



# Radial Acceleration Relation & Mass-Velocity Dispersion Relation on BCG-cluster Scales

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Reference: [Tian et al. \(2020\), ApJ, 896, 70](#); [Tian et al. \(2021a\), ApJ, 910, 56](#); [Tian et al. \(2021b\), ApJL, 917, L24](#)

# Outline

- **Introduction**
- **Scaling Relations in Galaxies**
  - ❑ Radial Acceleration Relation (RAR)
  - ❑ Baryonic Tully-Fisher Relation (BFTR)
- **Scaling Relations on BCG-Cluster Scales**
  - ❑ Radial Acceleration Relation (RAR)
  - ❑ Mass-Velocity Dispersion Relation (MVDR)
- **Remarks**

# “Missing Mass Problem”

- ❖ Definitions of two accelerations:

**Newton’s law**

$$g_{obs} \equiv |-\nabla\Phi_{obs}| \quad =?$$

**Newton’s gravity**

$$g_{bar} \equiv \frac{GM_{bar}(<r)}{r^2}$$

- ❖ Assuming  $g_{obs} = g_{bar}$ , mass discrepancy is expected.  
“dark matter” is introduced to resolve the insufficient baryonic mass.
- ❖ **What if  $g_{obs} \neq g_{bar}$ ? Acceleration discrepancy instead.**



## Radial Acceleration Relation in Rotationally Supported Galaxies

Stacy S. McGaugh and Federico Lelli

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(Received 18 May 2016; revised manuscript received 7 July 2016; published 9 November 2016)

Independent measurements  
 for two-axis:

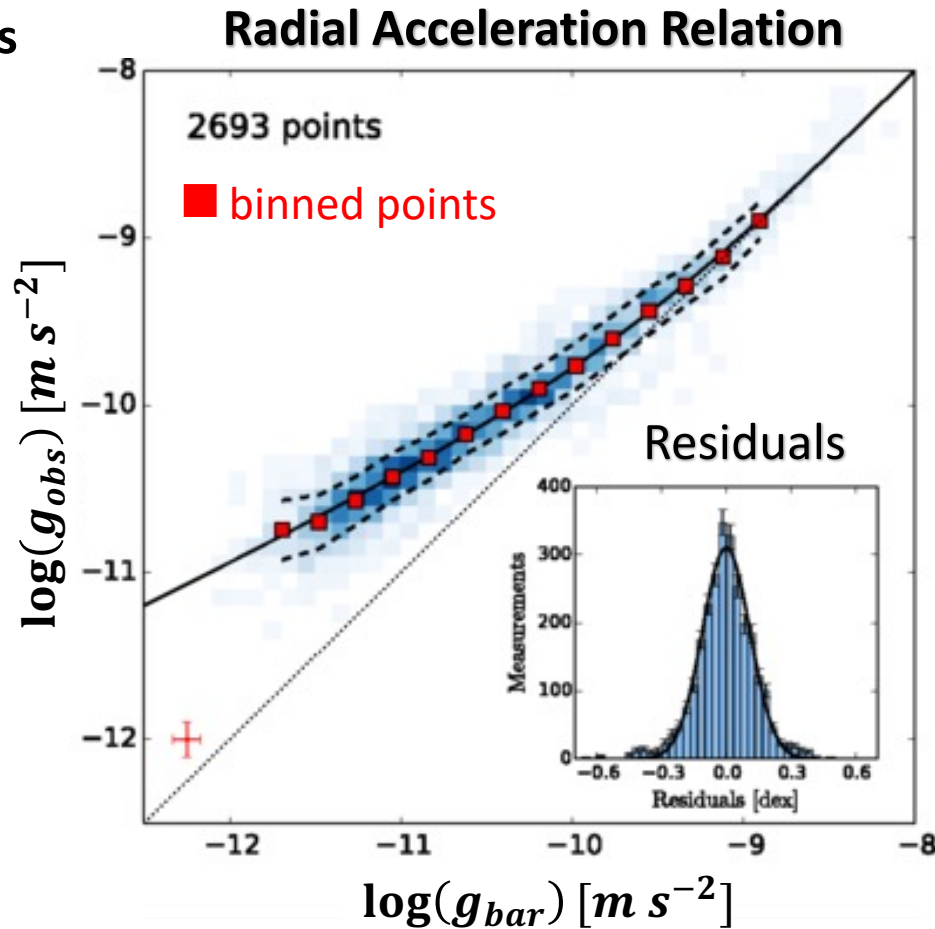
□ Vertical axis ( $g_{obs}$ )

$$g_{obs} = \frac{v^2}{r}$$

□ Horizontal axis ( $g_{bar}$ )

$$g_{bar} = \frac{GM_{bar}(< r)}{r^2}$$

$$M_{bar} = M_{star} + M_{gas}$$

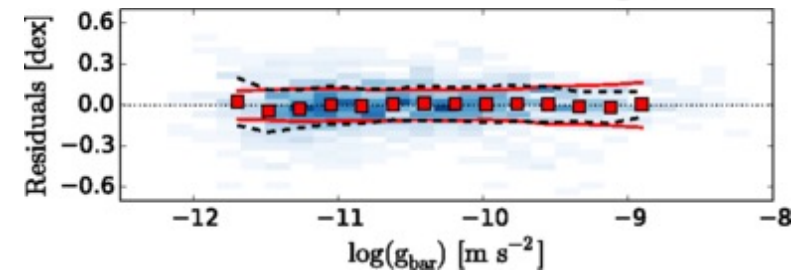


The tight relation gives

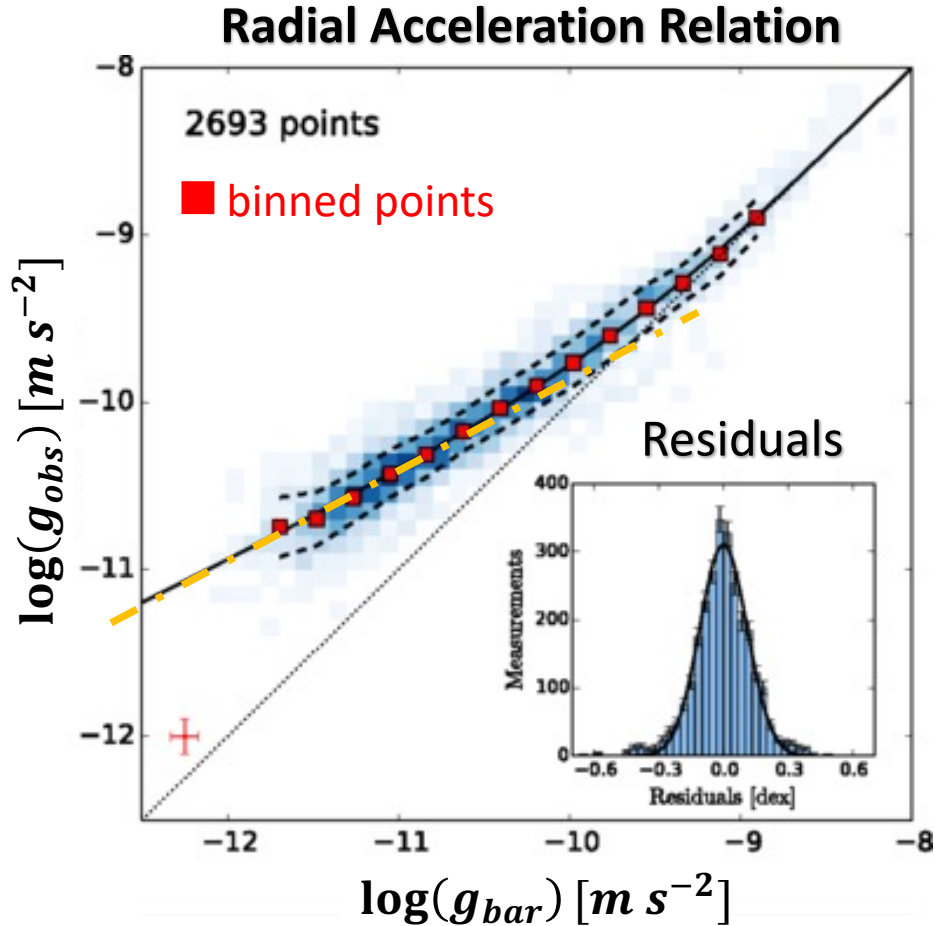
$$g_{obs} = \frac{g_{bar}}{1 - e^{-\sqrt{g_{bar}/g_{\dagger}}}}$$

$$g_{\dagger} = (1.20 \pm 0.02) \times 10^{-10} m s^{-2}$$

The residuals of  $\log(g_{bar})$



# Baryonic Tully-Fisher Relation



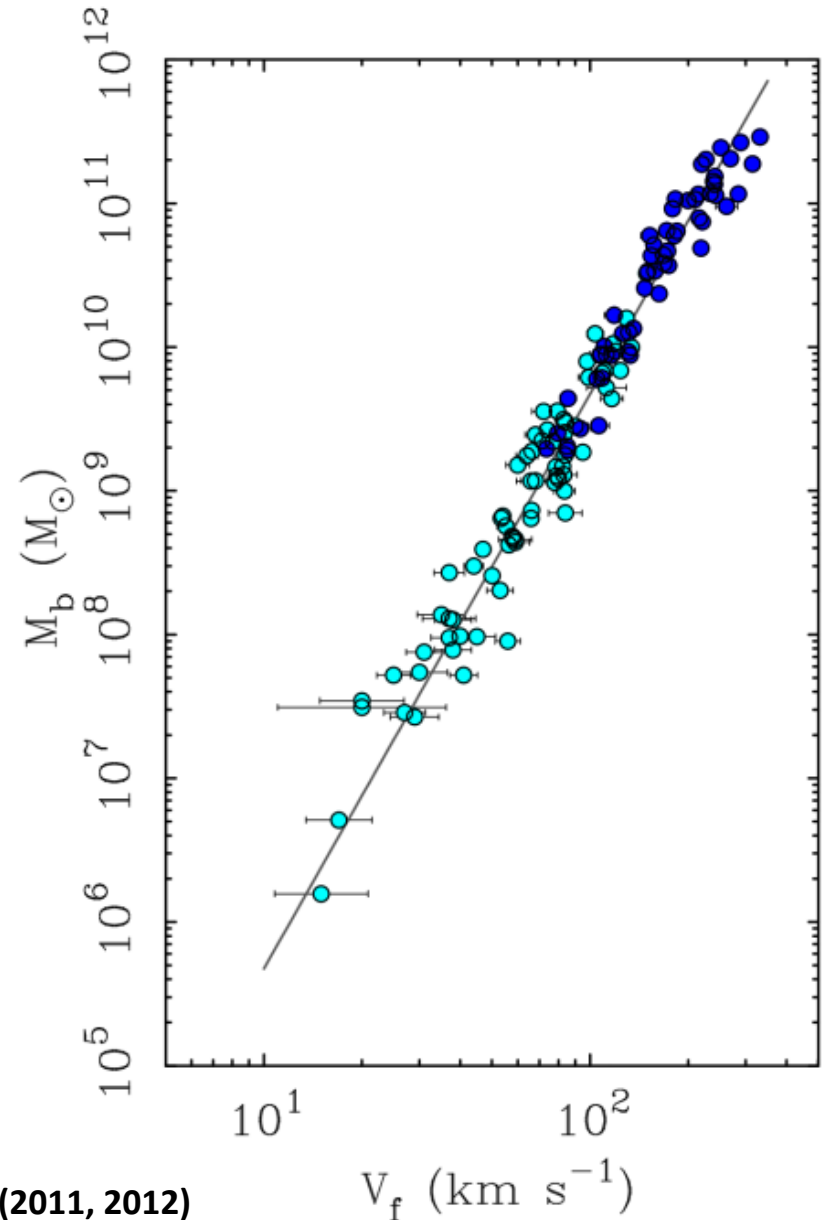
[McGaugh et al. \(2016\), PRL, 117, 201101](#)

