaxy.

E. Corbelli; P. Salucci (2000). "The extended rotation curve and the dark matter halo of M33". Monthly Notices of the Royal Astronomical Society. 311 (2): 441447. arXiv:astro-ph/9909252. doi:10.1046/j.1365-8711.2000.03075.x.

Mass Conservation

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0$$

$$\frac{\partial^2 \rho}{\partial t^2} + \vec{\nabla} \cdot (\frac{\partial \rho}{\partial t} \vec{v} + \rho \frac{\partial \vec{v}}{\partial t}) = 0.$$

$$\frac{1}{c^2} \ddot{M} = \frac{1}{c^2} \int \frac{\partial^2 \rho}{\partial t^2} d^3 x = \oint d\vec{S} \cdot \left(\frac{\vec{v}}{c} \left(\frac{\vec{v}}{c} \cdot \vec{\nabla} \rho + \rho \vec{\nabla} \cdot (\frac{\vec{v}}{c}) \right) - \rho \frac{1}{c} \frac{\partial \frac{\vec{v}}{c}}{\partial t} \right).$$

"Dark Mass"

$$M_d(r) = \frac{r^2}{2} \oint d\vec{S} \cdot \left(\frac{\vec{v}}{c} \left(\frac{\vec{v}}{c} \cdot \vec{\nabla}\rho + \rho \vec{\nabla} \cdot \left(\frac{\vec{v}}{c}\right) \right) - \rho \frac{1}{c} \frac{\partial \frac{\vec{v}}{c}}{\partial t} \right).$$

$$\left[\frac{M_d(r)}{M}\right] \approx \left(\frac{v}{c}\right)^2 \left[\frac{r}{l_\rho} + \frac{r}{l_v} + \frac{r}{l_d}\right]$$

$$\left(\frac{v}{c}\right)^2 \approx 10^{-6}$$

$$l_\rho \equiv \frac{\rho}{|\vec{\nabla}\rho|}, \qquad l_v \equiv \frac{v}{|\vec{\nabla} \cdot v|}, \qquad l_d \equiv \frac{v^2}{|\partial_t v|}$$

$$\frac{r}{l} \approx 10^6$$

Retardation vs. Newtonian Forces

$$\frac{1}{l_t} = \left[\frac{1}{l_\rho} + \frac{1}{l_v} + \frac{1}{l_d}\right]$$

$$\frac{F_r}{F_N} = \left[\frac{M_d(r)}{M}\right] \approx \left(\frac{v}{c}\right)^2 \left[\frac{r}{l_t}\right]$$

Acceleration Condition

MOND corrections are needed for small acceleration, so let us write the conditions needed for retardation corrections in the language of acceleration.

$$a = \frac{v^2}{r}$$

$$fr \equiv \frac{F_r}{F_N}$$

⇒

$$\frac{v^2}{c^2}\frac{r}{l_t} > fr$$

⇒

$$a = \frac{v^2}{r} > a_c(r) \equiv fr\frac{c^2 l_t}{r^2}$$

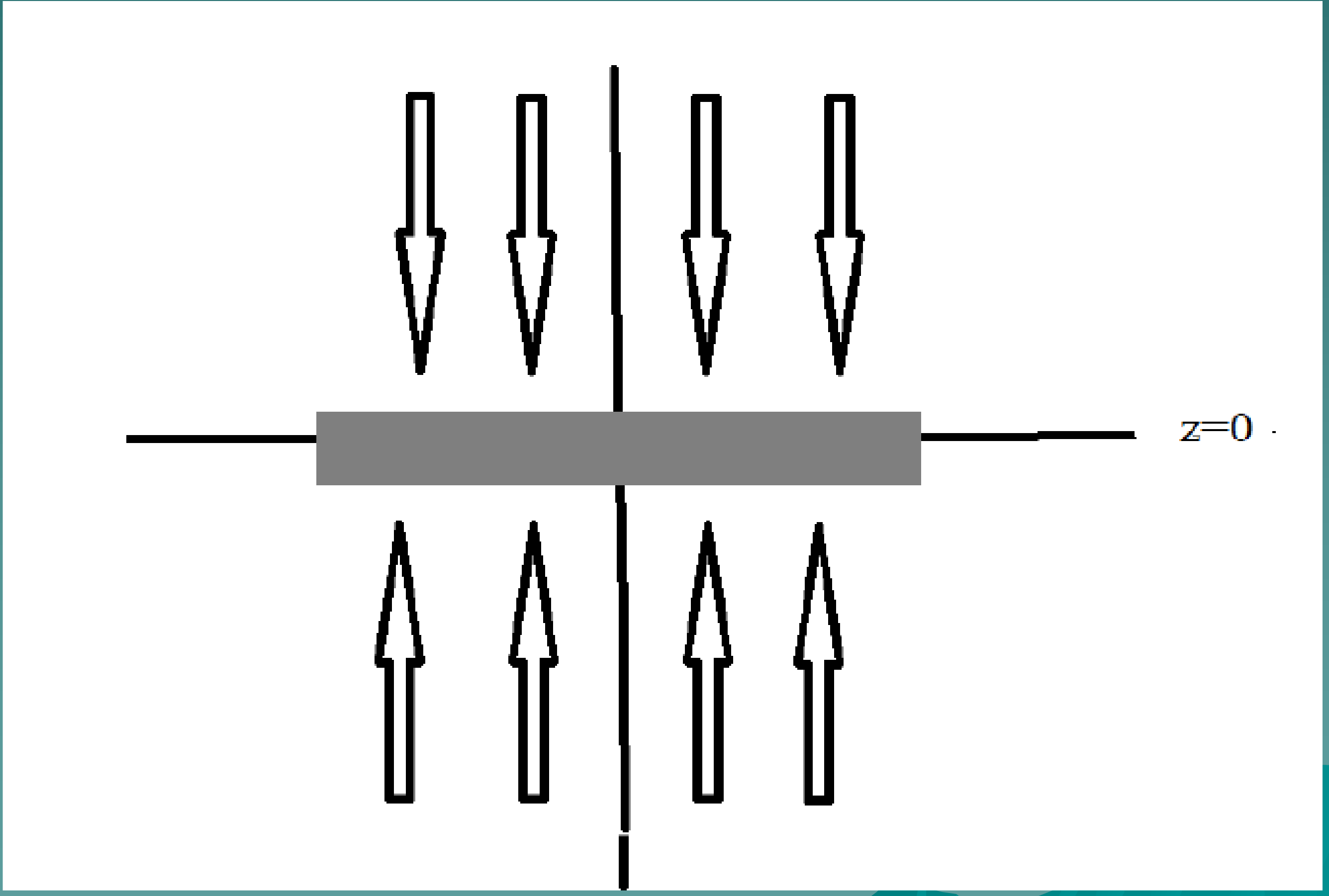
Acceleration Condition

$$a = \frac{v^2}{r} > a_c(r) \equiv f r \frac{c^2 l_t}{r^2}$$

However, as $a_c(r)$ decreases as r^{-2} this means that this inequality will be satisfied more easily for larger r. Thus "dark matter" effects can be interpreted in terms of acceleration as MOND postulates.

However, acceleration does not need to be small or equal with respect to a_0 in order to have "dark matter" effects, rather acceleration must be bigger than some critical acceleration $a_c(r)$ which depends on radial distance. Thus, the inequality becomes easier to satisfy at large radial distances, in which case the acceleration <u>is indeed quite small as MOND suggests</u>.

Depletion



$$(\vec{v}\cdot\vec{\nabla})\vec{v} = -\frac{\vec{\nabla}p(\rho)}{\rho} - \vec{\nabla}\phi$$

$$\vec{v}\cdot\vec{\nabla} = v_z\frac{\partial}{\partial z} + \frac{v_\theta}{\bar{r}}\frac{\partial}{\partial\theta}$$

$$\frac{v_\theta^2}{\bar{r}} \simeq \frac{\partial\phi}{\partial\bar{r}},$$

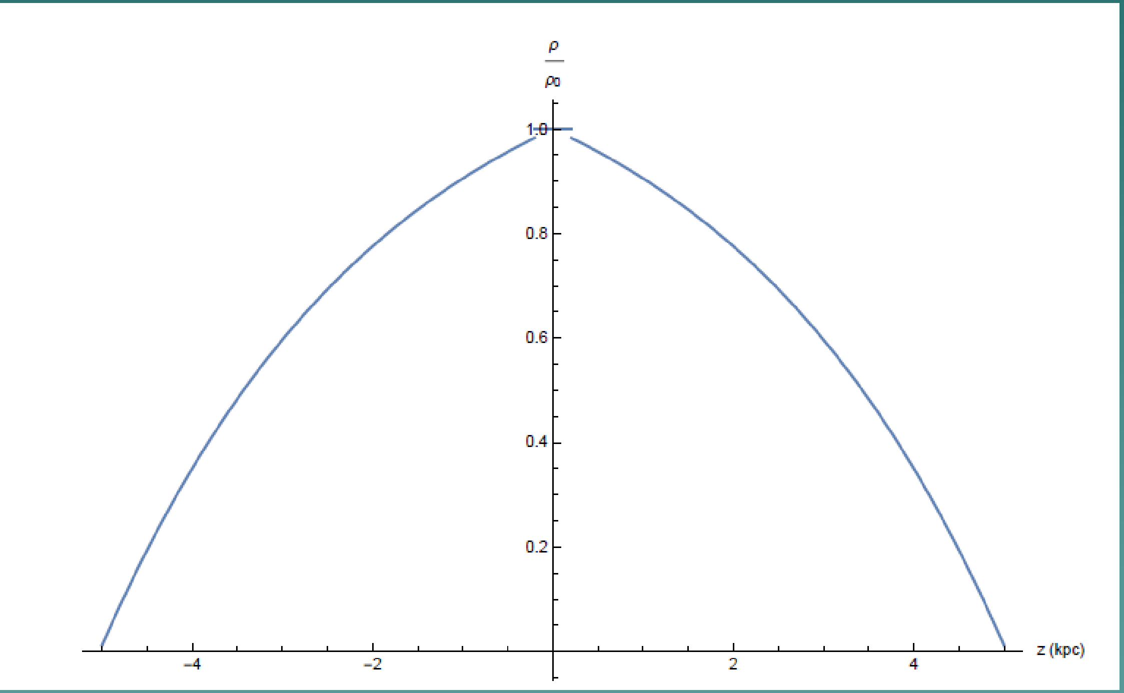
$$\frac{1}{2}v_z^2 + w(\rho) + \phi = C(r,t).$$

$$w(\rho) = \int\frac{dP}{\rho}$$

Dynamical Equilibrium

$$v_z = \begin{cases} -|v_z| & z > 0 \\ |v_z| & z < 0 \end{cases}$$

Depletion



$$\rho_o(\bar{r}, z, 0) = re(z)\left[\rho_1(\bar{r}) + \rho_2(\bar{r})e^{k|z|}\right],$$

$$re(z) = \begin{cases} 1 & |z| < z_i \\ 0 & |z| \geq z_i \end{cases}$$

Density distribution outside the galaxy

$$\rho_o(\bar{r},z,t) = \frac{\gamma}{v_z} = re(z - v_z t)[\rho_1(\bar{r}) + \rho_2(\bar{r})e^{k|z - v_z t|}]$$

Mass outside the galaxy

$$M_o(t) = 2\pi \left[\int_{-z_i}^{-\frac{1}{2}\Delta z} dz \int_0^{r_m} d\bar{r}\bar{r}\rho_o(\bar{r}, z, t) + \int_{\frac{1}{2}\Delta z}^{z_i} dz \int_0^{r_m} d\bar{r}\bar{r}\rho_o(\bar{r}, z, t) \right]$$

Mass inside the galaxy

$$M(t) = M_T - M_o(t)$$

$$\dot{M}(t) = -\dot{M}_o(t), \qquad \ddot{M}(t) = -\ddot{M}_o(t)$$

Mass inside the galaxy

$$\ddot{M}(t) = \ddot{M}(0)e^{\frac{t}{\tau}}$$

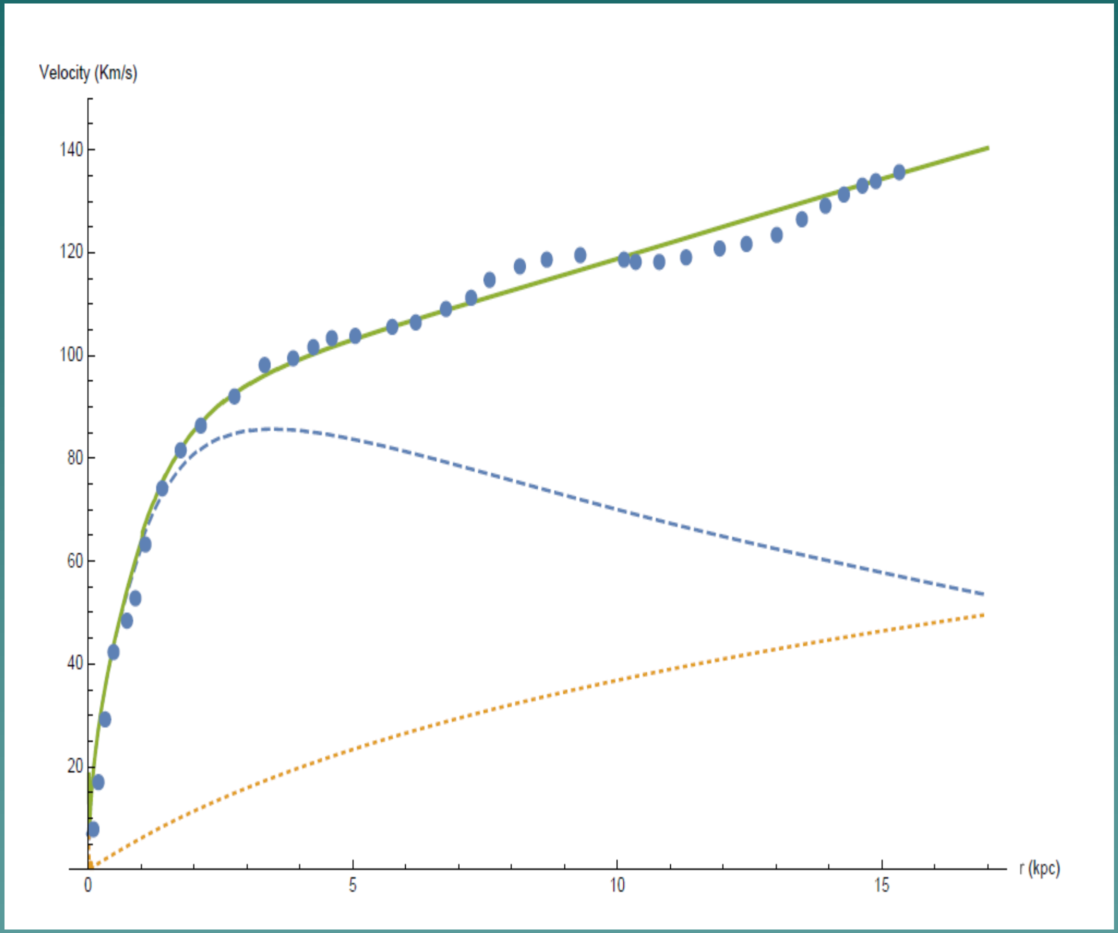
$$\alpha \equiv k|v_z|, \qquad \tau \equiv \frac{1}{\alpha}$$

