Quaternionic mass quantisation

Robert A. Wilson

Queen Mary University of London

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THE BEGINNING

Introduction

- In 2015 I discovered the mass equation:

$$e^- + \mu^- + \tau^- + 3p^+ = 5n^0$$

$$.511 + 105.658 + (1776.86 \pm .12) \\ +3 \times 938.272 = 5 \times 939.565(1)$$

  See "Remarks on the group-theoretical foundations of particle physics", *International Journal of Geometric Methods in Modern Physics* **19** (2022), 2250164.
- This equation may have profound consequences for quantum gravity, on every scale, and therefore for the modifications of Newtonian dynamics (MOND) that are necessary to explain astronomical observations.
- It tells us that 'mass' is not a real scalar (as in classical physics) or a complex number (as in quantum physics), but a quaternion, with four independent coordinates.

MATHEMATICAL LAWS

Newton's laws

Philosophiæ Naturalis Principia Mathematica (1687).

- Inertial mass m is defined by $\mathbf{F} = m\mathbf{a}$, so m is a scalar.
- Active gravitational mass $\mathbf{m}$ is defined by $\mathbf{a} = \mathbf{m}/\mathbf{r}.\mathbf{r}$, so $\mathbf{m}$ is a vector, pointing in the same direction as $\mathbf{a}$.
- The weak equivalence principle states that $m = |\mathbf{m}|$, and if also $a = |\mathbf{a}|$ then $m\mathbf{a} = a\mathbf{m}$. Since $\mathbf{m} \times \mathbf{a} = \mathbf{0}$, we have

$$2\mathbf{F} = m\mathbf{a} + a\mathbf{m} + \mathbf{m} \times \mathbf{a}. \tag{2}$$

- If $F = |\mathbf{F}|$ then $F = ma = \mathbf{m}.\mathbf{a}$ so

$$\begin{aligned} 2F &= ma + \mathbf{m}.\mathbf{a} \\ 0 &= ma - \mathbf{m}.\mathbf{a} \end{aligned} \tag{3}$$

Hamilton's quaternions

- Hamilton (1843) unified a scalar $m = m_0$ and a vector $\mathbf{m} = (m_1, m_2, m_3)$ into a quaternion

$$\mathcal{M} = m + \mathbf{m} = m_0 + m_1 i + m_2 j + m_3 k \tag{4}$$

with multiplication defined by

$$i^2 = j^2 = k^2 = ijk = -1. \tag{5}$$

- Then the generalised Newtonian law becomes essentially

$$\mathcal{F} = \mathcal{M}\mathcal{A} \tag{6}$$

where $\mathcal{A} = a + \mathbf{a}$ and $\mathcal{F} = F + \mathbf{F}$.

- Formulations of MOND can be obtained by relaxing any or all of the Newtonian rules

$$\begin{aligned} &\mathbf{a}.\mathbf{a} = a^2, \quad \mathbf{m}.\mathbf{m} = m^2, \quad \mathbf{F}.\mathbf{F} = F^2, \\ &\mathbf{m} \times \mathbf{a} = \mathbf{0}, \quad \mathbf{m} \times \mathbf{F} = \mathbf{0}, \quad \mathbf{F} \times \mathbf{a} = \mathbf{0}. \end{aligned} \tag{7}$$

Gravitational charges

- Attempts to quantise a (real or complex) scalar mass have failed consistently.
- But it is easy to quantise a quaternionic mass for the five fundamental gravitating particles: neutron, proton, electron, muon and tau.
- For example, the following choice of a scalar electric charge plus a vector gravitational charge exhibits the symmetry between the three generations of electron:

$$\begin{aligned} n &= 0 + (1,1,1) && = i + j + k \\ p &= 1 + (1,1,1) && = 1 + i + j + k \\ e &= -1 + (1,1,0) && = -1 + i + j \\ \mu &= -1 + (0,1,1) && = -1 + j + k \\ \implies \tau &= -1 + (1,0,1) && = -1 + k + i \end{aligned} \tag{8}$$

PHYSICAL THEORIES

A mass gauge

- To get a mass $\mathcal{M} = m + \mathbf{m}$ from a charge $\mathcal{Q} = q + \mathbf{q}$, we need a quaternionic gauge $\mathcal{G} = g + \mathbf{g}$ so that

$$\begin{aligned} \mathcal{M} &= -\mathcal{G}\mathcal{Q} \\ \implies m &= -gq + \mathbf{g}.\mathbf{q} \end{aligned} \tag{9}$$

- Substituting in the values of $\mathcal{Q}$ and m for the four particles p, n, e and μ we can solve for $\mathcal{G}$ in MeV/c^2:

$$\mathcal{G} = 1.29333 + 835.20037i - 835.98271j + 940.34775k \tag{10}$$

- The k term is roughly the astronomical approximation to baryonic mass. The real part is the mass difference between neutron and proton, and both the other terms involve the muon, so give leptonic mass.