$$g_{obs} = \sqrt{g_{bar} g_{\dagger}}$$

$$g_{obs} = \frac{v^2}{r}; \quad g_{bar} = \frac{GM_{bar}}{r^2}$$

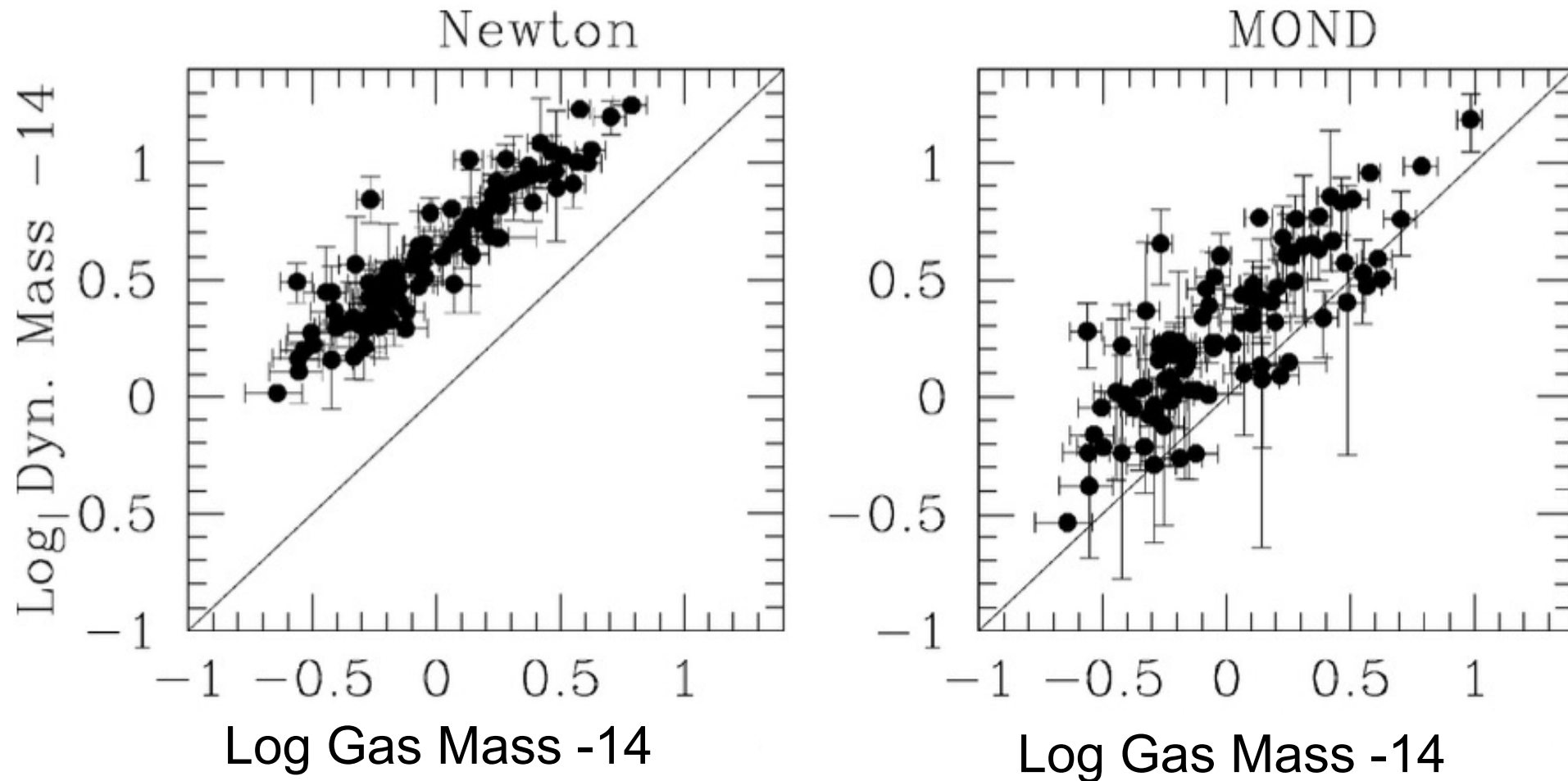
$$\rightarrow \frac{v^4}{\cancel{r^2}} = \frac{GM_{bar} g_{\dagger}}{\cancel{r^2}}$$

$$\rightarrow v^4 = GM_{bar} g_{\dagger}$$



McGaugh (2011, 2012)

# MOND in Galaxy Clusters



**Figure 10** (Left) the Newtonian dynamical mass of clusters of galaxies within an observed cutoff radius ( $r_{out}$ ) vs. the total observable mass in 93 X-ray-emitting clusters of galaxies (White et al. 1997). The solid line corresponds to  $M_{dyn} = M_{obs}$  (no discrepancy). (Right) the MOND dynamical mass within  $r_{out}$  vs. the total observable mass for the same X-ray-emitting clusters. From Sanders (1999).

# Lensing RAR on Cluster Scales?

# Galaxy Cluster: IDCS J1426

X-ray gas

member galaxies

Brightest Cluster  
Galaxy (BCG)

Components	Mass fraction
Galaxies	1%
Intergalactic gas	9%
Dark matter	90%



# Ingredients in Galaxy Clusters

❑ **Cluster Lensing And Supernova survey with Hubble (CLASH)**

❑ **Observational Mass (Lensing Mass)**

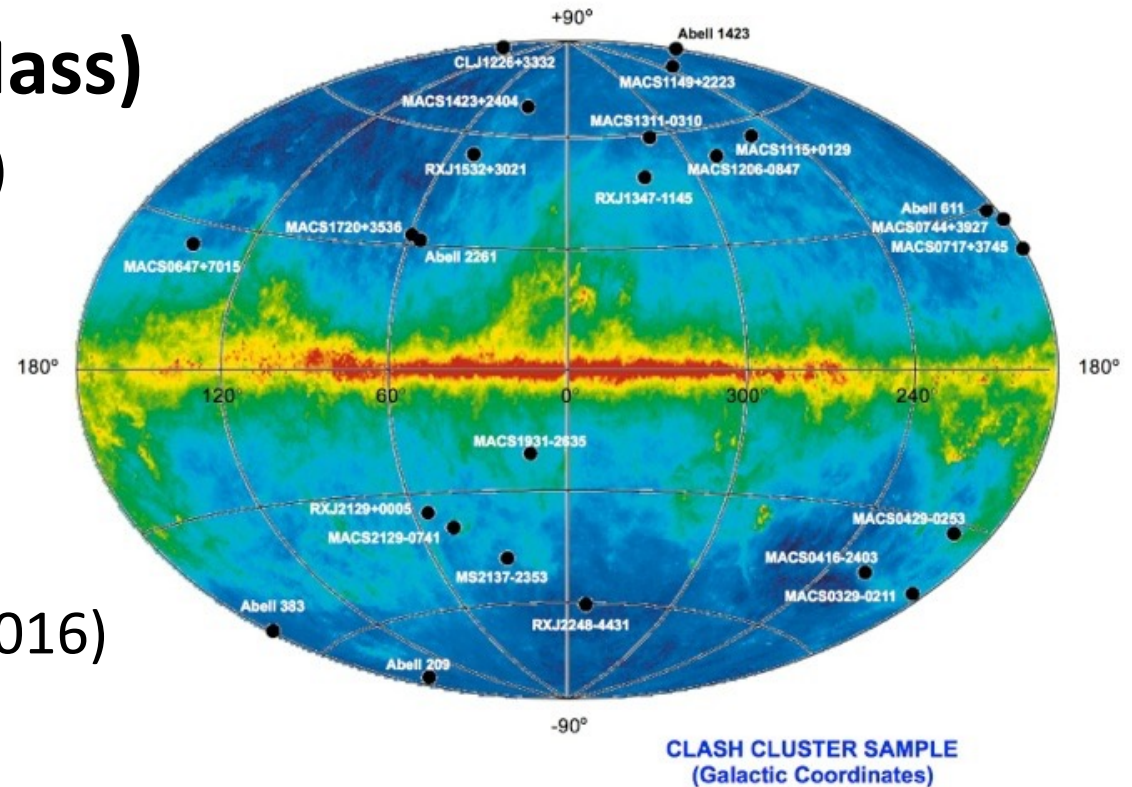
■ Strong & Weak-Lensing (Umetsu+ 2016)

❑ **Baryonic Mass**

■ X-ray gas (Donahue+ 2014)

■ Stellar mass (Chiu+ 2018)

■ Brightest Cluster Galaxy (BCG; Cooke+ 2016)





## The Radial Acceleration Relation in CLASH Galaxy Clusters

Yong Tian<sup>1</sup> , Keiichi Umetsu<sup>2</sup> , Chung-Ming Ko<sup>1,3</sup> , Megan Donahue<sup>4</sup> , and I-Non Chiu<sup>2</sup>

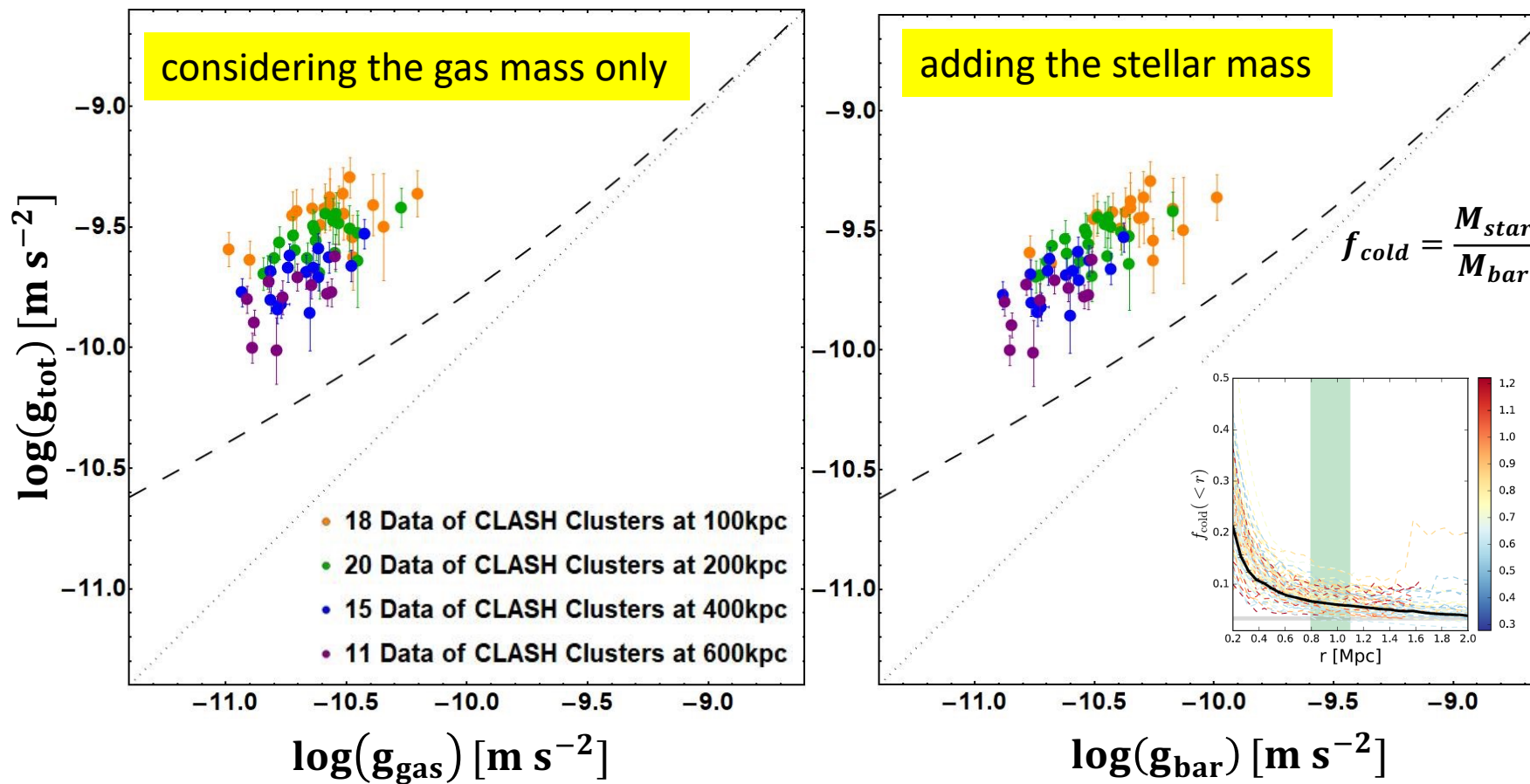
<sup>1</sup>Institute of Astronomy, National Central University, Taoyuan 32001, Taiwan; [yongtian@gm.astro.ncu.edu.tw](mailto:yongtian@gm.astro.ncu.edu.tw)

<sup>2</sup>Academia Sinica Institute of Astronomy and Astrophysics (ASIAA), No. 1, Section 4, Roosevelt Road, Taipei 10617, Taiwan; [keiichi@asiaa.sinica.edu.tw](mailto:keiichi@asiaa.sinica.edu.tw)

<sup>3</sup>Department of Physics and Center for Complex Systems, National Central University, Taoyuan 32001, Taiwan; [cmko@gm.astro.ncu.edu.tw](mailto:cmko@gm.astro.ncu.edu.tw)

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# The Radial Acceleration Relation in CLASH Galaxy Clusters