Mass inside the galaxy

$$\dot{M}(t) = \dot{M}(0) + \tau\ddot{M}(0)\left(e^{\frac{t}{\tau}} - 1\right) = \dot{M}(0) - \tau|\ddot{M}(0)|\left(e^{\frac{t}{\tau}} - 1\right).$$

$$M(t) = M(0) + \left(\dot{M}(0) - \tau\ddot{M}(0)\right)t + \tau^2\ddot{M}(0)\left(e^{\frac{t}{\tau}} - 1\right), \quad \tau > 0.$$

Some Results for Various Galaxies



M/L =1

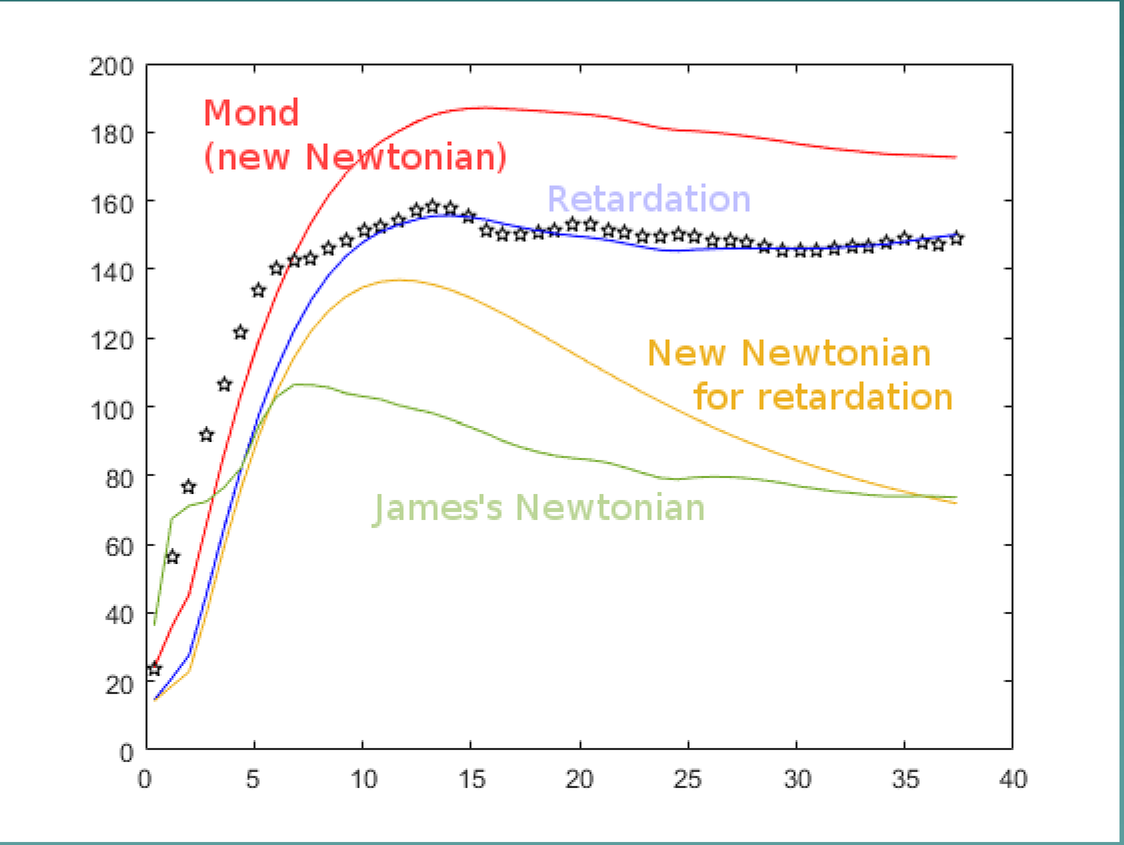
M/L is not a fitting parameter as most authors assume.

$$|\ddot{M}| = \frac{M}{t_r^2} \simeq 9.12 \times 10^{16}\ \mathrm{kg/s^2}$$

the full line describe the complete rotation curve which is the sum of the dotted line describing the retardation contribution and the dashed line which is the Newtonian contribution.

Work by Michal Wagman former PhD student at Ariel University.

NGC 3198



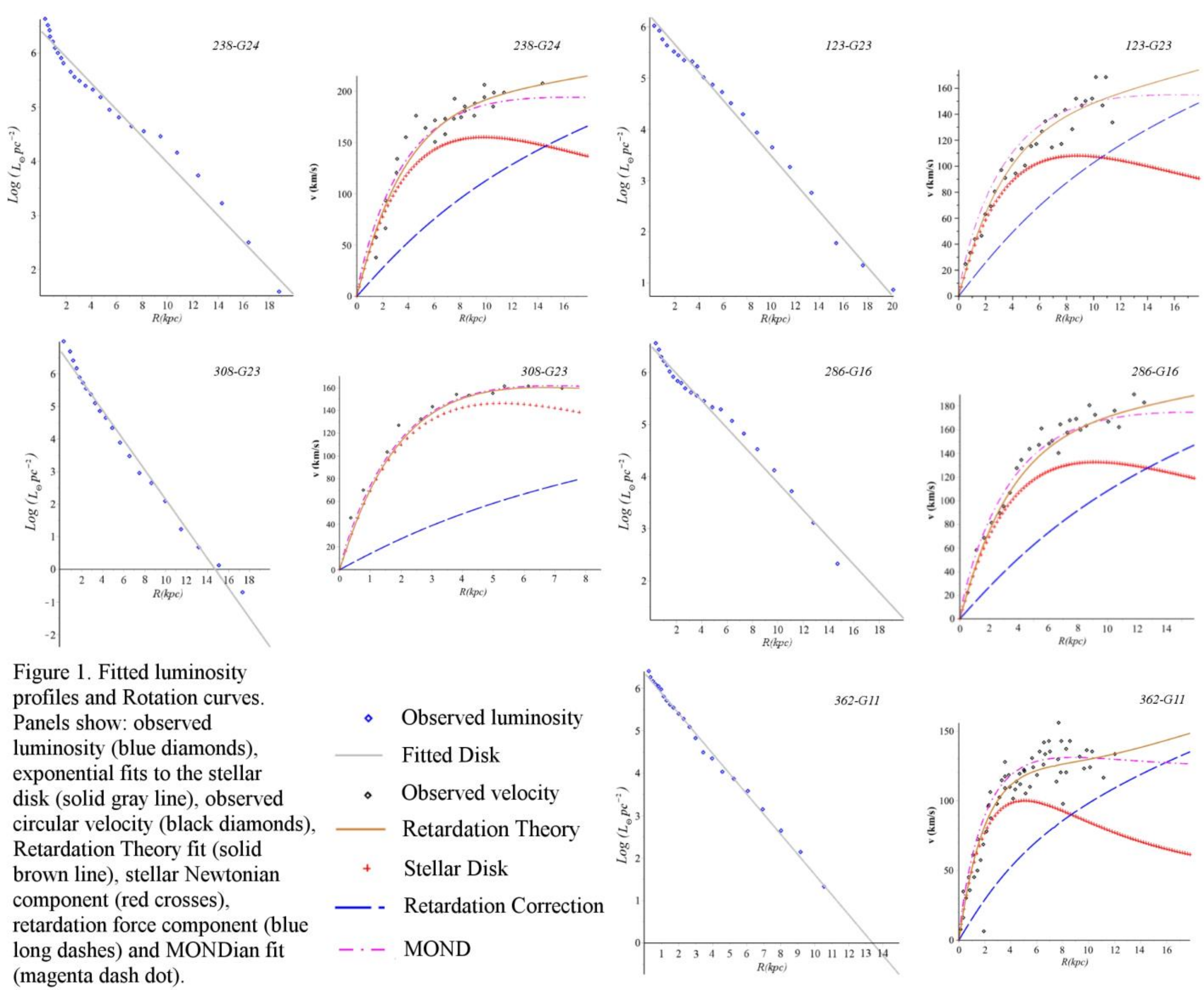


Figure 1. Fitted luminosity profiles and Rotation curves. Panels show: observed luminosity (blue diamonds), exponential fits to the stellar disk (solid gray line), observed circular velocity (black diamonds), Retardation Theory fit (solid brown line), stellar Newtonian component (red crosses), retardation force component (blue long dashes) and MONDian fit (magenta dash dot).

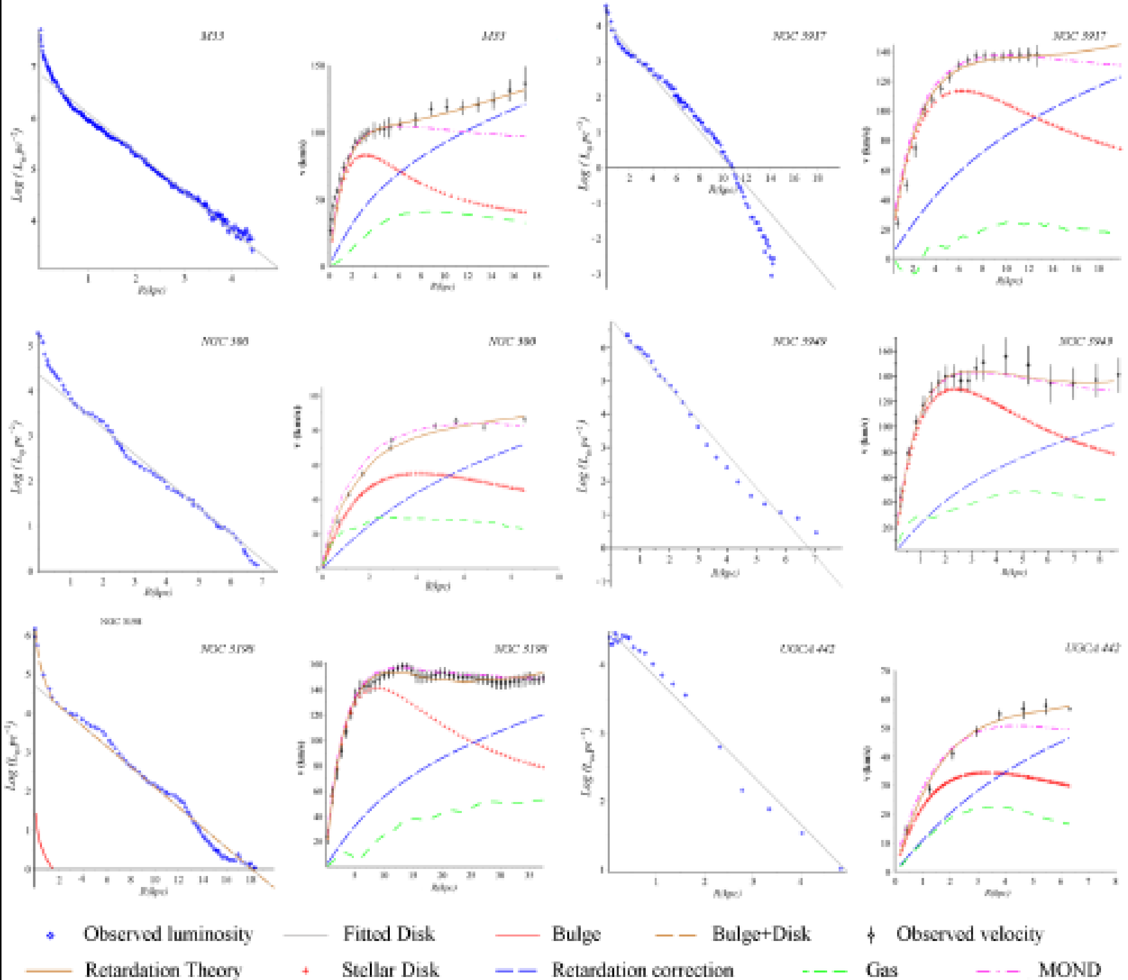
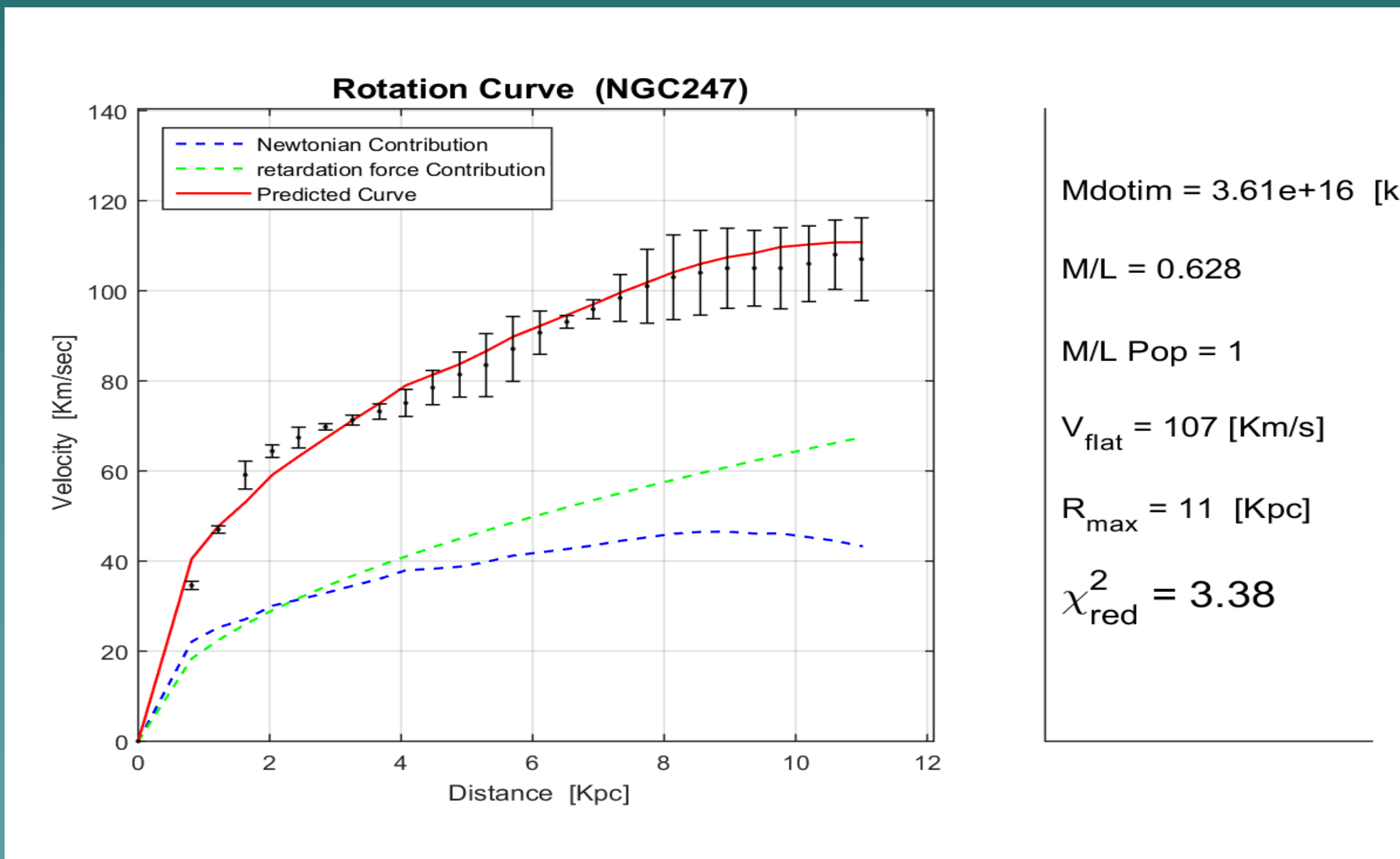
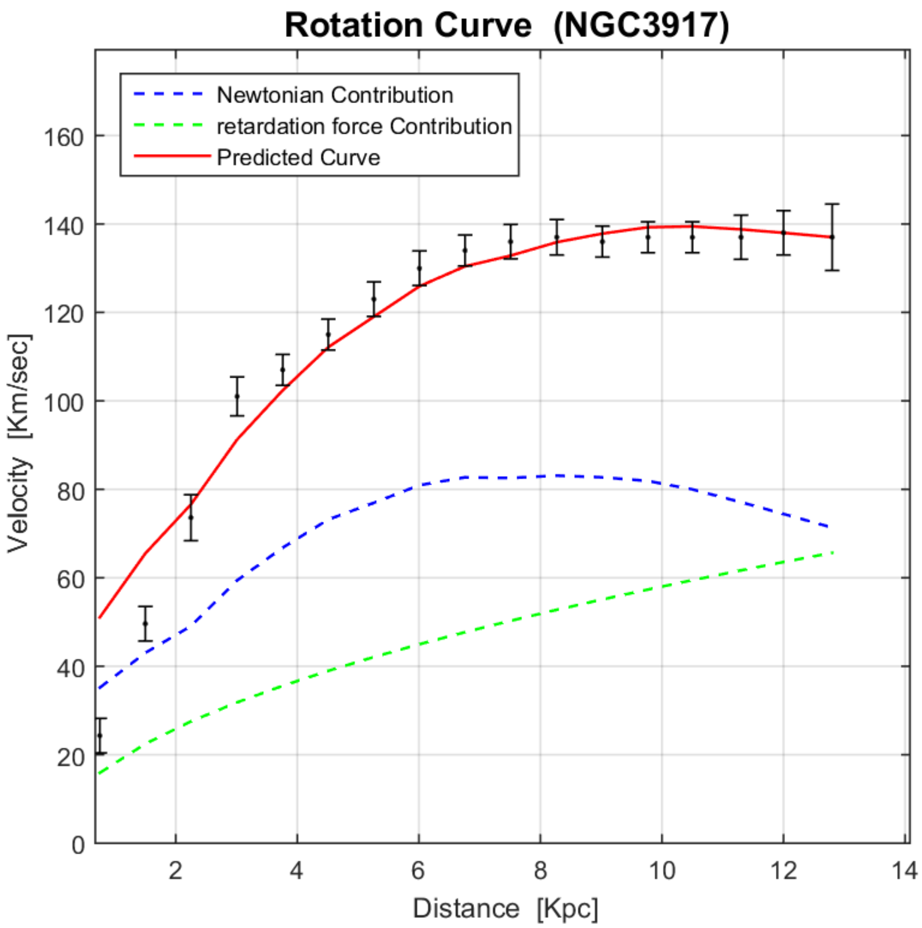