Relation to the weak force

- The three dimensionless parameters that describe $\mathcal{G}$ in the standard model of particle physics are

$$\begin{aligned} (g_0 + g_1 + g_2)/g_0 &\approx .395103 \approx \cos(66.7276^\circ) \\ (g_1 - g_2 - g_3)/g_3 &\approx .77719687 \approx \cos^2(28.165516^\circ) \\ (g_1 + g_2 + g_3)/g_3 &\approx .99916804 \approx \cos(2.337325^\circ) \end{aligned} \tag{11}$$

- The second of these angles is the electro-weak mixing angle θ_W, and the other two come from the CKM matrix, namely the CP-violating phase δ_{13} and the mixing angle θ_{23} between the second and third generations of quarks.
- Experimental values are

$$\begin{aligned} \delta_{13} &= 68.8^\circ \pm 4.5^\circ \\ \sin^2\theta_W &= .22290 \pm .00030 \\ \theta_{23} &= 2.38^\circ \pm .06^\circ \end{aligned} \tag{12}$$

Second quantisation

- In particular, gravitational mass is intimately linked to properties of the weak nuclear force. Weak interactions are always associated with the emission or absorption of a neutrino and/or an antineutrino.
- An interaction (at a distance) between two quantised masses $\mathcal{P}$ and $\mathcal{Q}$ involves 16 individual terms. Modulo scalars, there are five types of (massless) particles emitted or absorbed by $\mathcal{P}$ and $\mathcal{Q}$, each in three directions in space. These are photons, in two polarisations, and neutrinos, in three generations.
- The gravitational part of the interaction comes from the imaginary parts $\mathbf{p}$ and $\mathbf{q}$, and consists of 9 individual products, combining into 4 results $\mathbf{p}.\mathbf{q} + \mathbf{p} \times \mathbf{q}$, leaving 5 dimensions of null products. These null products form a spin 2 representation.

ASTRONOMICAL OBSERVATIONS

Perturbations of the mass gauge?

- MOND requires $\mathcal{G}$ not to be constant, in order for gravitational and inertial mass to diverge significantly.
- Here I present evidence that the gauge $\mathcal{G}$ used in particle physics is strongly dependent on peculiarities of Solar System astronomy in the mid-20th century.
- First, we convert mass ratios to frequency ratios, using the fundamental equations

$$\begin{aligned} E &= m\mathbf{c}.\mathbf{c} \\ E &= \mathbf{h}.\mathbf{f} \end{aligned} \tag{13}$$

- The two fundamental (vector!) frequencies on Earth are 1/year and 1/day, so we have two fundamental dimensionless parameters:

$$\begin{aligned} \text{1 Solar year} &= 365.24 \text{ Solar days} \\ \text{Earth's axial tilt} &\approx 23.44^\circ \end{aligned} \tag{14}$$

Coincidence or not?

- It is extremely surprising to find the two most fundamental mass ratios of particle physics in the formulae:

$$\begin{aligned} 1 + 1/2 \times 365.24 &\approx 1.0013690 \\ m(n)/m(p) &= 1.00137842 \\ 2 \times 365.24/\sin 23.44^\circ &\approx 1836.35 \\ m(p)/m(e) &= 1836.152 \end{aligned} \tag{15}$$

- The value of 1836.152 was first adopted as standard in 1973, and requires an angle of tilt of

$$\arcsin(.3978320) \approx 23.44272^\circ \tag{16}$$

which was attained (only!) in 1957, 1967 and 1973.

Further coincidences

- Possibly the small discrepancy (of 9ppm) in $m(n)/m(p)$ may be due to the gravitational effect of the rest of the Solar System, predominantly Jupiter.
- The orbital parameters of Jupiter are

$$\begin{aligned} \text{1 Jovian year} &= 11.862 \text{ Earth years} \\ \text{Jupiter's orbital inclination} &\approx 1.31^\circ \end{aligned} \tag{17}$$

- We find

$$\begin{aligned} 11.862 \sin 23.44^\circ/\sin 1.31^\circ &\approx 206.4 \\ m(\mu)/m(e) &\approx 206.768 \end{aligned} \tag{18}$$

THE PUDDING

Experimental support for MOND?

Since these formulae are not constant, they are inconsistent with the equivalence principle. Experiments that may already contradict the EP include:

- Inconsistent measurements of G, at 10^{-4},
- Inconsistent measurements of $m(e)/m(p)$ using classical electrodynamics in the 1950s and 1960s, at 10^{-4}, before QED abolished the variation and fixed this (inertial mass) ratio at its 1973 gravitational value.
- Inconsistent measurements of the W/Z mass ratio, at 10^{-4}, approximately equal to $(g_1 + g_2)/g_3$.
- The muon $g-2$ anomaly, at 2×10^{-6}, equal to the change in gravitational field direction (in natural units, i.e. radians) across the 15 metres of the experiment.
- CP violation of neutral kaon decay, also at 2×10^{-6}, across a 17 metre experiment.

Conclusion

- I have sketched some fundamental mathematical facts that suggest that inertial and gravitational mass are entirely different concepts.
- I have pointed out (approximately) ten suspicious coincidences, which back up this claim.
- Of course, I have not proved that they are not just coincidences, pure and simple.
- But I (and everybody else who has attempted it) have failed to disprove that they are not coincidences. That is, we have failed to prove that they are coincidences.
- That is the best one can ever hope for in physics, where the inviolable rule is that

  the proof of the pudding is in the eating.

A toast

- This conference is obviously mainly about

  Gravity Of Large Features (GOLF)

- But we can't completely avoid

  Weak Hypercharge and Isospin, the Strong force, Kaons, and Yang-Mills theory (WHISKY)

  an intoxicating liquor that causes visions of **dark matter**, **dark energy** and pink elephants
- On the other hand, the Gaelic form of the word just means pure (mathematical) water:

  Unified Inertia, Spacetime, Gravity and Everything (UISGE)

- Slainte mhath!