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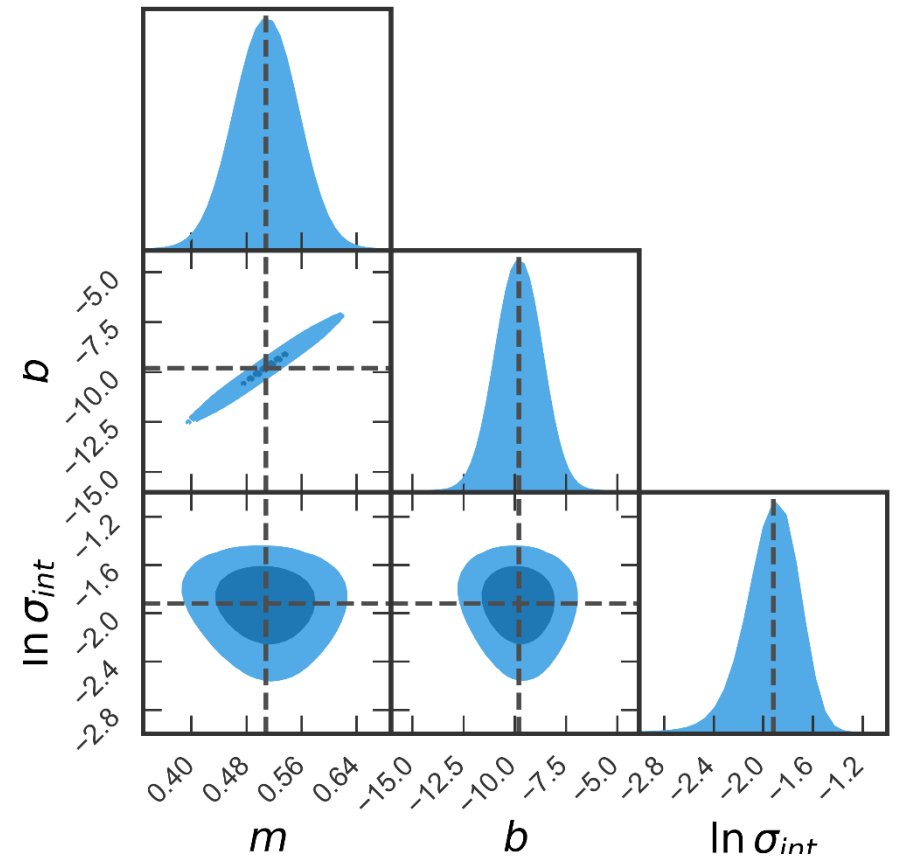
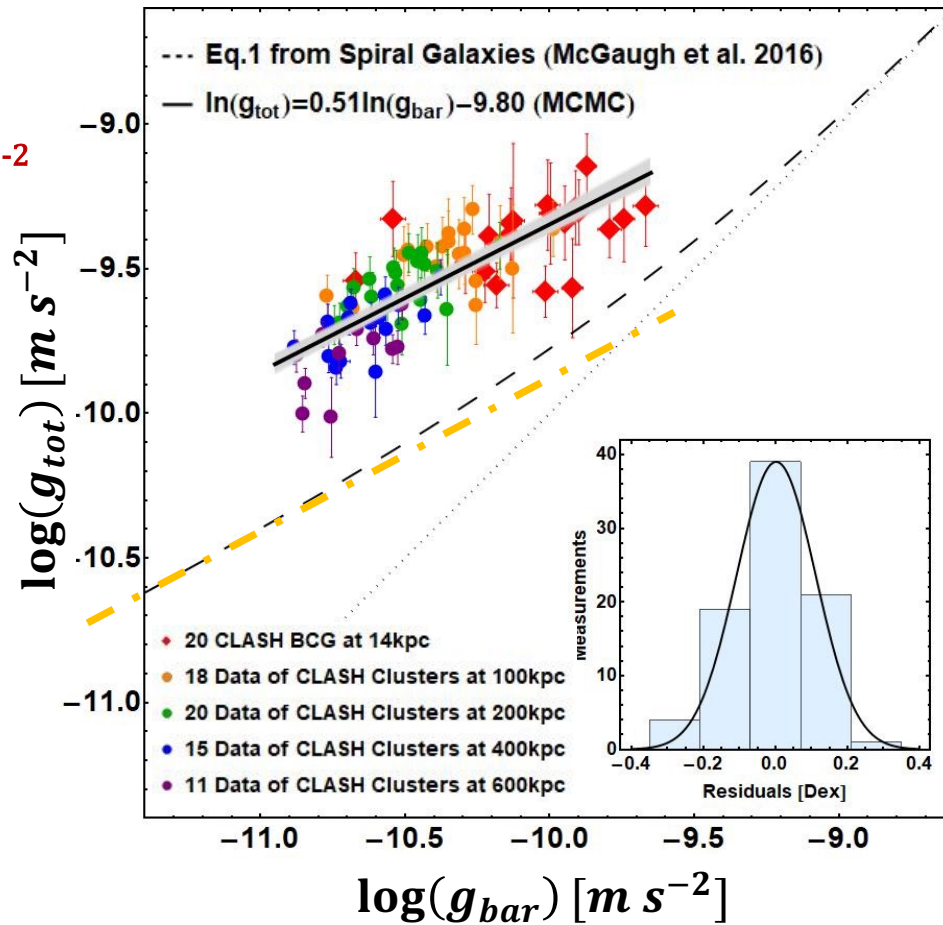
<sup>3</sup>Department of Physics and Center for Complex Systems, National Central University, Taoyuan 32001, Taiwan; [cmko@gm.astro.ncu.edu.tw](mailto:cmko@gm.astro.ncu.edu.tw)

<sup>4</sup>Physics and Astronomy Department, Michigan State University, East Lansing, MI 48824, USA

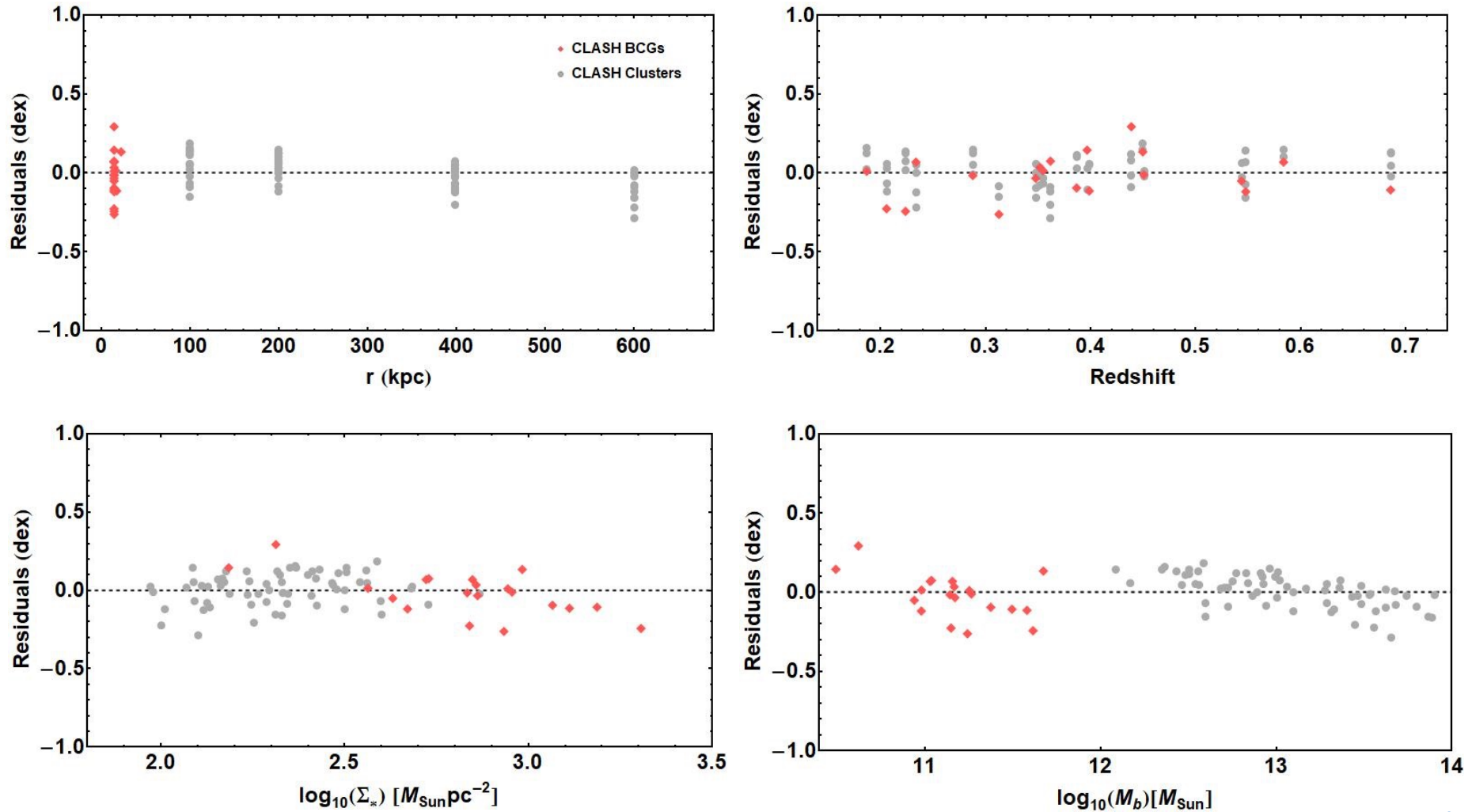
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$$g_{\text{tot}} = \sqrt{g_{\text{bar}} g_{\ddagger}}$$

$$g_{\ddagger} = (2.0 \pm 0.1) \times 10^{-9} \text{ ms}^{-2}$$



# Residuals in CLASH RAR



[Tian et al. \(2020\), ApJ, 896, 70](#)

# Four Issues in CLASH RAR

## Four Issues in RAR (Desmond 2017; Lelli+ 2017)

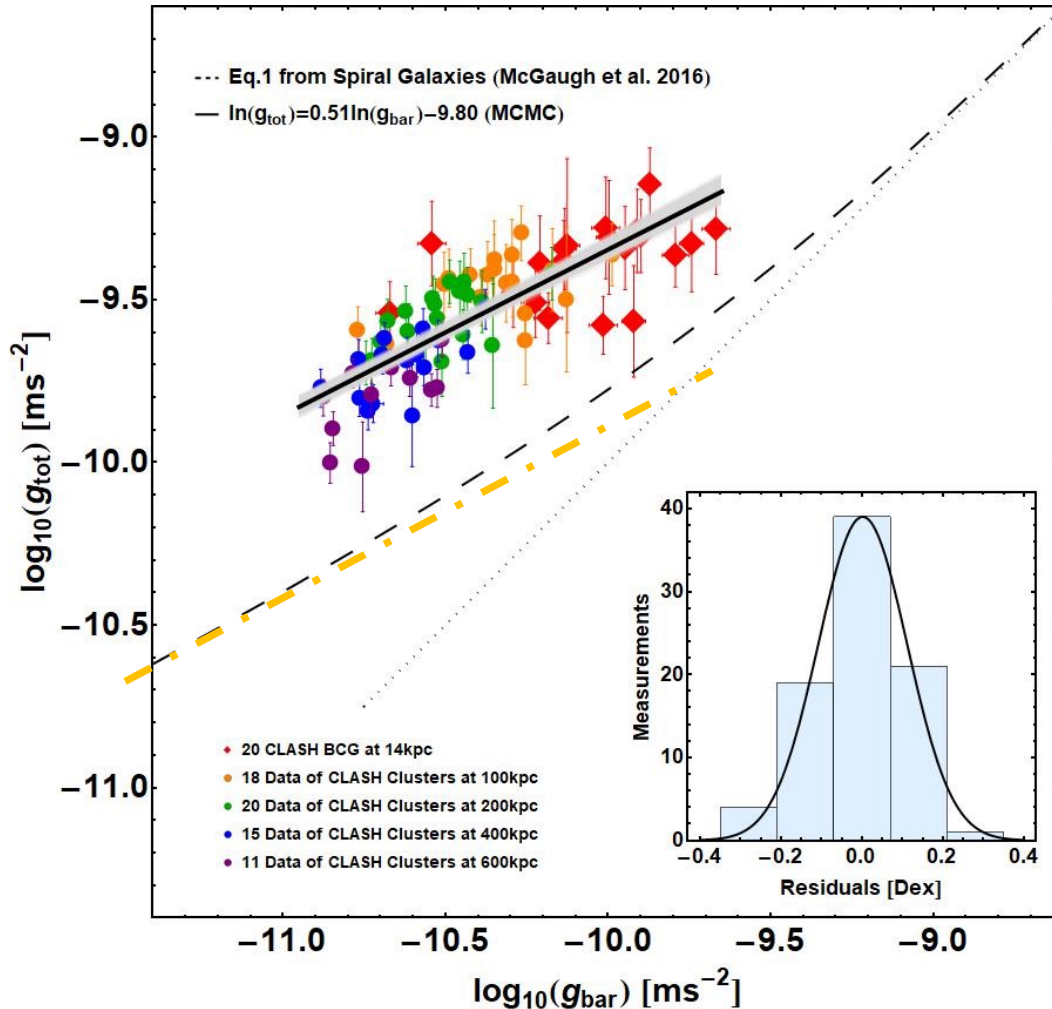
- i. the acceleration scale  $g_{\dagger} = 1.20 \times 10^{-10} \text{ ms}^{-2}$ ;
- ii. the low-acceleration slope (0.5);
- iii. the intrinsic tightness ( $\leq 0.11$  dex);
- iv. no correlations between residuals and other galaxy properties.