Figure 2. Fitted luminosity profiles and Rotation curves. Panels show: observed luminosity (blue diamonds), exponential fits to the stellardisk (solid gray line), bulge fit (red solid line), sum of bulge+disk (long brown dashes), observed circular velocity (black diamonds w/ error bar), Retardation Theory fit (solid brown line), stellar Newtonian component (red crosses), retardation force component (blue long dashes), gas component (short green dashes) and MONDian fit (magenta dash dot).

Work by Tomer Zimmerman & Roy Gomel
PhD students at Tel Aviv University





Mdotim = 2.94e+16 [kg

M/L = 0.912

M/L Pop = 1.3

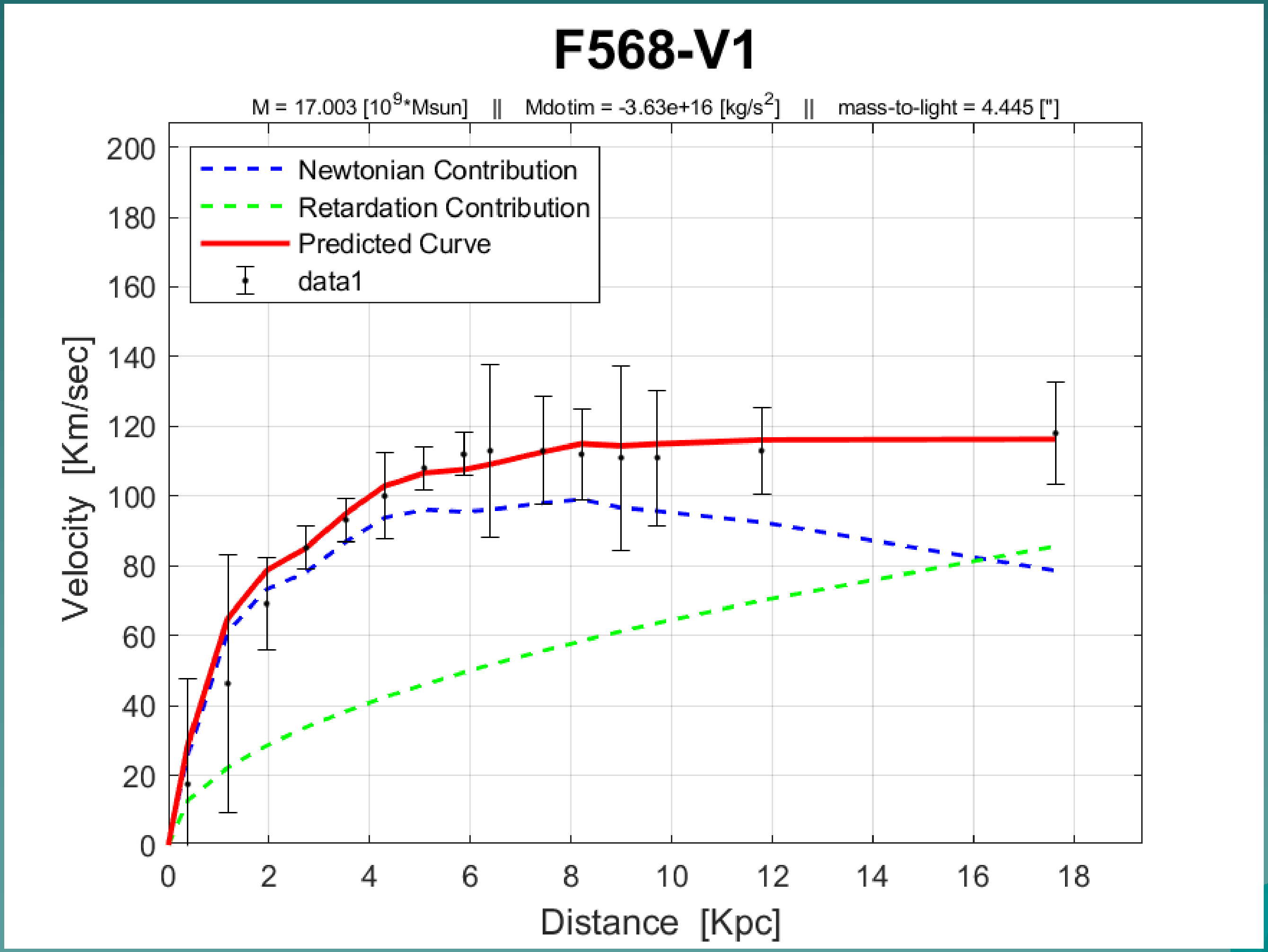
V_{flat} = 135 [Km/s]

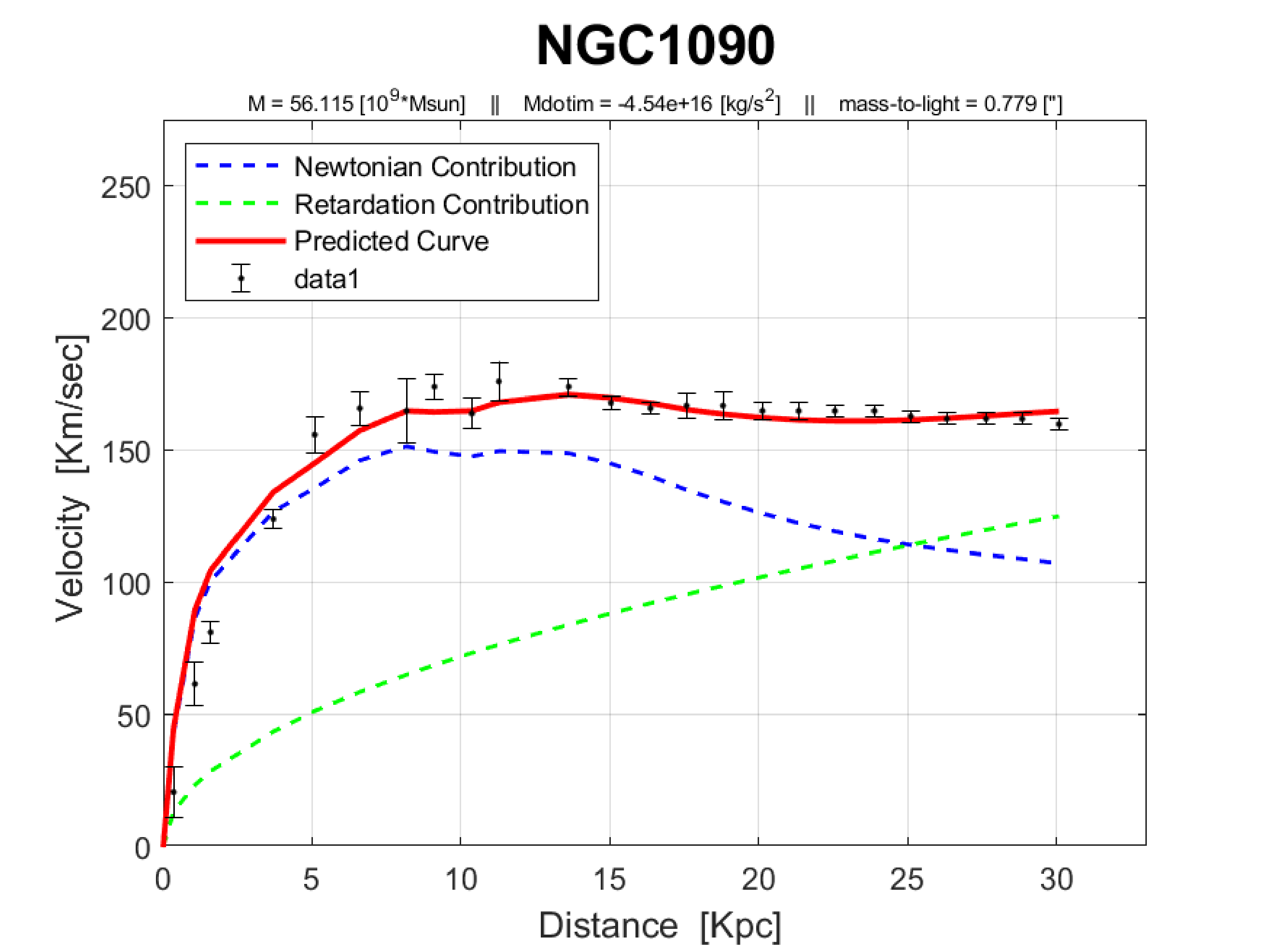
R_{max} = 12.8 [Kpc]

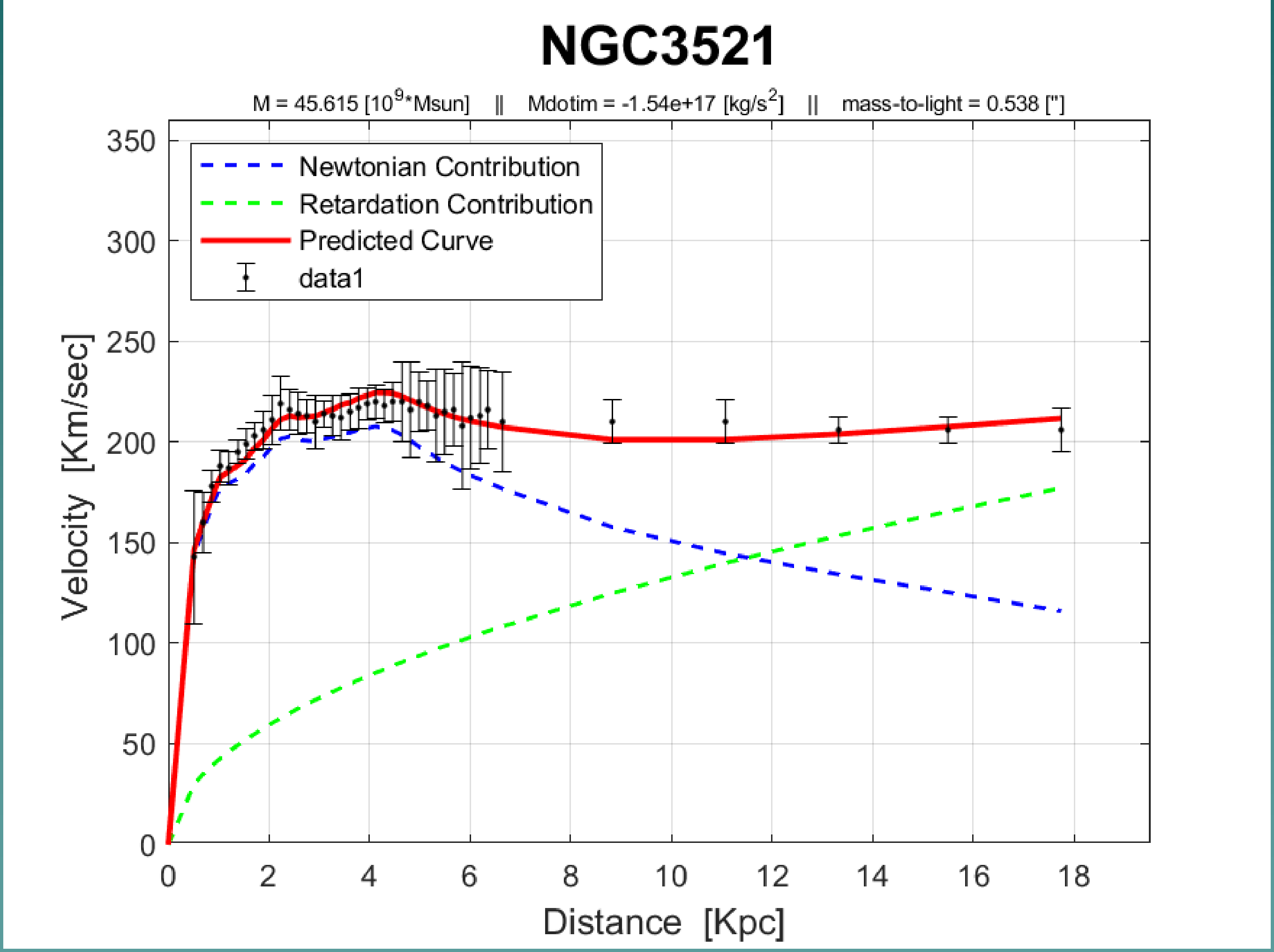
χ^2_{red} = 5.11

Yuval Glas

1. **Automation of the fitting process**
2. **143 Galaxies in 20 minutes.**


F568-V1
M = 17.003 [10^9*Msun] || Mdotim = -3.63e+16 [kg/s^2] || mass-to-light = 4.445 ["]
Newtonian Contribution
Retardation Contribution
Predicted Curve
data1
Velocity [Km/sec]
200
180
160
140
120
100
80
60
40
20
0
0
2
4
6
8
10
12
14
16
18
Distance [Kpc]


NGC1090
M = 56.115 [10^9*Msun] || Mdotim = -4.54e+16 [kg/s^2] || mass-to-light = 0.779 ["]
Newtonian Contribution
Retardation Contribution
Predicted Curve
data1
Velocity [Km/sec]
Distance [Kpc]
0
50
100
150
200
250
0
5
10
15
20
25
30


NGC3521
M = 45.615 [10^9*Msun] || Mdotim = -1.54e+17 [kg/s^2] || mass-to-light = 0.538 ["]
Newtonian Contribution
Retardation Contribution
Predicted Curve
data1
Velocity [Km/sec]
Distance [Kpc]
0
50
100
150
200
250
300
350
0
2
4
6
8
10
12
14
16
18

How not to interpret retardation theory

$$|\ddot{M}| = \frac{M}{t_r^2} \simeq 9.12 \times 10^{16}\ \mathrm{kg/s^2}$$

Is not constant over large time scales!

$$\ddot{M}(t) = \ddot{M}(0)e^{\frac{t}{\tau}}$$

How not to interpret retardation theory

If it is assumed constant:

$$\dot{M}(t) = \dot{M}(0) + t\ddot{M} = \dot{M}(0) - t|\ddot{M}|$$

$$\dot{M}(0) = T|\ddot{M}|$$

How not to interpret retardation theory

If it is assumed constant:

$$M(t) = M(0) + \dot{M}(0)t + \frac{1}{2}\ddot{M}t^2$$

$$M(t) = \ddot{M}t(\frac{1}{2}t - T) \Rightarrow M(T) = -\frac{1}{2}\ddot{M}T^2 = \frac{1}{2}|\ddot{M}|T^2$$

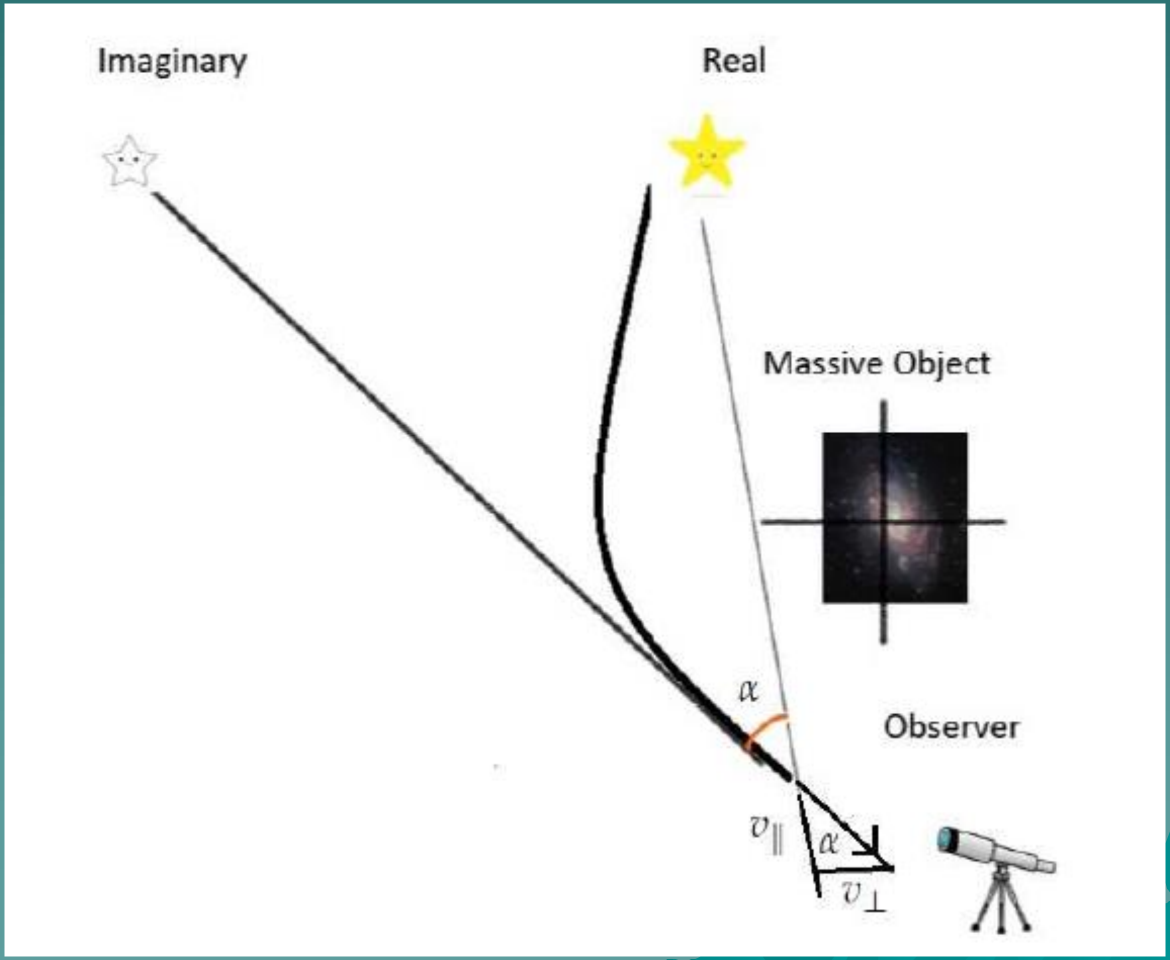
How not to interpret retardation theory

By plugging in the mass accumulation decrease rate, we arrive at $M(T) \simeq 7.66 \times 10^{51}$ kg, which is clearly 11 orders of magnitude greater than the known mass of the galaxy.

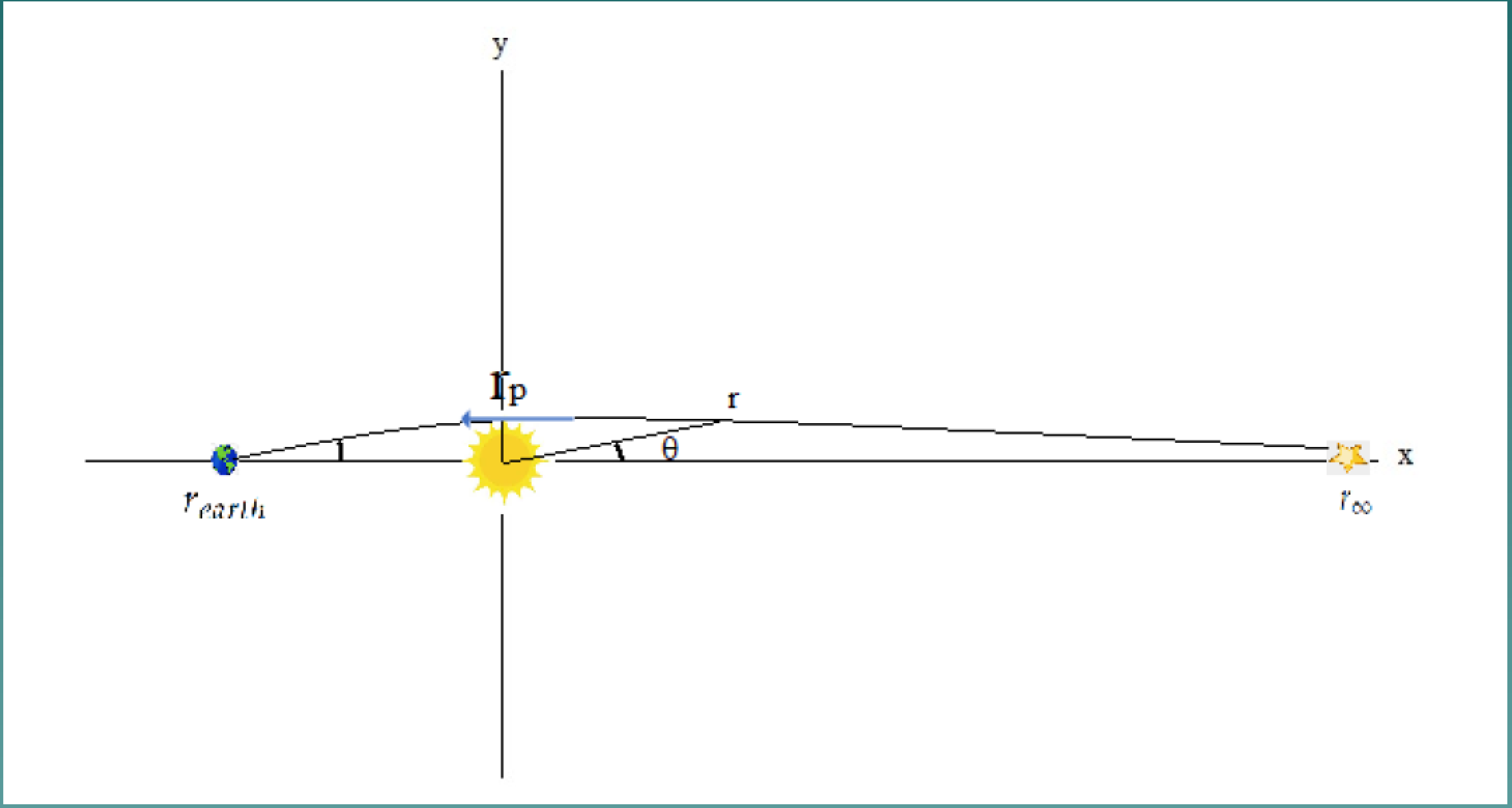
However, the second derivative of M is clearly not constant as dictated by the dynamics.

$$\ddot{M}(t) = \ddot{M}(0)e^{\frac{t}{\tau}}$$

Lensing


Imaginary
Real
Massive Object
α
Observer
$v_{\|}$
α
$v_{\perp}$

$$\frac{d\vec{v}_\perp}{dt} = \vec{F}_\perp, \qquad \vec{F}_\perp \equiv -2\vec{\nabla}_\perp \phi$$


y
rp
r
θ
x
r_{earth}
r_∞


EINSTEIN
AND
EDDINGTON

$$\alpha \simeq \frac{2}{e} \Rightarrow \alpha_E \simeq \frac{2}{e_E} = \frac{2r_s}{r_p} = 2\alpha_N.$$

$$\alpha_E \simeq 8.47 \times 10^{-6} \text{ radians} = 1.75 \text{ arcseconds},$$
$$\alpha_N \simeq 4.27 \times 10^{-6} \text{ radians} = 0.87 \text{ arcseconds}.$$

Eddington, A.S. *The Mathematical Theory of Relativity*; Cambridge University Press: Cambridge, UK, 1923.

$$\frac{d\vec{v}_\perp}{dt} = \vec{F}_\perp, \qquad \vec{F}_\perp \equiv -2\vec{\nabla}_\perp \phi$$

$$\begin{aligned}
\vec{F}_\perp &= \vec{F}_{N\perp} + \vec{F}_{r\perp} \\
\vec{F}_{N\perp} &\simeq -2G\int \frac{\rho(\vec{x}',t)}{r^2}\hat{r}_\perp d^3x' = -\frac{2GM}{r^2}\hat{r}_\perp, \qquad \hat{r}_\perp \equiv \hat{r} - \hat{v}_0(\hat{v}_0 \cdot \hat{r}) \\
\vec{F}_{r\perp} &\simeq \frac{G}{c^2}\int \rho^{(2)}(\vec{x}',t)\hat{r}_\perp d^3x' = \frac{G}{c^2}\ddot{M}\hat{r}_\perp.
\end{aligned}$$

$$\vec{F}_{\perp} \simeq -[\frac{2GM}{r^2} - \frac{G}{c^2}\ddot{M}]\hat{r}_{\perp} = -\frac{2G(M + M_d(r))}{r^2}\hat{r}_{\perp}$$

$$M_d(r) \equiv \frac{r^2|\ddot{M}|}{2c^2}$$

The same as in the case of Galactic rotation curves.

Tully-Fisher Relations

An empirical relationship between the mass or intrinsic luminosity of a spiral galaxy and its asymptotic rotation velocity or emission line width. It was first published in 1977 by astronomers Tully and Fisher. The relation states that baryonic galactic mass is proportional to velocity to the power of roughly four.