## Four Issues in CLASH RAR (Tian+ 2020)

- i. a new acceleration scale  $g_{\ddagger} = (2.0 \pm 0.1) \times 10^{-9} \text{ ms}^{-2}$ ;
- ii. the acceleration slope (0.5);
- iii. lognormal intrinsic scatter  $14.7_{-2.8}^{+2.9}\%$ ;
- iv. a **small correlation** between residuals and radius.

# Implication by the CLASH RAR

## The CLASH RAR



## Implication:

## Mass-Velocity Dispersion Relation

$$g_{\text{obs}} = \sqrt{g_{\text{bar}} g_{\ddagger}}$$

$$g_{\text{obs}} \propto \frac{\sigma^2}{r}; \quad g_{\text{bar}} = \frac{GM_{\text{bar}}}{r^2}$$

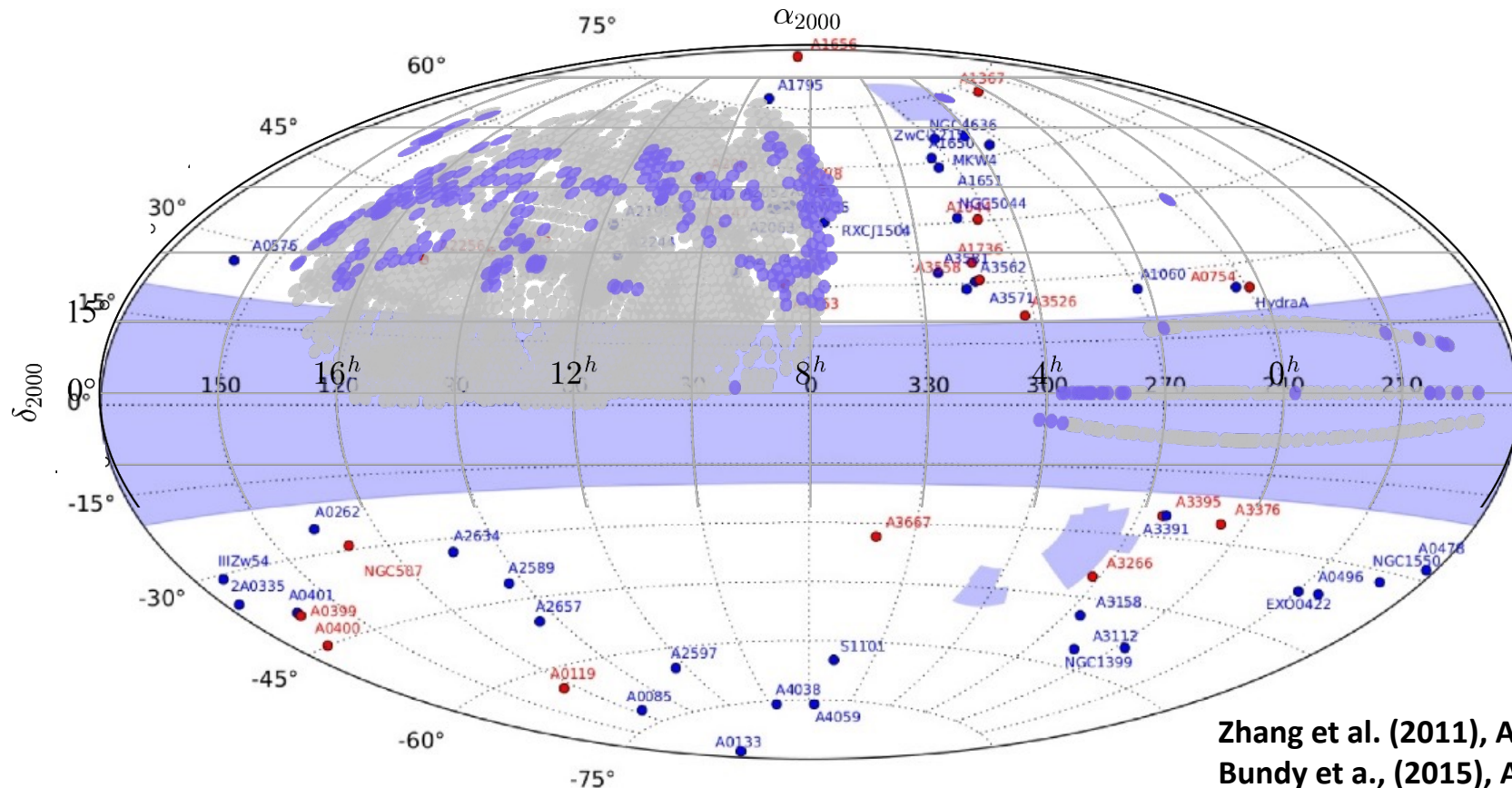
$$\rightarrow \sigma^4 \propto GM_{\text{bar}} g_{\ddagger}$$

[Tian et al. \(2020\), ApJ, 896, 70](#)

# The MVDR on BCG-Cluster Scales?

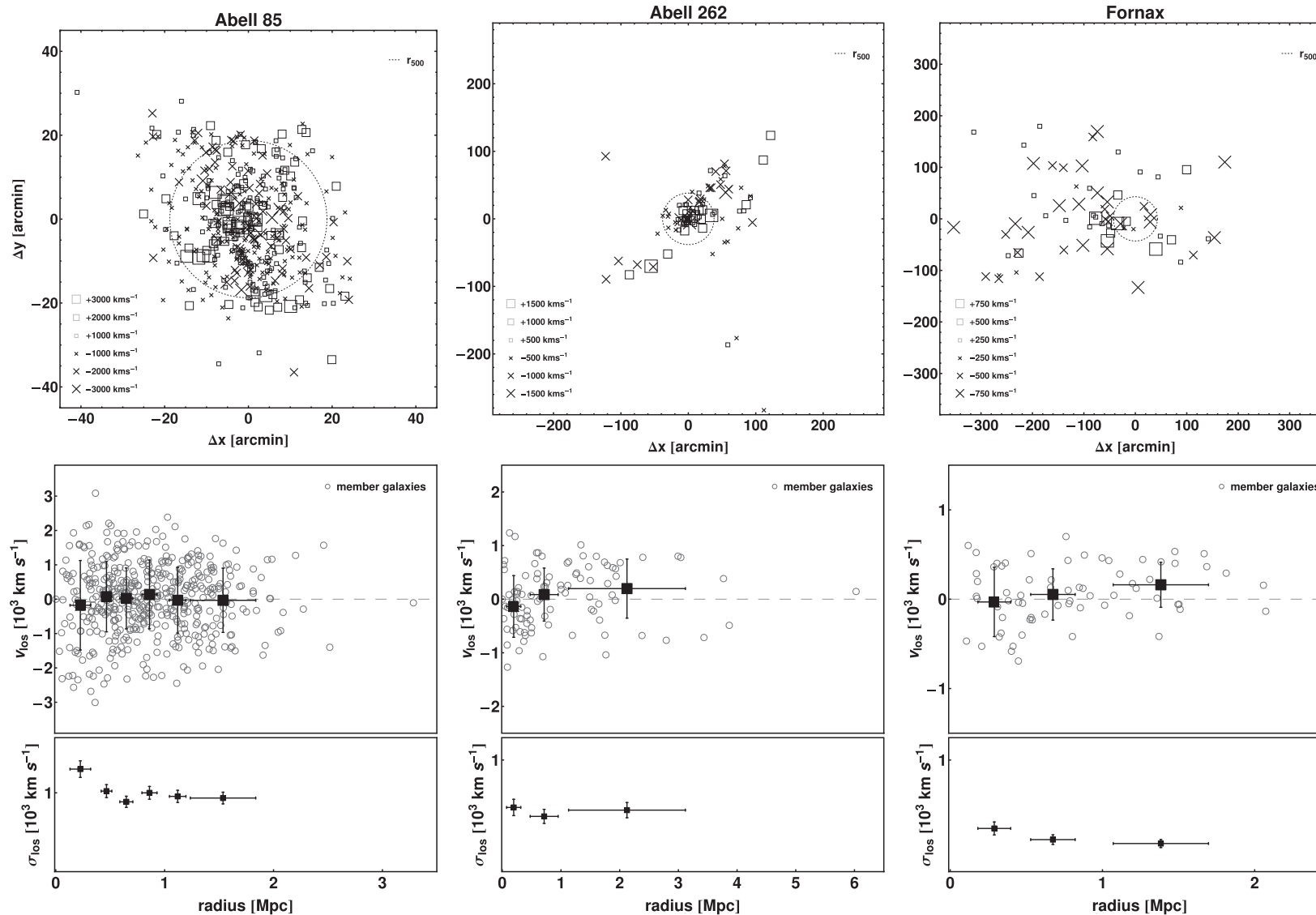
# BCG and galaxy cluster samples

- 29 galaxy clusters in the HIFLUGCS ([Tian et al. 2021a, ApJ, 910, 56](#))
- 54 BCGs in MaNGA MPL-7 ([Tian et al. 2021b, ApJL, 917, L24](#))





# Flat Velocity Dispersion Profiles



**Three examples of HIFLUGCS clusters.**

Upper panel: the spatial distributions are relative to the center point of the BCG.

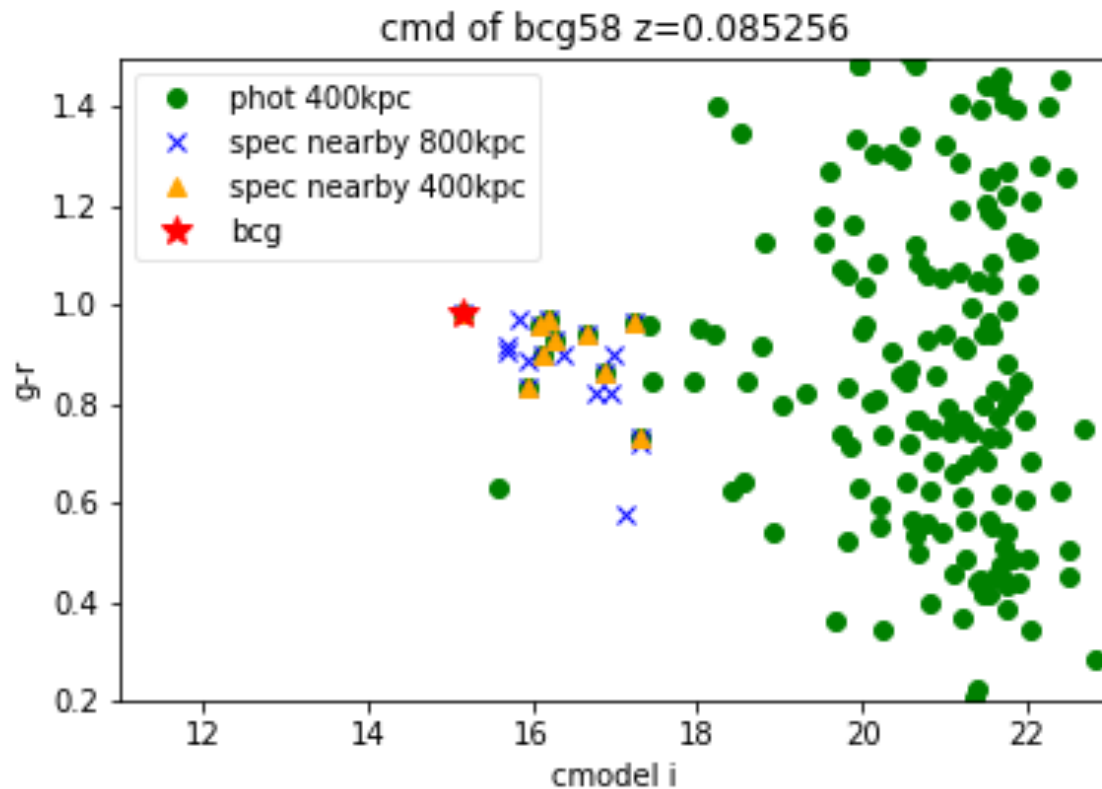
Middle panel: the relative  $l\sigma_8$  velocity ( $V_{los}$ ) distribution is in terms of the projected radius relative to the BCG.

Lower panel: the  $l\sigma_8$  velocity dispersion ( $\sigma_{los}$ ) presents a flat tail for each cluster.

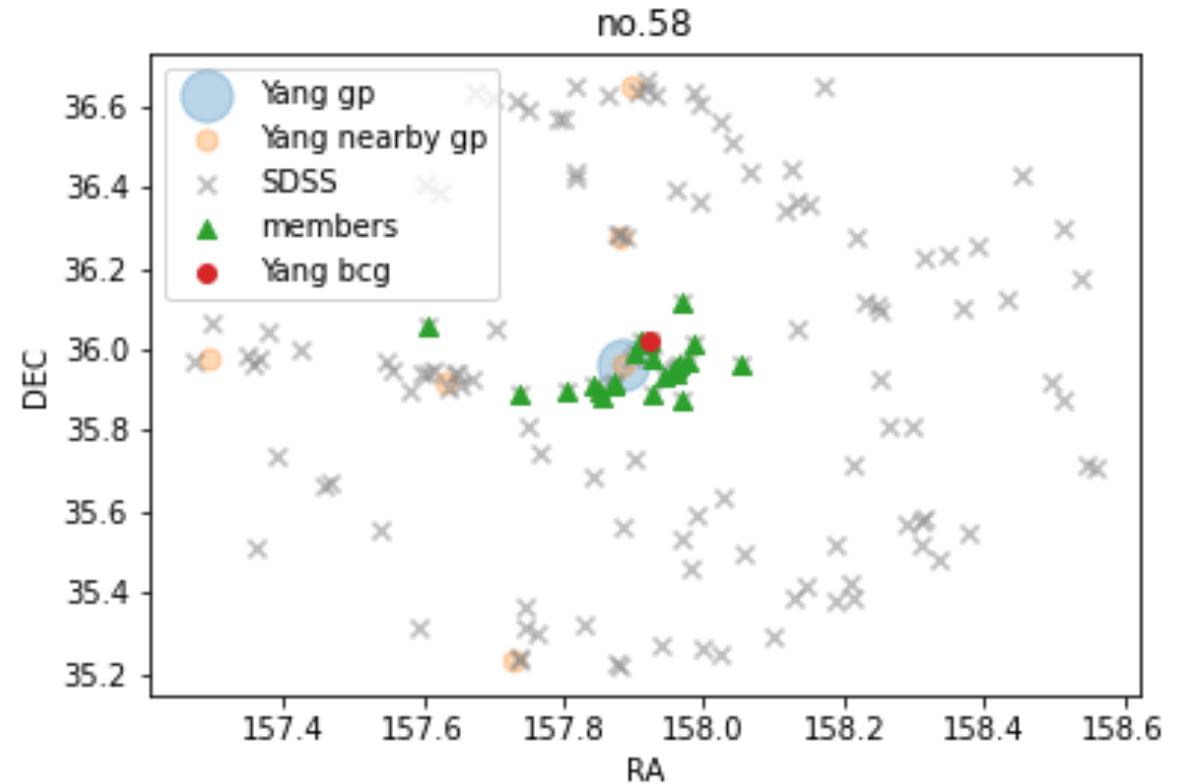
[Tian et al. \(2021a\), ApJ, 910, 56](#)

# Identifying BCGs in MaNGA

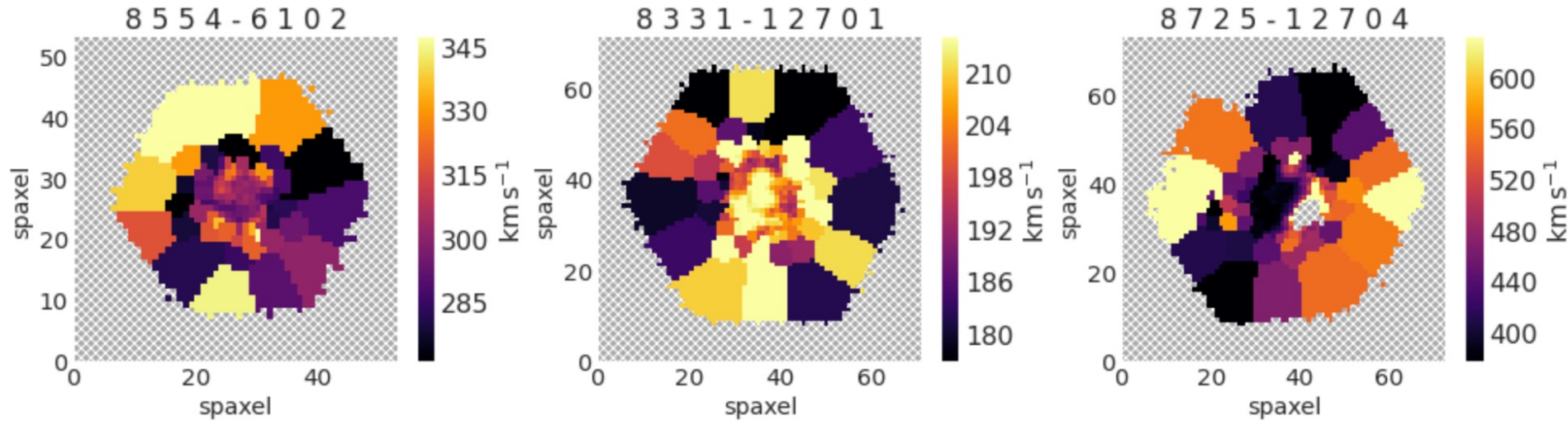
## Color-Magnitude



## Membership Distribution



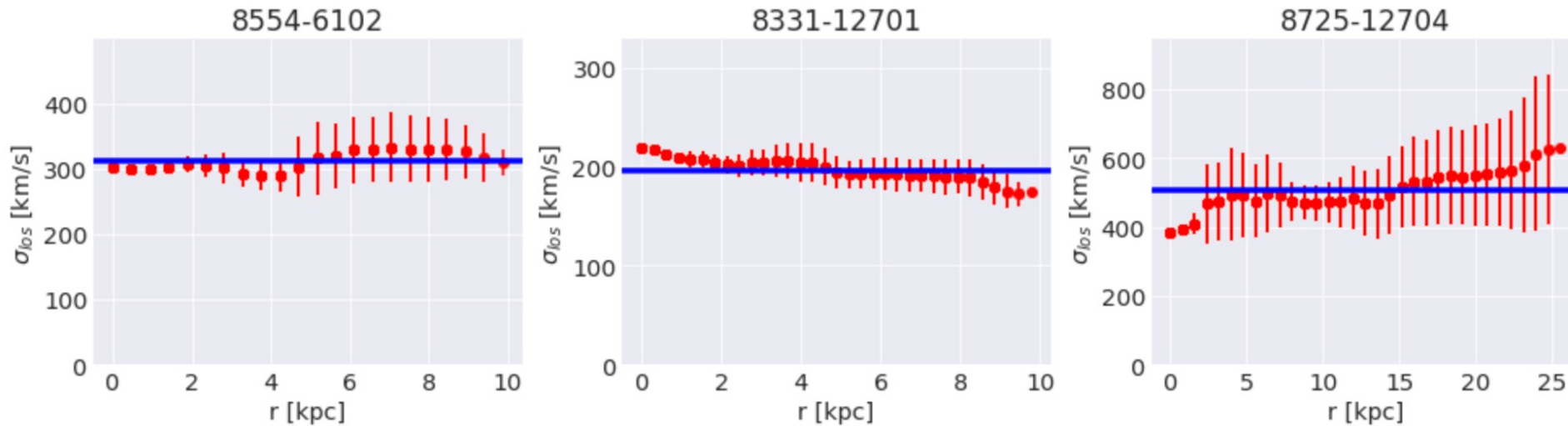
# Flat Velocity Dispersion Profiles



## Three examples of MaNGA BCGs.

Upper panel: the map plot of Spaxel data for the stellar velocity dispersion.

Lower panel: velocity dispersion profiles in terms of radius. MaNGA BCGs present a flat velocity dispersion profile.



[Tian et al. \(2021b\), ApJL, 917, L24](#)

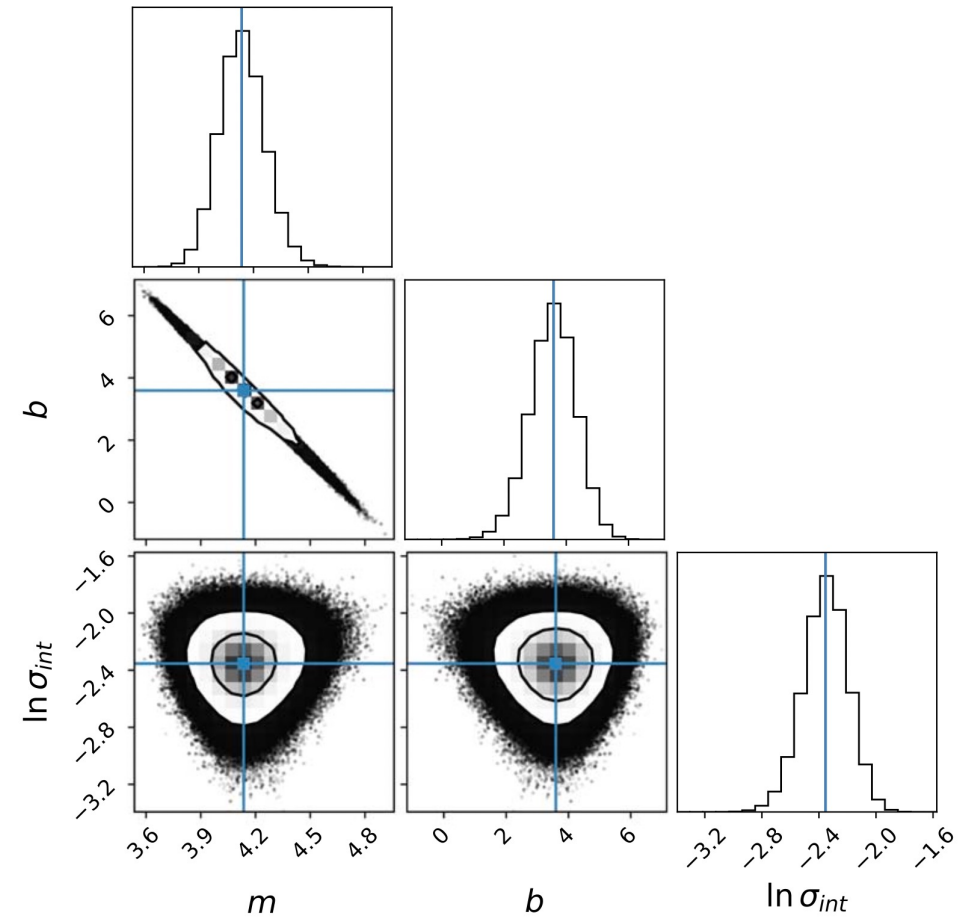
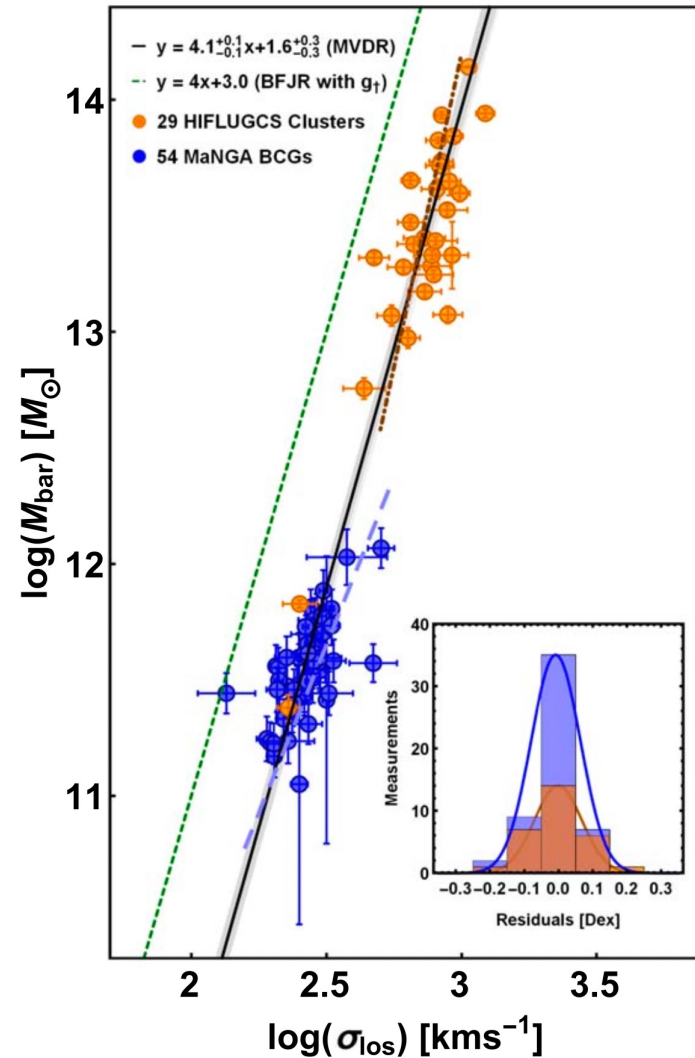


## Mass–Velocity Dispersion Relation in MaNGA Brightest Cluster Galaxies

Yong Tian<sup>1</sup> , Han Cheng<sup>1</sup>, Stacy S. McGaugh<sup>2</sup> , Chung-Ming Ko<sup>1,3,†</sup> , and Yun-Hsin Hsu<sup>4,5</sup>





**The MVDR of both BCGs and clusters.** Left panel: the blue circles represent 54 MaNGA BCGs while the orange circles indicate 29 HIFLUGCS clusters in Tian et al. (2021a). Right panel: triangle diagrams of the regression parameters. Black contours represent 1 $\sigma$  and 2 $\sigma$  confidence regions.