R. B. Tully and J. R. Fisher, *Astron. Astrophys.* **54** (1977) 661.

Derivation of the Tully Fisher Relations

$$\vec{F} = \vec{F}_N + \vec{F}_r = -\frac{GM}{r^2}\hat{r}\left(1 + \frac{|\ddot{M}|}{2Mc^2}r^2\right) = -\frac{GM}{r^2}\hat{r}\left(1 + \frac{r^2}{2R_r^2}\right),$$

$$R_r \equiv c\sqrt{\frac{M}{|\ddot{M}|}}$$

$$v_\theta^2 = \frac{GM}{r}\left(1 + \frac{r^2}{2R_r^2}\right).$$

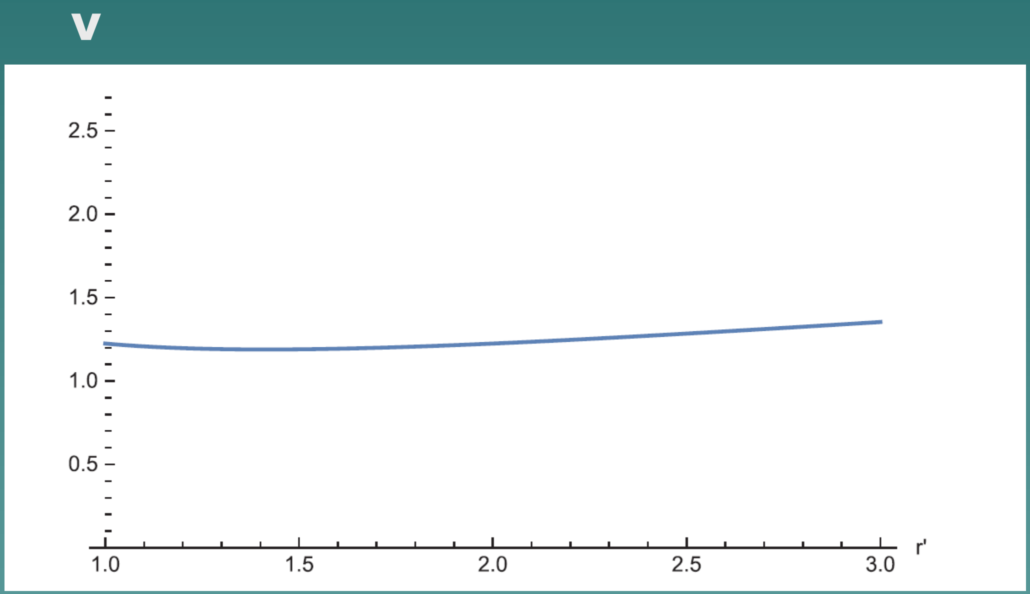
Derivation of the Tully Fisher Relations

$$v_{ty} \equiv \sqrt{\frac{GM}{R_r}}$$

$$v \equiv \frac{v_\theta}{v_{ty}}, r' \equiv \frac{r}{R_r}$$

$$v^2 = \frac{1}{r'}\left(1 + \frac{r'^2}{2}\right)$$

Derivation of the Tully Fisher Relations



Notice that the plot is almost flat, however, it has a shallow minimum at $r'_{\min} = \sqrt{2}$ for which $v_{\min} \approx 1.19$. The maximal value in this range is obtained for $r'_{\max} = 3$ and is 14% higher. Thus we can assume that roughly $v \approx 1$ and hence

Derivation of the Tully Fisher Relations

$$v_\theta \approx v_{ty} = \sqrt{\frac{GM}{R_r}} = \sqrt{\frac{GM}{c\sqrt{\frac{M}{|\ddot{M}|}}}} = \sqrt{\frac{G}{c}} M^{\frac{1}{4}} |\ddot{M}|^{\frac{1}{4}}$$

$$\Rightarrow \quad M = k v_\theta^4, \quad k \approx \frac{c^2}{G^2 |\ddot{M}|}$$

Higher order effects

Are retardation effects result of a Taylor series approximation? No.

$$\frac{d\vec{v}}{dt} = -\vec{\nabla}\phi = \vec{F}$$

$$\begin{aligned}\phi &= \frac{c^2}{4}\bar{h}_{00} = -\frac{G}{c^2}\int \frac{T_{00}(\vec{x}', t - \frac{R}{c})}{R} d^3x' \\ &= -G\int \frac{\rho(\vec{x}', t - \frac{R}{c})}{R} d^3x'\end{aligned}$$

The force is always partitioned to a retarded Newtonian force and a retardation force that has no parallel in Newtonian theory.

$$\vec{F} = \vec{F}_{Nr} + \vec{F}_r$$

$$\vec{F}_{Nr} = -G\int \frac{\rho(\vec{x}', t - \frac{R}{c})}{R^2}\hat{R} d^3x', \qquad \hat{R} \equiv \frac{\vec{R}}{R}$$

$$\vec{F}_r \equiv -\frac{G}{c}\int \frac{\rho^{(1)}(\vec{x}', t - \frac{R}{c})}{R}\hat{R} d^3x', \qquad \rho^{(n)} \equiv \frac{\partial^n \rho}{\partial t^n}.$$

Conservation of Mass

Question:

How do we know that the correction of the gravitational potential created by the negative second derivative of the mass of the galaxy is not cancelled out by the gravitational effect of the necessarily positive second derivative of the mass of the IGM.

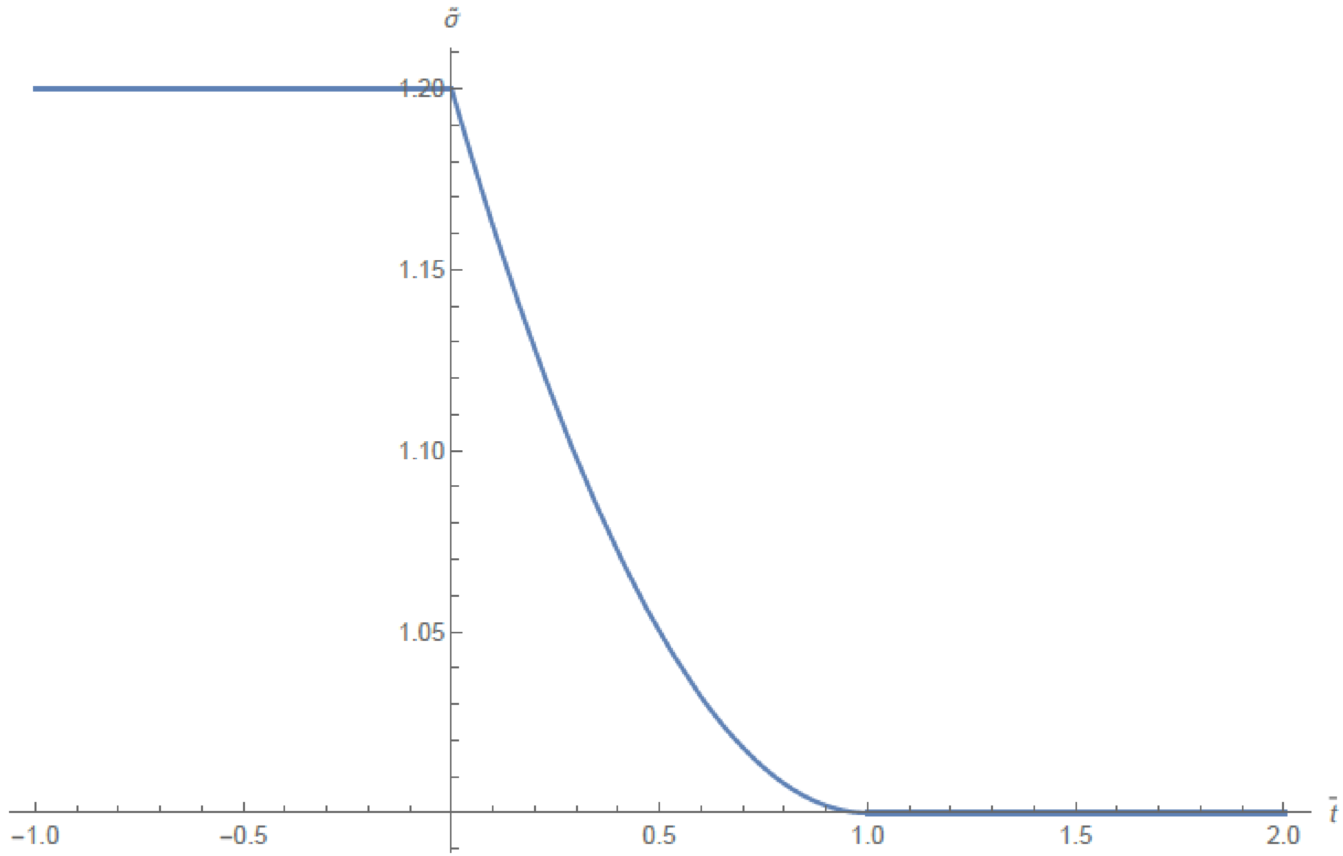
Answer:

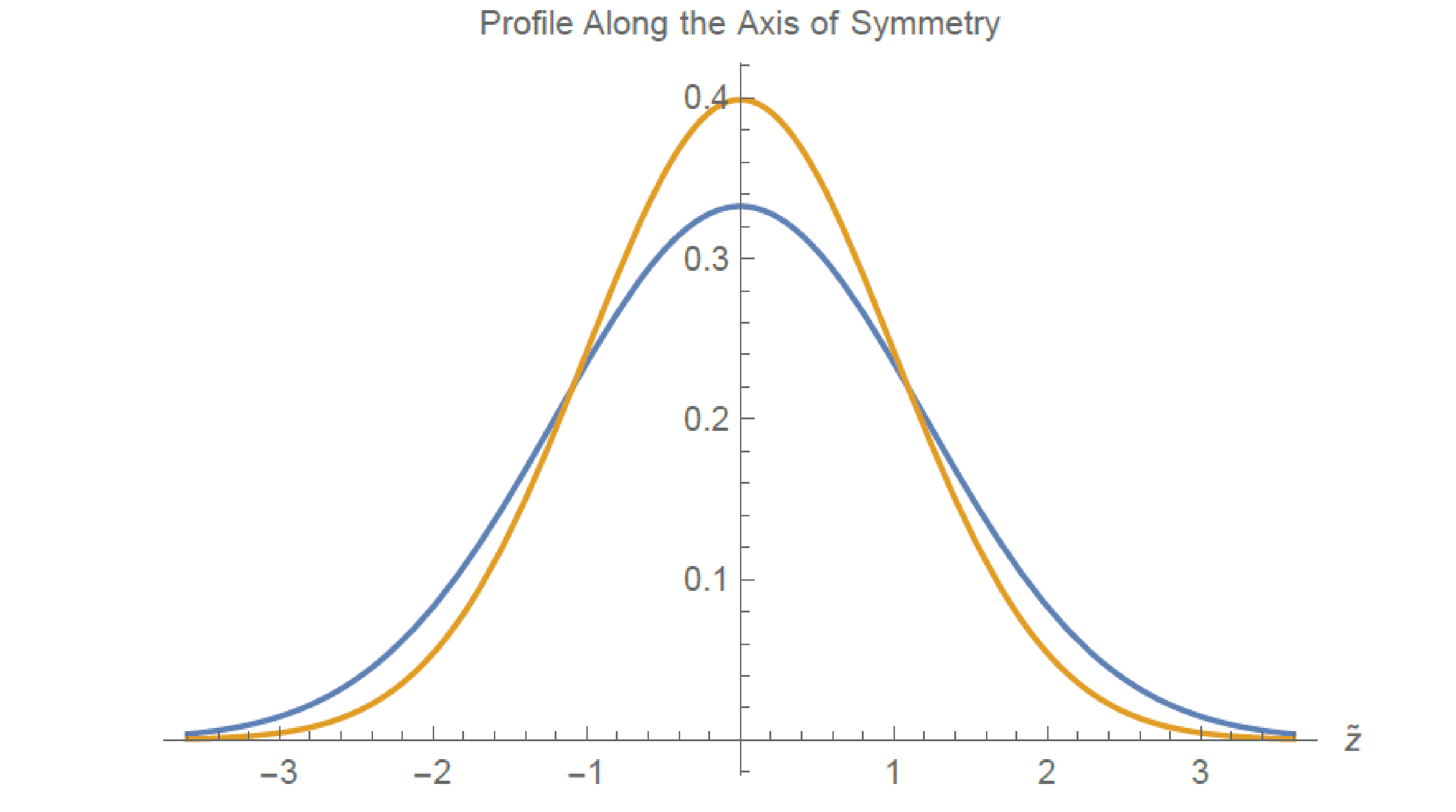
Using the non perturbative approach we can study the gravitational effect of the galactic matter and the IGM by considering both as one body with a time dependent density profile.

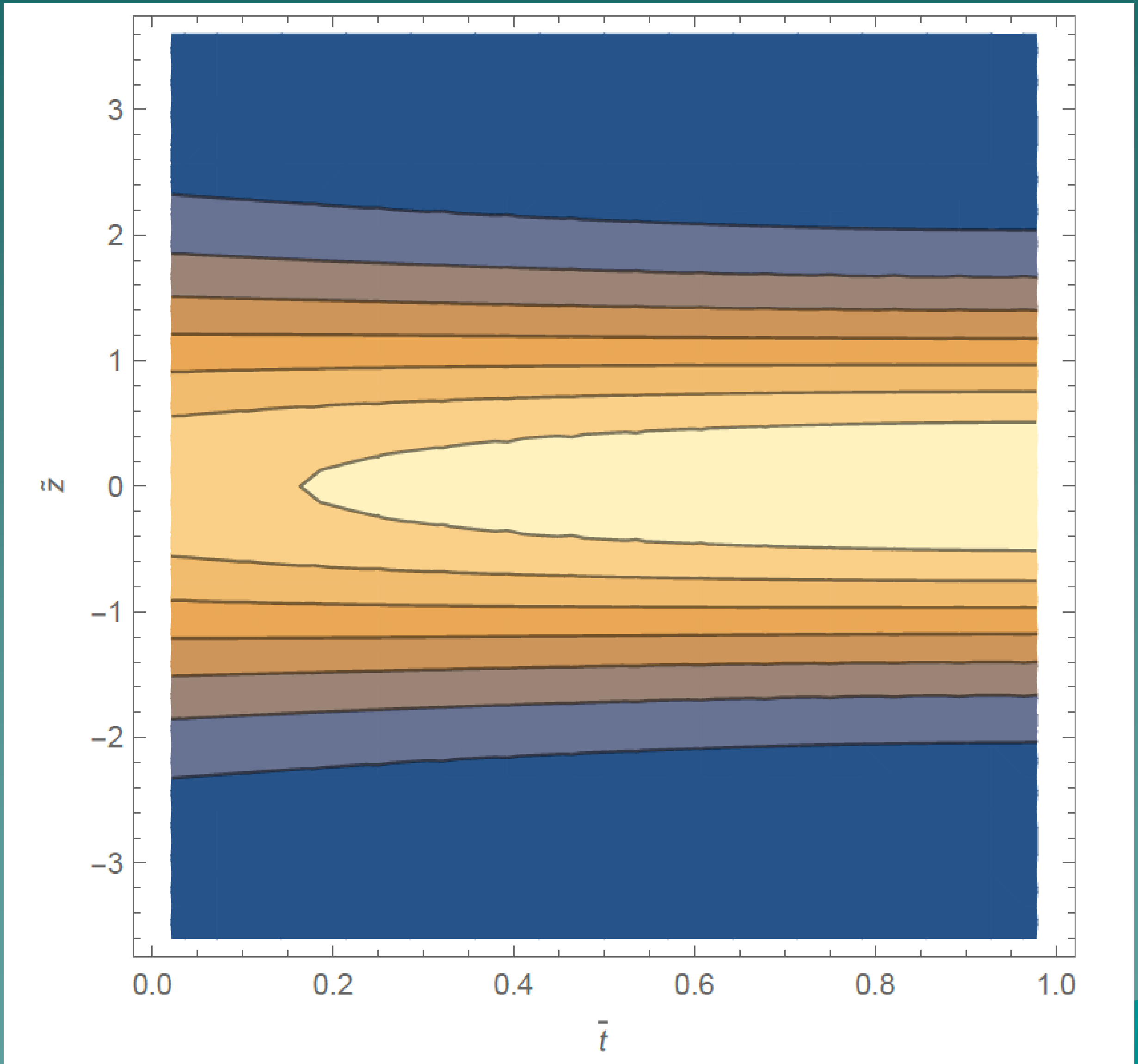
$$\rho = \rho_c \tilde{\rho}, \qquad \tilde{\rho} = \Sigma(\vec{x}_\perp) h(z,t)$$

$$h(z,t) = \frac{R_s}{\sqrt{2\pi}\sigma(t)} e^{-\frac{z^2}{2\sigma(t)^2}}.$$

$$\sigma(t) = \begin{cases} \sigma_i & \bar{t} \leq 0 \\ \sigma_i + (\sigma_f - \sigma_i)\bar{t}(2 - \bar{t}) & 0 < \bar{t} < 1 \\ \sigma_f & \bar{t} \geq 1 \end{cases} \qquad \bar{t} \equiv \frac{t}{t_f}$$