[Tian et al. \(2021b\), ApJL, 917, L24](#)



# Dynamical RAR on BCG-Cluster Scales?

# A Distinct Radial Acceleration Relation across Brightest Cluster Galaxies and Galaxy Clusters

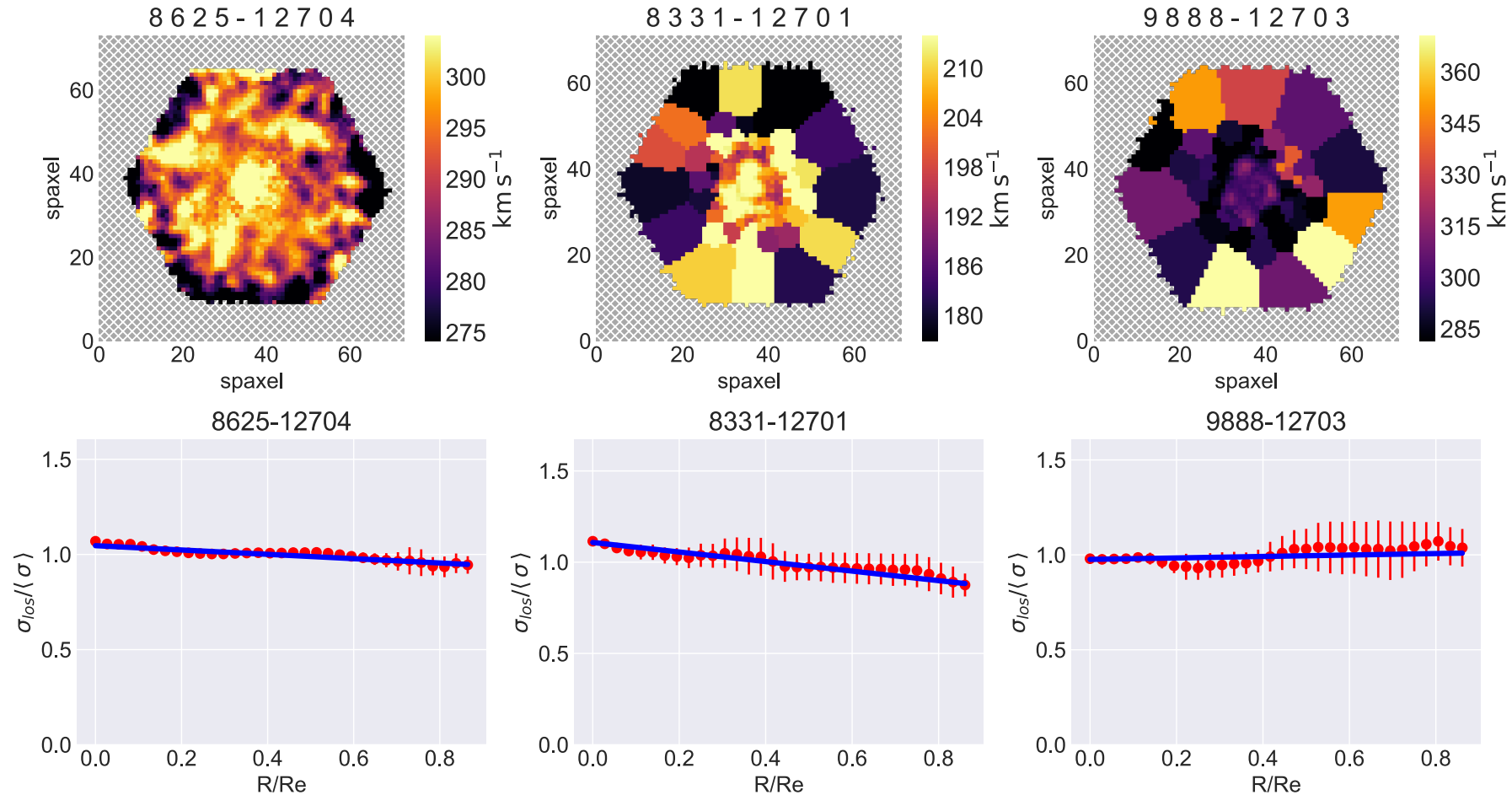
YONG TIAN <sup>1</sup>, CHUNG-MING KO <sup>1,2</sup>, PENGFEI LI <sup>3,\*</sup>, STACY S. MCGAUGH <sup>4,†</sup> AND SHEMILE L. POBLETE<sup>1</sup>

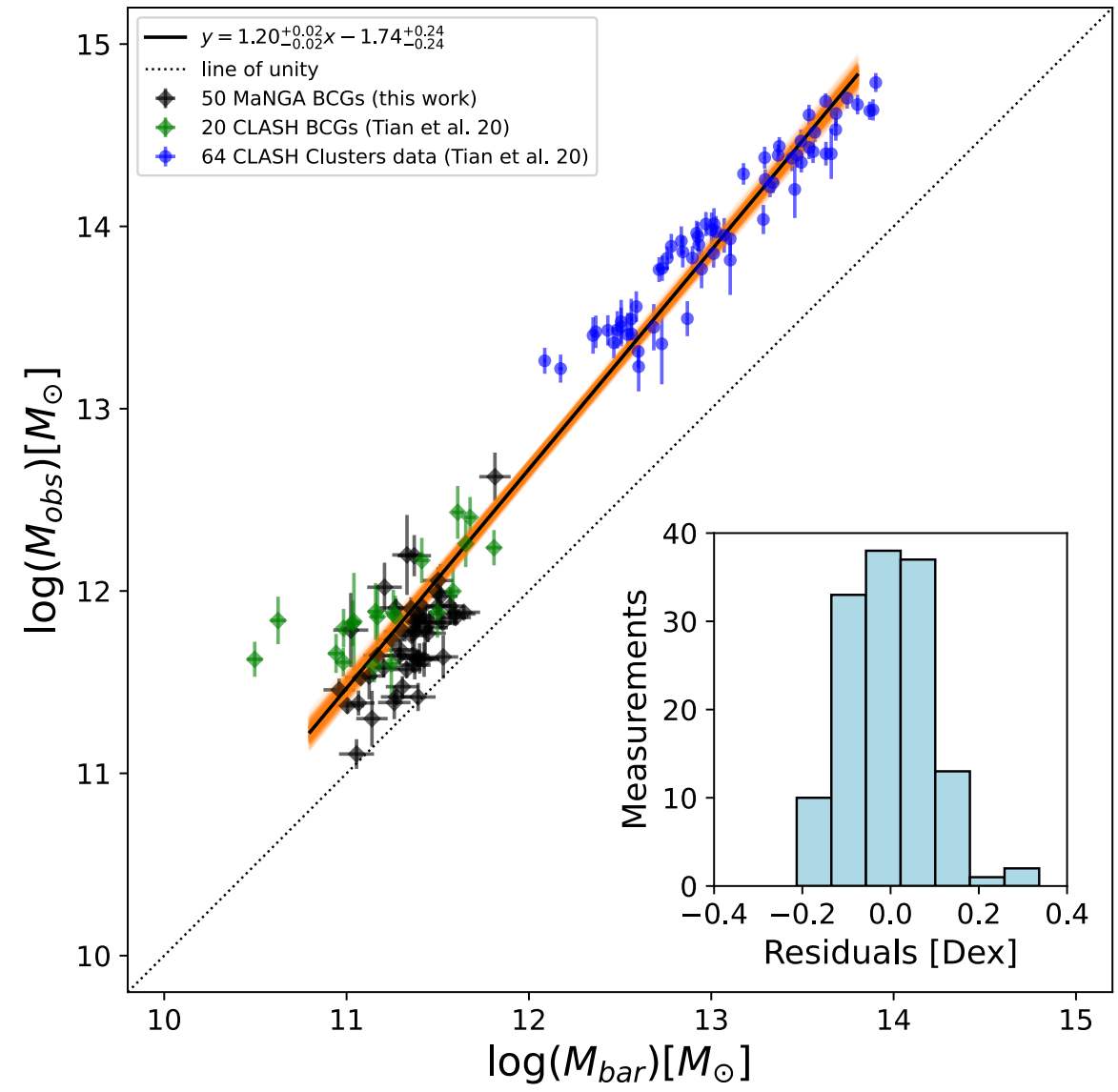
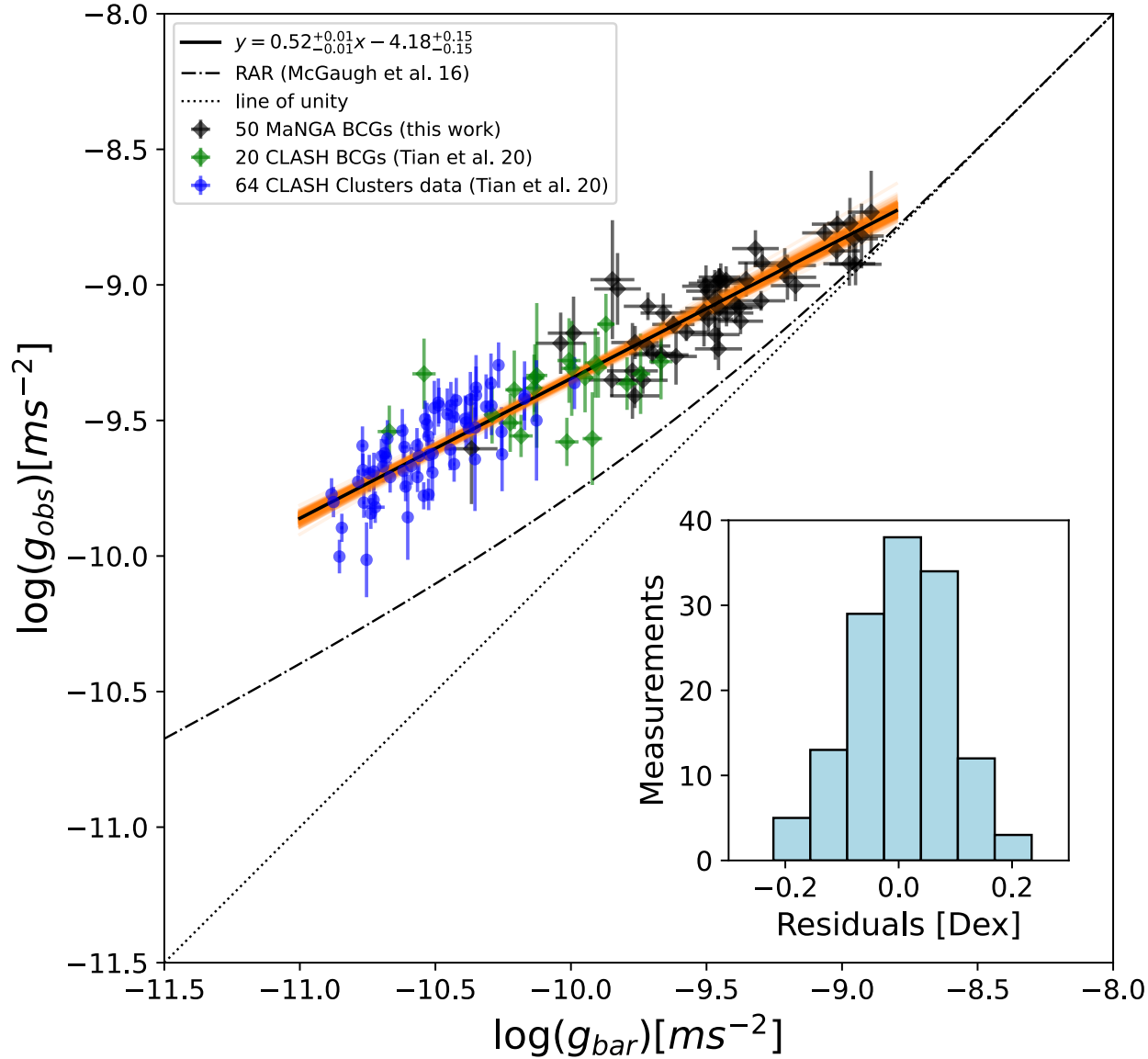
<sup>1</sup>*Institute of Astronomy, National Central University, Taoyuan 320317, Taiwan*

<sup>2</sup>*Department of Physics and Center for Complex Systems, National Central University, Taoyuan 320317, Taiwan*

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# The Implication by Acceleration Scale



# Scale Length Implied by Acceleration Scale

$$a \approx \frac{\sigma^2}{r_{\dagger}} \approx g_{\dagger}$$

$$a \approx \frac{\sigma^2}{r_{\ddagger}} \approx g_{\ddagger}$$

$$r_{\dagger} = \sigma^2 g_{\dagger}^{-1}$$

$$r_{\ddagger} = \sigma^2 g_{\ddagger}^{-1}$$

Milgrom (1984), ApJ, 287,571

[Tian et al. \(2021b\), ApJL, 917, L24](#)

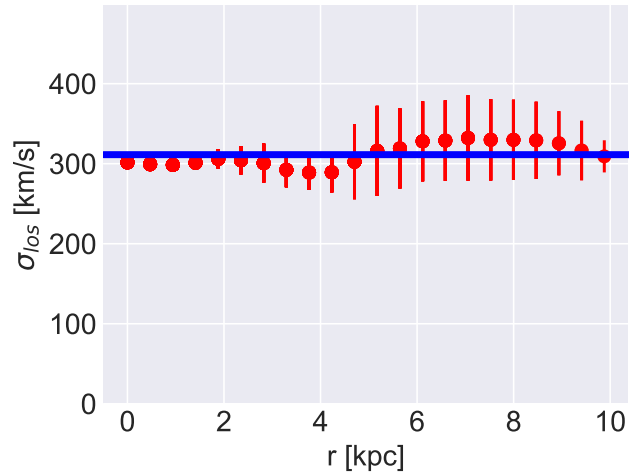
If  $\sigma$  ranges in 100-200 km/s for ellipticals and 200-300 km/s for BCGs, we have different scale lengths for the flat tails.