$\tilde{\sigma}$
1.20
1.15
1.10
1.05
−1.0
−0.5
0.5
1.0
1.5
2.0
$\bar{t}$


Profile Along the Axis of Symmetry
0.4
0.3
0.2
0.1
−3
−2
−1
1
2
3
z̃


3
2
1
0
−1
−2
−3
z̃
0.0
0.2
0.4
0.6
0.8
1.0
t̄

$$\phi = -\frac{GM}{r}\psi_r$$

$$\phi_N = -\frac{GM}{r}\psi_N$$

$$\Delta\psi \equiv \psi_r - \psi_N \qquad \Rightarrow \qquad \lim_{r->\infty} \Delta\psi = 0$$

In other words, retardation is a near field effect.

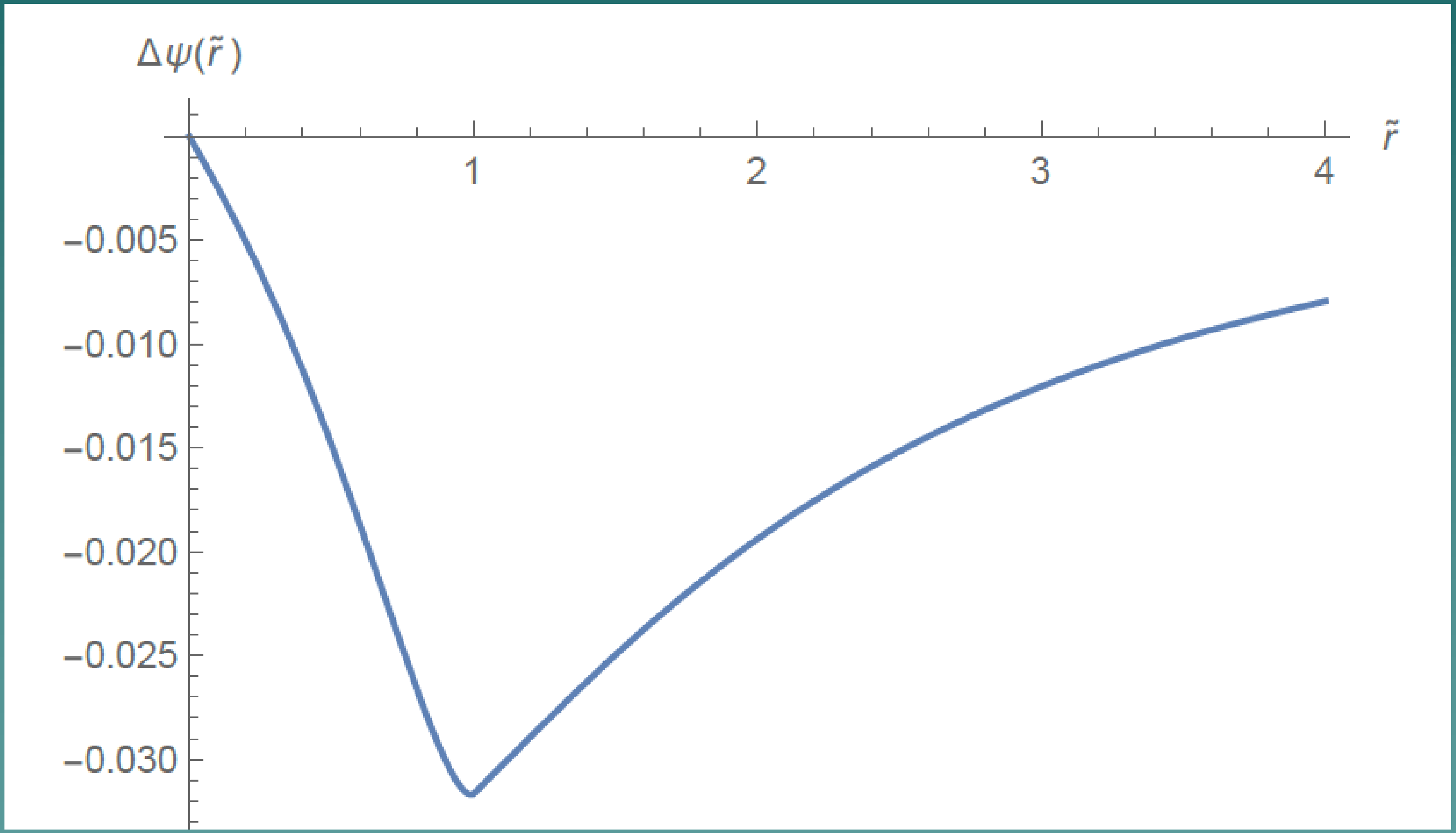
Why?

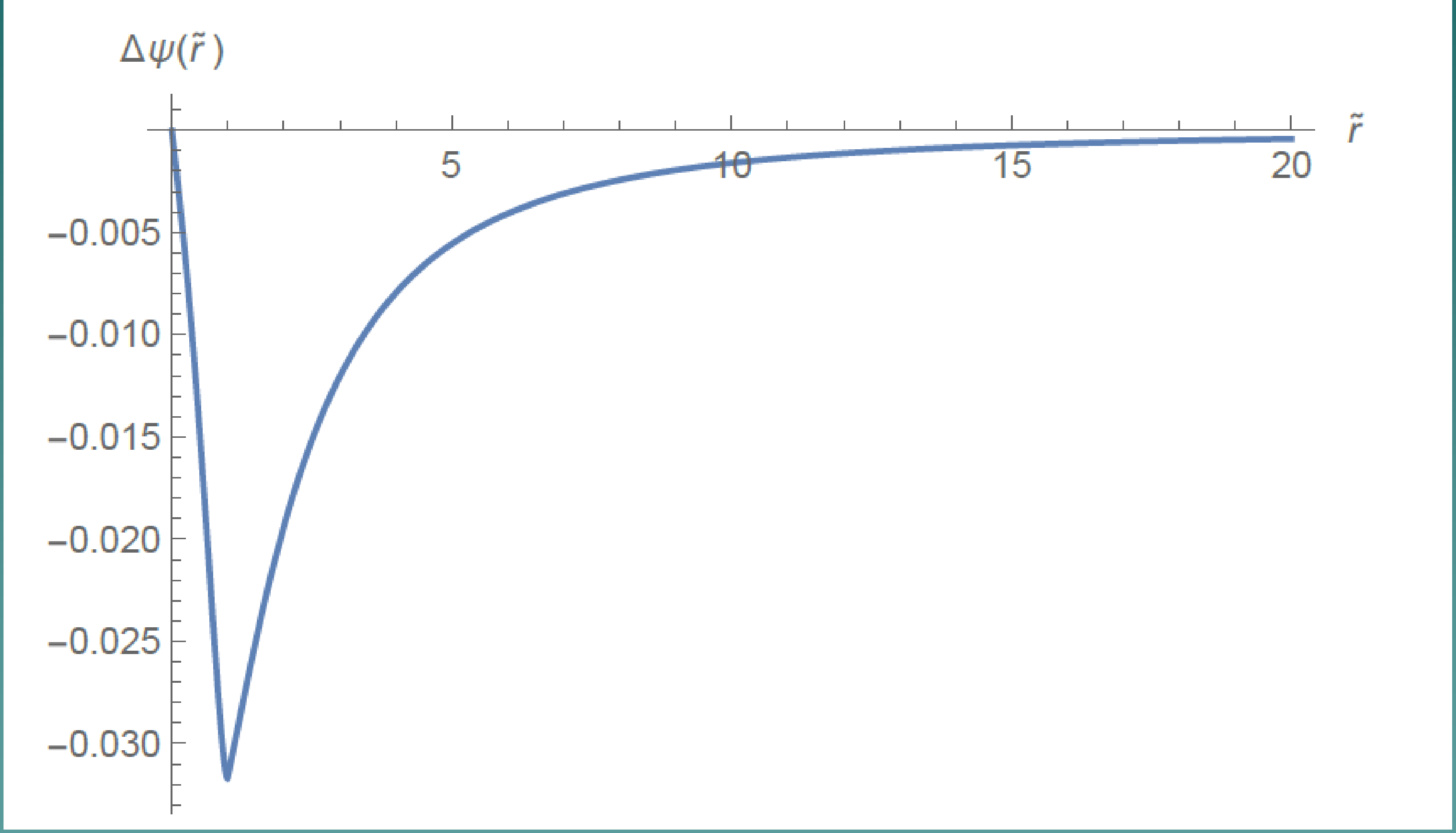
Let us look at the gravitational potential in the limit of large r, in which r is much bigger than a typical scale of the system: r>>Rs. In this limit, R is about the same as r.

$$\phi \simeq -G \int \frac{\rho(\vec{x}', t - \frac{r}{c})}{r} d^3x' = -\frac{G}{r} \int \rho(\vec{x}', t - \frac{r}{c}) d^3x' = -\frac{GM}{r},$$

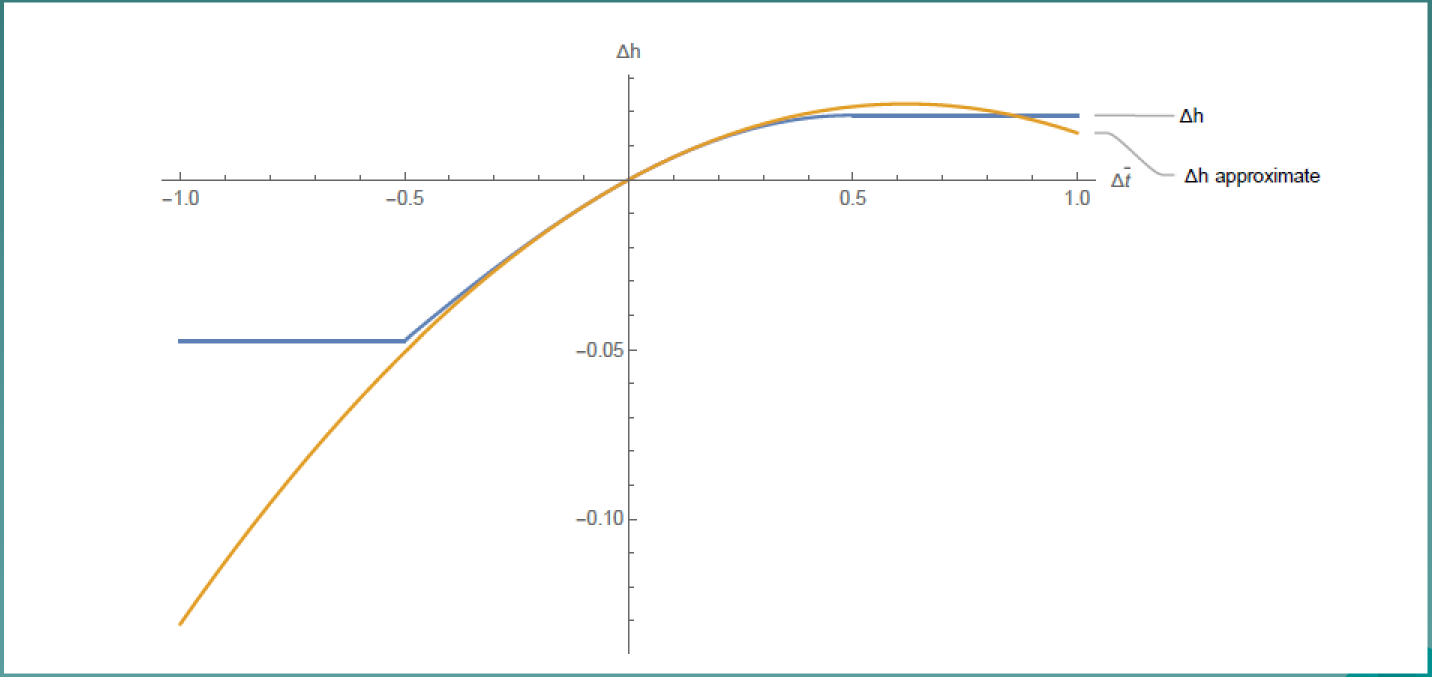
Which is the same as:

$$\phi_N \simeq -G \int \frac{\rho(\vec{x}', t)}{r} d^3x' = -\frac{G}{r} \int \rho(\vec{x}', t) d^3x' = -\frac{GM}{r}$$

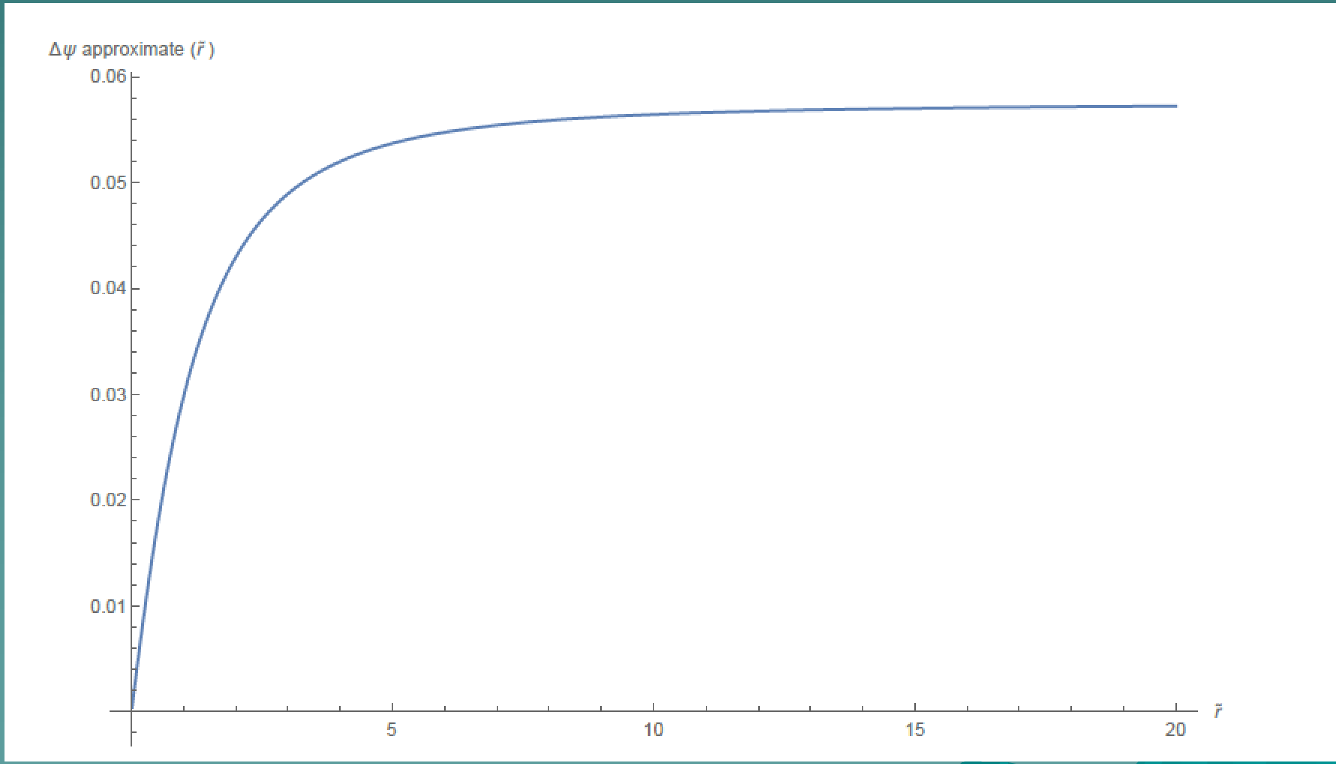

$\Delta\psi(\tilde{r})$
$\tilde{r}$
1
2
3
4
−0.005
−0.010
−0.015
−0.020
−0.025
−0.030


$\Delta\psi(\tilde{r})$
$\tilde{r}$
5
10
15
20
-0.005
-0.010
-0.015
-0.020
-0.025
-0.030

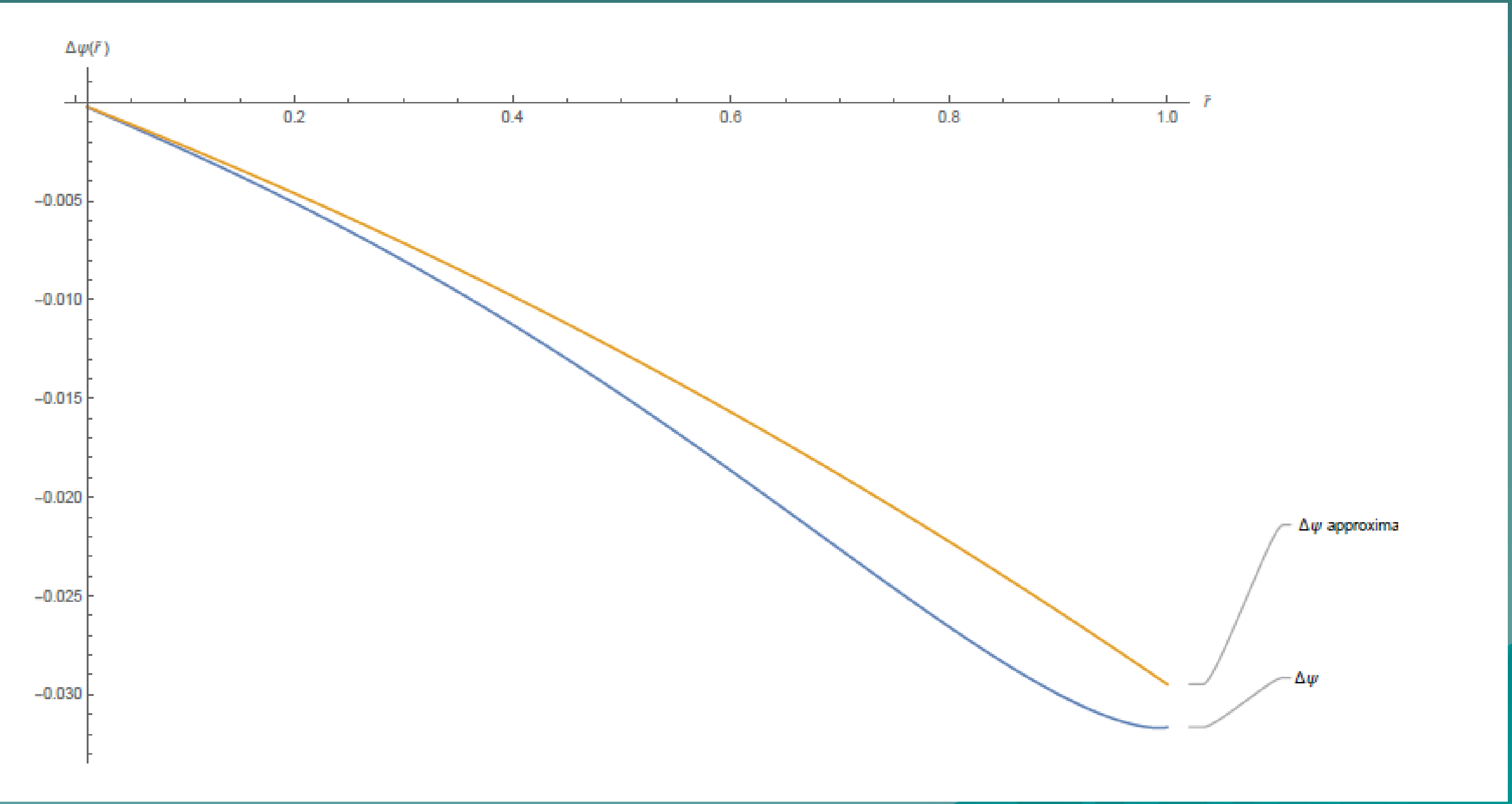
How good is the second order approximation?



Not very good, if one considers the galaxy + IGM



This is because the validity domain of the second order approximation has been violated. This can be amended by limiting the domain of integration to include only the galaxy.



The virial theorem and the Comma Cluster

The source of all "dark matter" evil

The Swiss-American astronomer Fritz Zwicky is arguably the most famous and widely cited pioneer in the field of dark matter.

He studied (1933) the redshifts of various galaxy clusters, as published by Hubble and Humason, and noticed a large scatter in the apparent velocities of eight galaxies within the Coma Cluster, with differences that exceeded 2000 km/s.

The fact that Coma exhibited a large velocity dispersion with respect to other clusters had already been noticed by Hubble and Humason, but Zwicky went a step further, applying the virial theorem to the cluster in order to estimate its mass.

A crucial difference between Newtonian and Retarded Forces

$$\vec{f} = \vec{f}_N + \vec{f}_r$$

$$\vec{f}_N \equiv -\vec{\nabla}\phi_N = -G\int \frac{\rho(\vec{x}', t)}{R^2}\hat{R}d^3x', \quad \hat{R} \equiv \frac{\vec{R}}{R},$$

$$\vec{f}_r \equiv -\vec{\nabla}\phi_r = -\frac{G}{2c^2}\int \rho^{(2)}(\vec{x}', t)\hat{R}d^3x'$$

A crucial difference between Newtonian and Retarded Forces

$$\rho_j = m_j \delta^{(3)}(\vec{x}' - \vec{r}_j(t))$$

$$\phi_{Nj} = -G\frac{m_j}{R_j(t)}, \qquad \vec{R}_j(t) = \vec{x} - \vec{r}_j(t), \qquad R_j(t) = |\vec{R}_j(t)|$$

A crucial difference between Newtonian and Retarded Forces

$$\phi_{rj} = -\frac{Gm_j}{2c^2}\frac{\partial^2}{\partial t^2}R_j(t) = \frac{Gm_j}{2c^2}\left(\hat{R}_j\cdot\vec{a}_j - \frac{\vec{v}_j^2 - (\vec{v}_j\cdot\hat{R}_j)^2}{R_j(t)}\right),$$

$$\hat{R}_j \equiv \frac{\vec{R}_j}{R_j},\ \vec{v}_j \equiv \frac{d\vec{r}_j}{dt},\ \vec{a}_j \equiv \frac{d\vec{v}_j}{dt}.$$

The gravitational force generated by particle j on particle k

$$\vec{F}_{j,k} = \vec{F}_{Nj,k} + \vec{F}_{rj,k}$$

$$\vec{F}_{Nj,k} = -G\frac{m_j m_k}{R_{k,j}^2}\hat{R}_{k,j}, \qquad \vec{R}_{k,j} \equiv \vec{r}_k - \vec{r}_j, \ R_{k,j} \equiv |\vec{R}_{k,j}(t)|, \ \hat{R}_{k,j} \equiv \frac{\vec{R}_{k,j}}{R_{k,j}},$$

$$\vec{F}_{rj,k} = \frac{G m_j m_k}{2R_{k,j}^2 c^2}\left(R_{k,j}\vec{a}_{\perp j,k} + \hat{R}_{k,j}\vec{v}_{\perp j,k}^2 - 2(\vec{v}_{j,k}\cdot\hat{R}_{k,j})\vec{v}_{\perp j,k}\right)$$

$$\vec{a}_{\perp j,k} \equiv \vec{a}_j - (\vec{a}_j\cdot\hat{R}_{k,j})\hat{R}_{k,j}, \qquad \vec{v}_{\perp j,k} \equiv \vec{v}_j - (\vec{v}_j\cdot\hat{R}_{k,j})\hat{R}_{k,j}.$$

Newton's third law

$$\vec{F}_{N\,k,j} = -\vec{F}_{N\,j,k}$$

$$\vec{F}_{r\,k,j} \neq -\vec{F}_{r\,j,k}$$

Acceleration and velocity of particles j and k are unrelated.

The Virial Theorem – Newtonian Version

$$\left\langle v^2 \right\rangle_t = \frac{\langle |V_{NT}| \rangle}{M} = \frac{GM}{r_g},$$

$$r_g = \frac{GM^2}{|V_{NT}|}$$

The Correct Virial Theorem

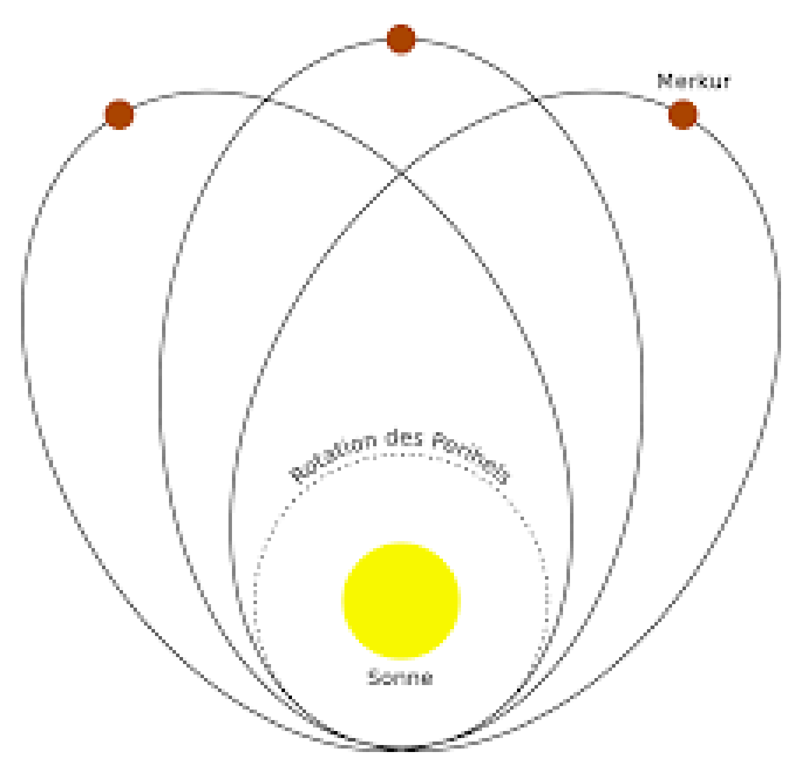
$$M_d \equiv \frac{|V_{rT}| r_g}{GM} \Rightarrow \left\langle v^2 \right\rangle_t = \frac{G(M+M_d)}{r_g} \Rightarrow M_d = \frac{r_g \left\langle v^2 \right\rangle_t}{G} - M.$$

The Virial Theorem

$$
\begin{aligned}
V_{rT} &= \int \rho(\vec{x})\vec{\nabla}\phi_r \cdot \vec{x} d^3x = -\frac{G}{2c^2}\int \rho(\vec{x})\vec{\nabla}\left(\int R\rho^{(2)}(\vec{x}')d^3x'\right)\cdot \vec{x} d^3x \\
&= -\frac{G}{2c^2}\int\int d^3x d^3x' \rho(\vec{x})\frac{\vec{R}\cdot\vec{x}}{R}\rho^{(2)}(\vec{x}') = -G\int\int d^3x d^3x' \frac{\rho(\vec{x})\rho_d(\vec{x}')}{R}.
\end{aligned}
$$

$$
\rho_d \equiv \rho^{(2)}\frac{\vec{R}\cdot\vec{x}}{2c^2}
$$

Precession of the Perihelion for Mercury



Causes of the precession of perihelion for Mercury (arcsec/Julian century)	Cause
532.3035	Gravitational tugs of other solar bodies
0.0286	Oblateness of the Sun (quadruple moment)
42.9799	General Relativity effect (Schwarzschild - like)
−0.0020	Lense-Thirring precession
575.31	**Total predicted**
574.10 ± 0.65	**Observed**

U. Le Verrier (1859), (in French), "Lettre de M. Le Verrier à M. Faye sur la théorie de Mercure et sur le mouvement du périhélie de cette planète", Comptes rendus hebdomadaires des séances de l'Académie des sciences (Paris), vol. 49 (1859), pp.379–383.