$$r_{\dagger} \sim 2.6 - 10.7 \text{ kpc}$$

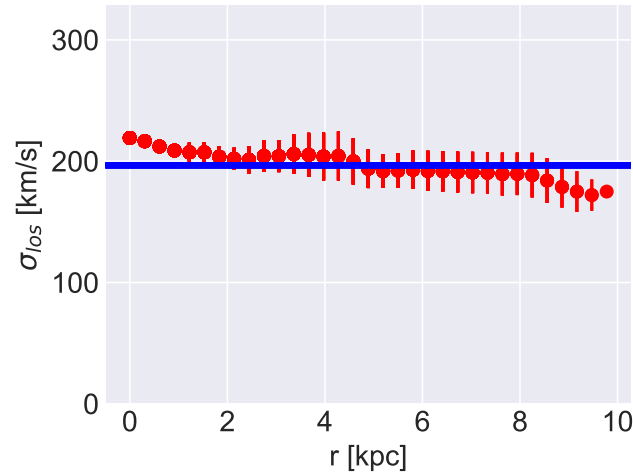
$$r_{\ddagger} \sim 0.6 - 1.5 \text{ kpc}$$

# Velocity Dispersion Profiles in Elliptical Galaxies

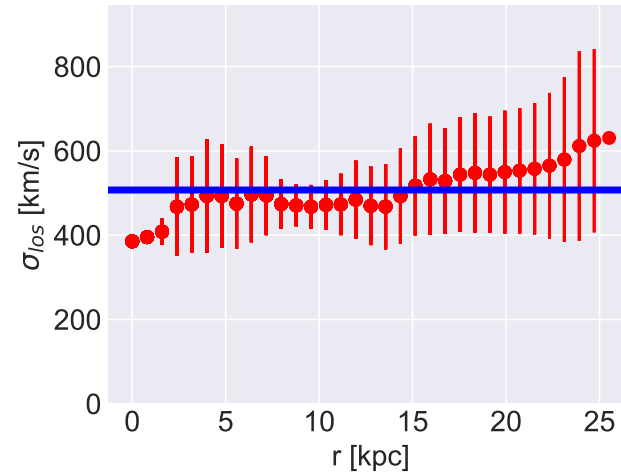
8554-6102



8331-12701



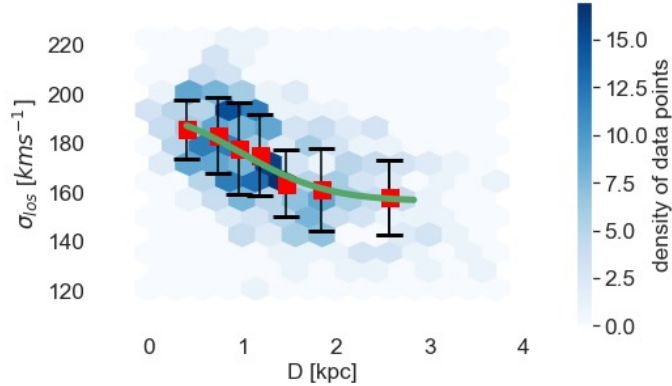
8725-12704



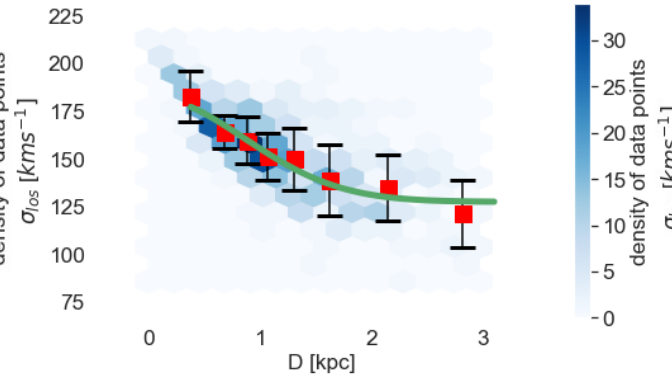
Three examples of  
MaNGA BCGs.

[Tian et al. \(2021b\), ApJL, 917, L24](#)

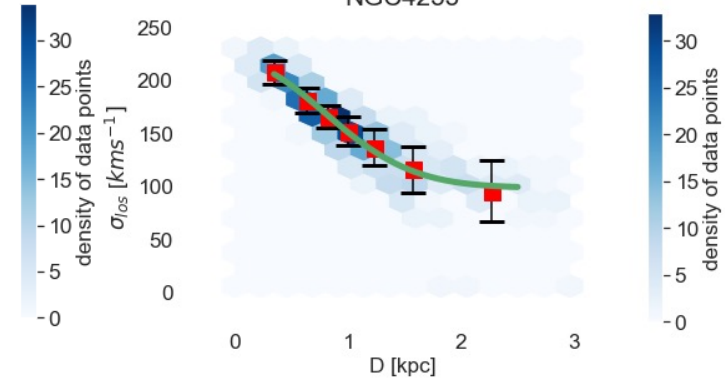
NGC0661



NGC0680



NGC4233



Three examples of  
ATLAS<sup>3D</sup> ellipticals.

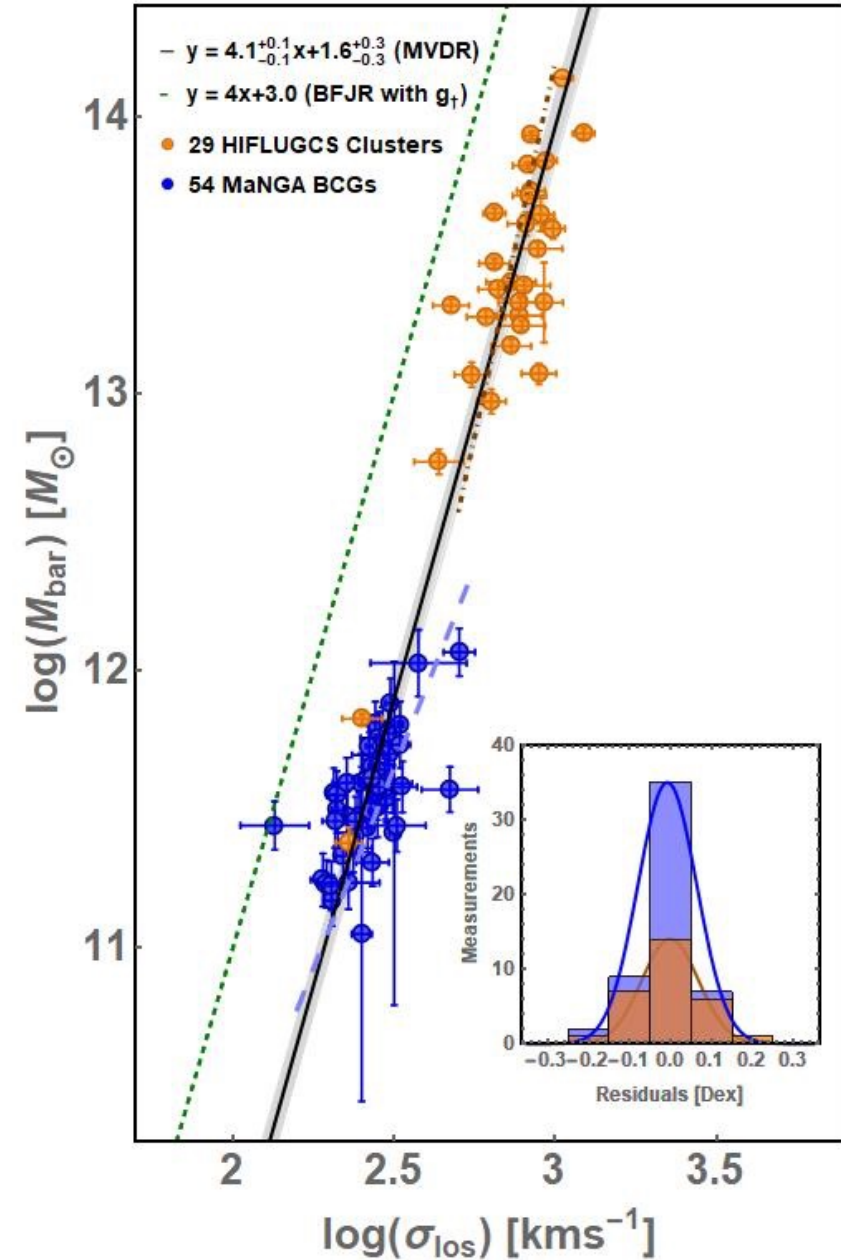
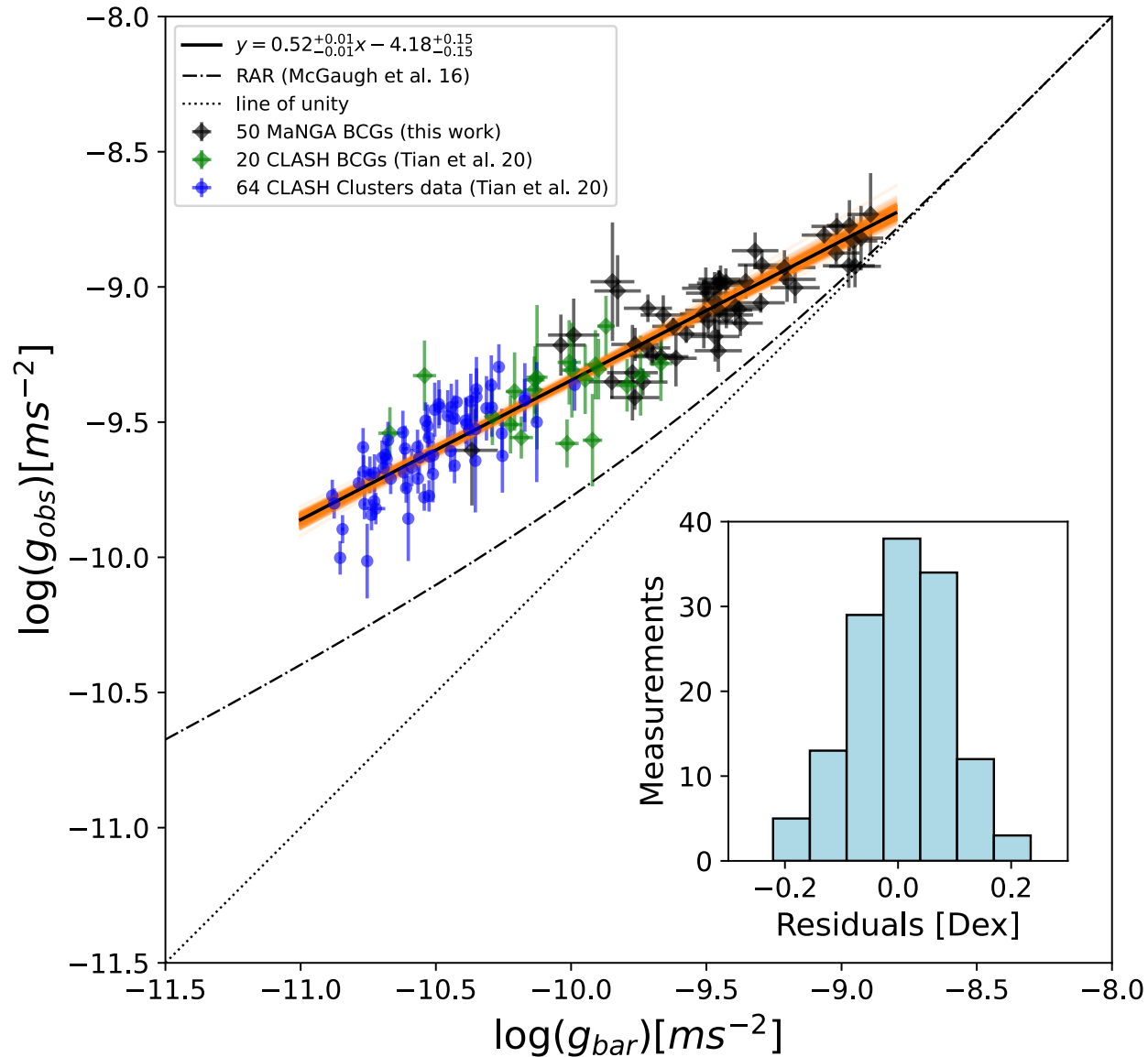
# Remarks

	Dynamics	Kinematics
Solar System	Newton's Law	Kepler's Law
Spirals	RAR	Tully-Fisher
Ellipticals	RAR	Faber-Jackson
<b>BCG-Cluster</b>	<b>CLASH RAR</b>	<b>MVDR</b>