Time independent GR

$$\delta\theta = \frac{6\pi G M}{a c^2 \left(1 - e^2\right)}$$

Einstein used the static Schwarzschild solution in his calculations and was thus unable to consider time dependent effects.

$$-c^2\, d\tau^2 = -\left(1 - \frac{r_s}{r}\right) c^2\, dt^2 + \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 + r^2 g_\Omega$$

$$g_\Omega = \left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

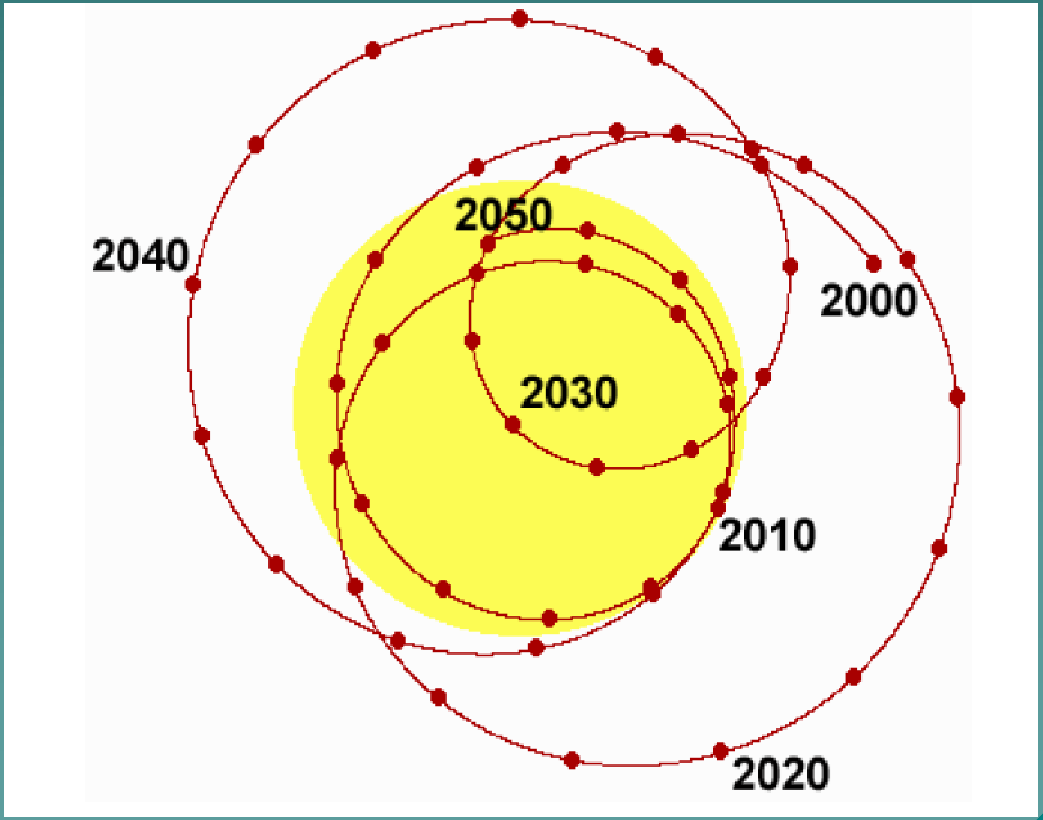
$$r_s = \frac{2GM}{c^2}$$

Lienard - Wiechert potential

$$\phi_{LWp} = -\frac{GM_p}{R_p(1-\hat{R}_p\cdot\vec{\beta}_p)}\bigg|_{t_{rp}}, \quad \hat{R}_p = \frac{\vec{R}_p}{R_p}, \quad t_{rp} = t - \frac{R_p(t_{rp})}{c}, \quad R_p(t) = |\vec{x}-\vec{x}_p(t)|$$

$$\Delta\phi \equiv \phi_{LW\mathrm{sun}} - \phi, \qquad \phi_{LW\mathrm{sun}} = \phi + \Delta\phi$$

The Solar System Center of Mass



Krizek, M.: Influence of celestial parameters on Mercury's perihelion shift. Bulg. Astron. J. 27 (2017), 41-56.

Lienard - Wiechert potential

$$\Delta\phi = \Delta\phi_\beta + \Delta\phi_{tr} + \Delta\phi_d,$$

$$\Delta\phi_d = -GM_{\text{sun}}\left(\frac{1}{R_{\text{sun}}(t)} - \frac{1}{r}\right), \qquad R_{\text{sun}}(t) = |\vec{x} - \vec{x}_{\text{sun}}(t)|,$$

$$\Delta\phi_{tr} = -GM_{\text{sun}}\left(\frac{1}{R_{\text{sun}}(t_r)} - \frac{1}{R_{\text{sun}}(t)}\right), \qquad t_r = t - \frac{R_{\text{sun}}(t_r)}{c},$$

$$\Delta\phi_\beta = -GM_{\text{sun}}\left(\frac{1}{(1 - \hat{R}_{\text{sun}}(t_r)\cdot\vec{\beta}_{\text{sun}}(t_r))R_{\text{sun}}(t_r)} - \frac{1}{R_{\text{sun}}(t_r)}\right)$$

Lienard - Wiechert potential

$$\Delta\phi_d \simeq -\phi\ (\hat{x}\cdot\hat{x}_{\rm sun})\frac{x_{\rm sun}}{r}, \qquad \hat{x}\equiv\frac{\vec{x}}{|\vec{x}|}=\frac{\vec{x}}{r}, \quad \hat{x}_{\rm sun}\equiv\frac{\vec{x}_{\rm sun}}{|\vec{x}_{\rm sun}|},$$

$$\Delta\phi_{tr} = -\Delta\phi_\beta = \phi\ (\hat{x}\cdot\vec{\beta}_{\rm sun}).$$

$$\phi_{LW\,{\rm sun}} = \phi\left(1-(\hat{x}\cdot\hat{x}_{\rm sun})\frac{x_{\rm sun}}{r}\right)$$

Lienard - Wiechert potential

$$\frac{x_{\text{sun}}}{r} \simeq \frac{2 \text{ Sun Radius}}{\text{Mercury Perihelion}} \simeq 2\frac{6.96 \cdot 10^{8}}{4.60 \cdot 10^{10}} \simeq 3\%.$$

Taking time dependent effects into account

$$\delta\theta = \frac{6\pi G M_{eff}}{ac^2\left(1-e^2\right)},$$

$$M_{eff} \equiv M_{sun}(1 \pm 0.03)$$

$$\delta\theta = 43\text{''}/\mathrm{cy} \pm 1.3\text{''}/\mathrm{cy}.$$

1. Weak Gravity suffices to describe gravity in the solar system (contrary to the claims the only strong gravity can explain Mercury's perihelion precession).

2. Precession of the perihelion of Mercury can be fully explained within error bars.

MOND & Retardation Theory

$$\vec{F}_M = -\frac{GM}{\mu(\frac{a}{a_0})r^2}\hat{r}$$

Standard Interpolation Function

$$\vec{F}_M = -\frac{GM}{\mu(\frac{a}{a_0})r^2}\hat{r}$$

$$\mu(x) = \frac{x}{\sqrt{1+x^2}} \quad \Rightarrow \quad \mu(\frac{a}{a_0}) = \frac{1}{\sqrt{1+(\frac{a_0}{a})^2}}$$

Deep MOND Regime

$$a_0 \gg a, \ \mu \simeq \frac{a}{a_0}$$

$$a = \frac{v^2}{r}$$

$$\vec{F}_M = -\frac{G M a_0}{v^2 \, r} \hat{r}$$

Deep MOND Regime = Retardation Force becomes significant

$$|\ddot{M}| = \frac{2 M a_0 c^2}{v^2 \, r}.$$

Problematic given a flat velocity as the left-hand side is spatial independent and the right-hand side depends on r, but can be used to calculate the mass secod derivative approximately.

$$a_0 = 1.2 \times 10^{-10} \mathrm{ms}^{-2}$$

The velocity at 15.33 kpc from the center of the M33 galaxy is 135,640 ms^{-1}.

We thus obtain $|\ddot{M}| \simeq 4.94 \times 10^{16}$ kgs^{-2}

Deep MOND Regime = Retardation Force becomes significant

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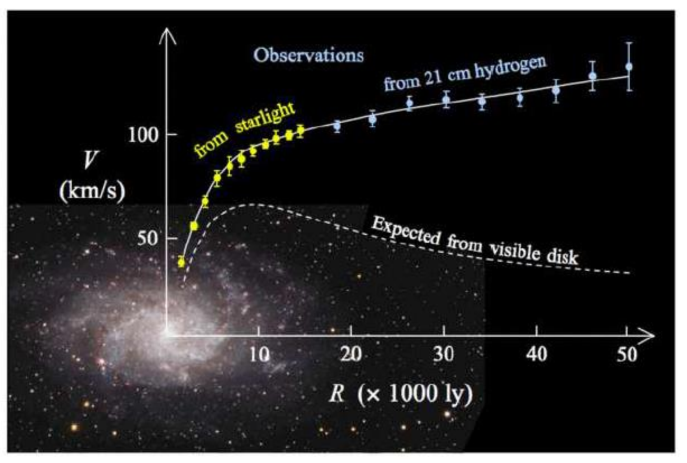
We thus obtain $|\ddot{M}| \simeq 4.94 \times 10^{16}$ kgs^{-2}.

Parameter Estimation

$$t_r = \sqrt{\frac{M}{|\ddot{M}|}} \simeq 6.35\ 10^{11}\ \mathrm{s}$$

$$t_r \simeq 20{,}129 \text{ years}$$



$$R_r = c t_r \simeq 20,129 \text{ light years}$$

Retardation Theory as a MOND type of theory

$$\vec{F}_M = -\frac{GM}{\mu(\frac{a}{a_0})r^2}\hat{r}$$

Non-Standard Interpolation Function

$$\vec{F}_M = -\frac{GM}{\mu(\frac{a}{a_0})r^2}\hat{r}$$

$$\mu(x) = \frac{x^2}{\sqrt{1+x^4}} = \frac{1}{\sqrt{1+x^{-4}}} \quad \Rightarrow \quad \mu(\frac{a}{a_0}) = \frac{1}{\sqrt{1+(\frac{a_0}{a})^4}}$$

Deep MOND Regime

$$a_0 \gg a, \; \mu \simeq \left(\frac{a}{a_0}\right)^2$$

$$a = \frac{v^2}{r}$$

$$\vec{F}_M = -\frac{G M a_0^2}{v^4}\hat{r}$$

Deep MOND Regime

$$|\ddot{M}| = \frac{2Ma_0^2c^2}{v^4}.$$

Tully-Fisher relation:

$$M = kv^4, \qquad k = \frac{|\ddot{M}|}{2a_0^2c^2}$$

a_0 calculated

$$|\ddot{M}| = \frac{2 M a_0^2 c^2}{v^4}.$$

$$a_0 = \frac{v^2}{c}\sqrt{\frac{|\ddot{M}|}{2M}}.$$

Retardation Theory is (approximately) a MOND type theory

$$\vec{F}_M = -\frac{GM}{\mu(\frac{a}{a_0})r^2}\hat{r} \simeq \vec{F}_N + \vec{F}_{ar}$$

But with a nonstandard interpolation function.

Conclusion

We show that galactic rotation curves, lensing effects, the Tully-Fisher relation, and the coma cluster virial high velocities are explained in the framework of standard GR as effects due to retardation without assuming any exotic matter or modifications of the theory of gravity.

Conclusion

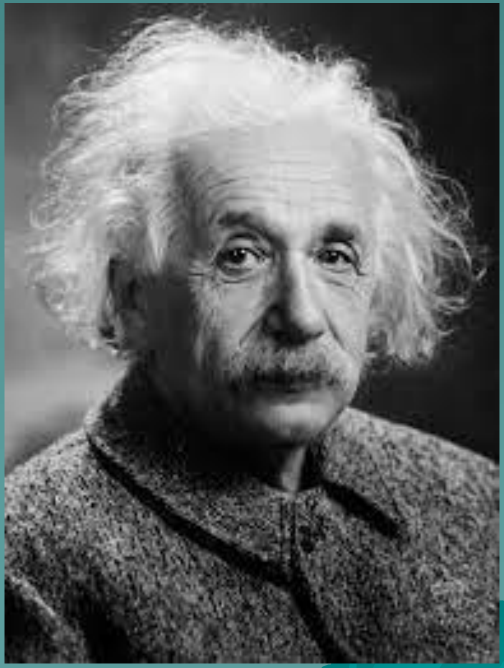
Retardation theory can be approximated as a MOND type theory with non-standard interpolation function.

What will happen if the mass outside the galaxy is totally depleted or not yet depleted?
In this case retardation force should vanish. This was indeed reported for the galaxies NGC1052-DF2 and NGC 1052-DF4.

Pieter van Dokkum, Shany Danieli, Yotam Cohen, Allison Merritt, Aaron J. Romanowsky, Roberto Abraham, Jean Brodie, Charlie Conroy, Deborah Lokhorst, Lamiya Mowla, Ewan OSullivan & Jielai Zhang "A galaxy lacking dark matter" Nature volume 555, pages 629632 (29 March 2018) doi:10.1038/nature25767.

More Fundamental Conclusions

“**The Lord God is subtle**, **but malicious** he is **not**. ” — Albert Einstein. Remark during visit to Princeton University (1921) - “dark matter” effects are subtle indeed.




BEWARE THE DARK SIDE

A Recent Paper

International Journal of Modern Physics D

Fitting of supernovae without dark energy

M. López-Corredoira[1,2] and J. I. Calvo-Torel[2]

[1] *Instituto de Astrofísica de Canarias,*
E-38205 La Laguna, Tenerife, Spain
E-mail: martin@lopez-corredoira.com
[2] *Dep. de Astrofísica, Universidad de La Laguna,*
E-38206 La Laguna, Tenerife, Spain

arXiv:2207.14688v1 [astro-ph.CO] 28 Jul 2022

- ΛCDM, $\Omega_M = 0.287$ ($Q = 0.678$).
- FLRW with curvature, $\Omega_M = 0.355$, $\Omega_\Lambda = 0.835$ ($Q = 0.696$).
- Einstein–de Sitter with extinction ($Q = 0.453$).
- Linear Hubble–Lemaître law static Euclidean with extinction ($Q = 0.333$).
- Static Euclidean with tired light with extinction ($Q = 0.275$).
- FLRW with curvature and with evolution $\Omega_M = 0.957$, $\Omega_\Lambda \approx 0$ ($Q = 0.702$).
- Einstein–de Sitter with evolution $\Omega_M = 1$, $\Omega_\Lambda = 0$ ($Q = 0.709$).

LOOKING FORWARD TO THE NEXT 40 YEARS OF MOND!