# Remarks

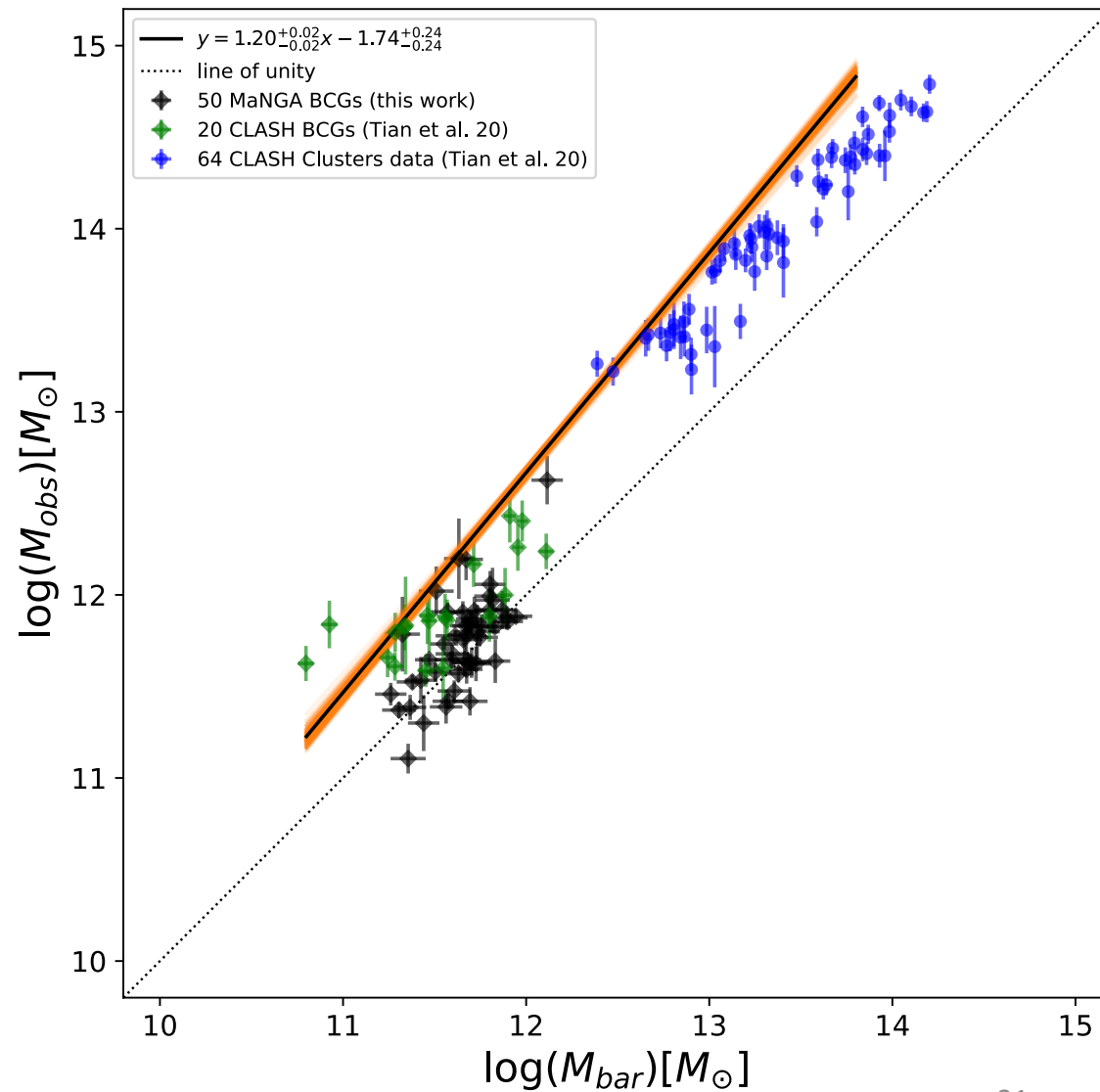
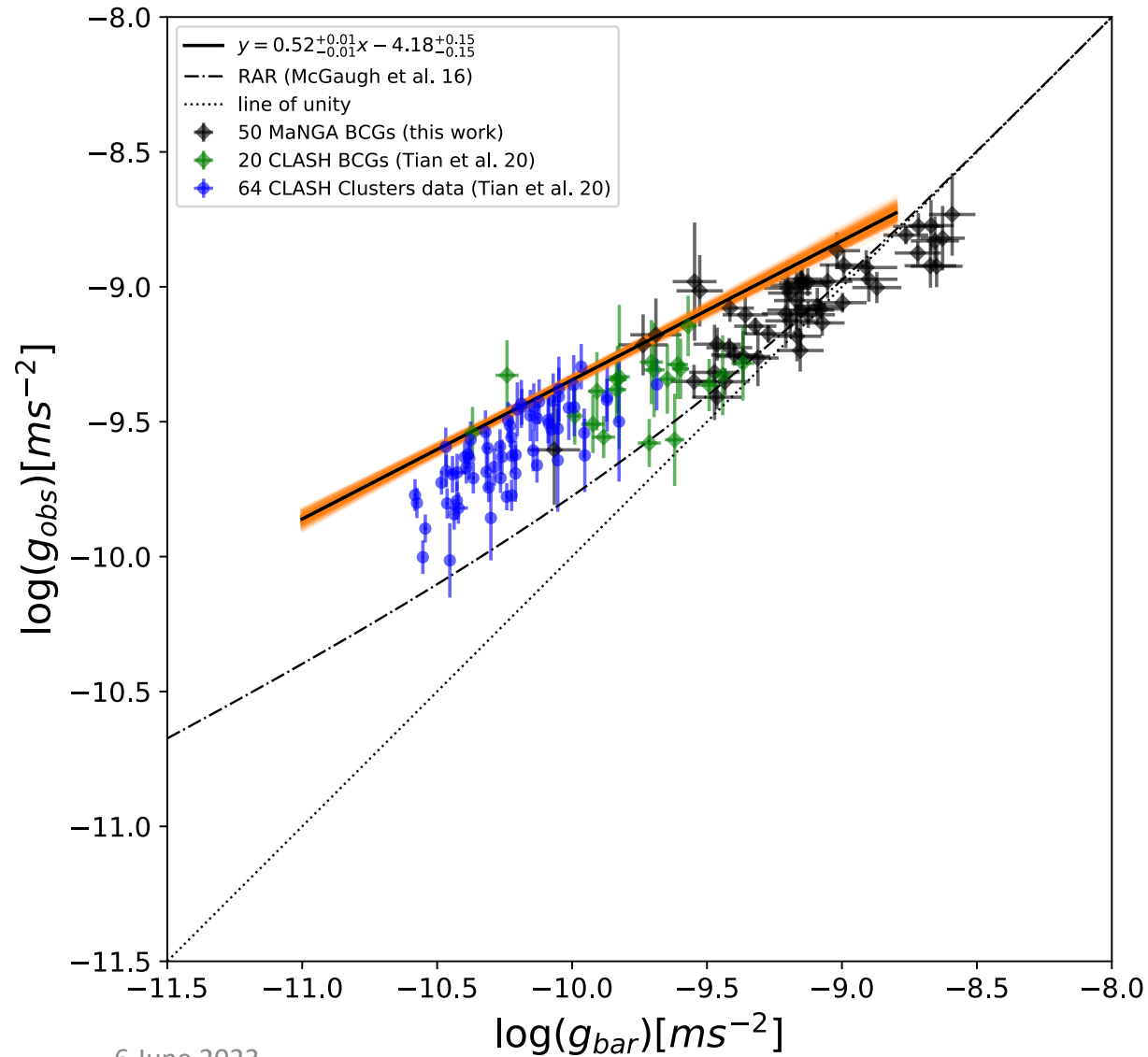
1. Nearly flat velocity dispersion profiles have been observed in both MaNGA Brightest Cluster Galaxies (BCGs) and HIFLUGCS clusters.
2. A distinct Radial Acceleration Relation (RAR), is represented as  $g_{\text{tot}} = \sqrt{g_{\text{bar}} g_{\ddagger}}$ , with  $g_{\ddagger} = (2.0 \pm 0.1) \times 10^{-9} \text{ms}^{-2}$ , is unveiled through the data gathered from 50 MaNGA BCGs, 20 CLASH BCGs, and 20 CLASH clusters with a tiny intrinsic scatter.
3. A Mass-Velocity Dispersion Relation, also known as a parallel Faber-Jackson Relation, is demonstrated by 50 MaNGA BCGs and 29 HIFLUGCS clusters. The relation aligns with the implications of the RAR on BCG-cluster scales.

# CLASH RAR & MVDR

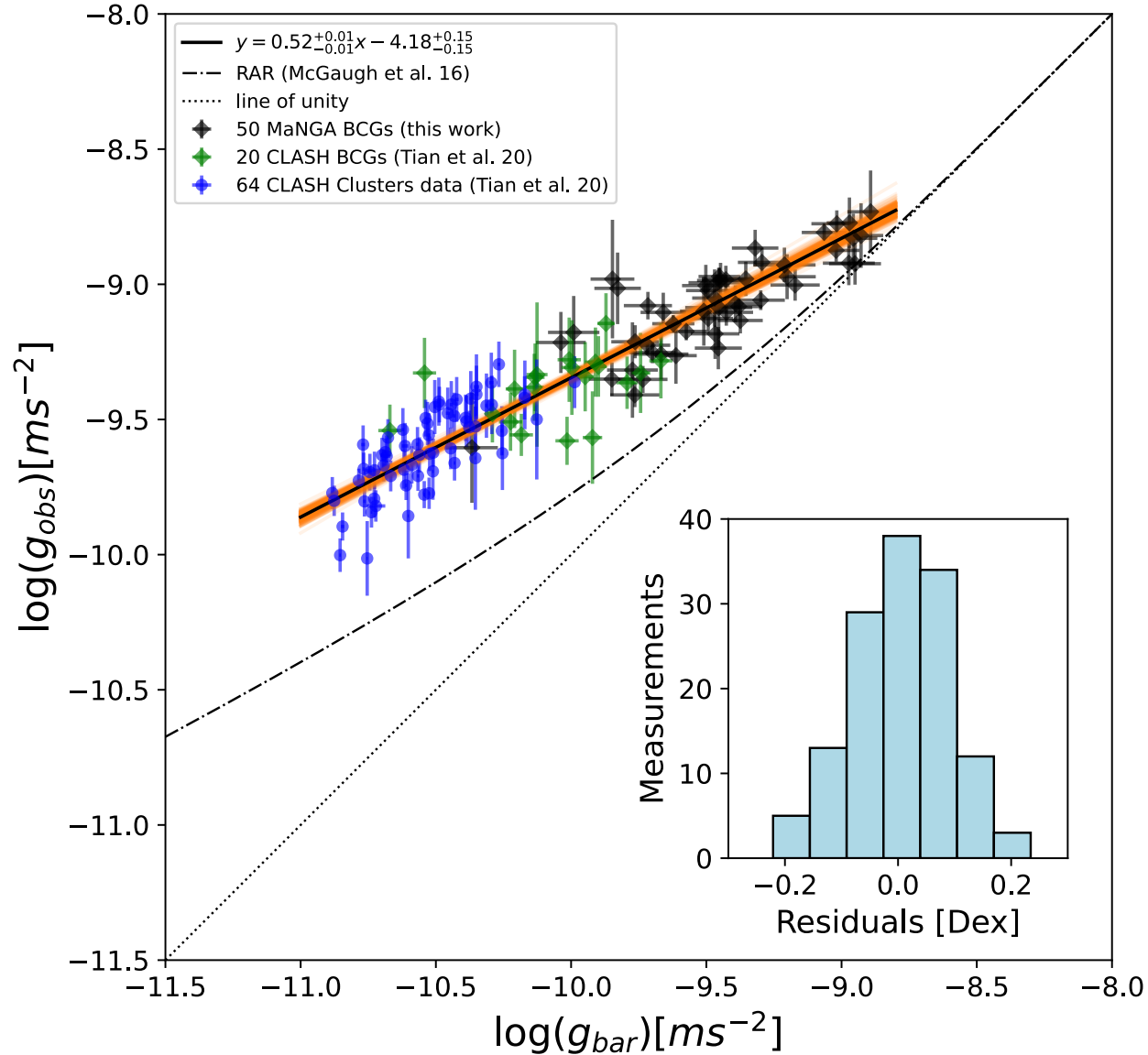


**Q & A**

# Residual Missing Mass



# Residual Missing Mass



$$\sqrt{g_{bar} g_{\ddagger}} \approx \frac{g_M}{1 - e^{-\sqrt{g_M/g_{\ddagger}}}}$$

The residual missing mass depends on baryonic acceleration.

For examples,

If  $g_{bar} = 2.1 \times 10^{-10} ms^{-2}$ ,  $M_M/M_{bar} \approx 2.7$ ;

If  $g_{bar} = 1.3 \times 10^{-11} ms^{-2}$ ,  $M_M/M_{bar} \approx 7.3$ .



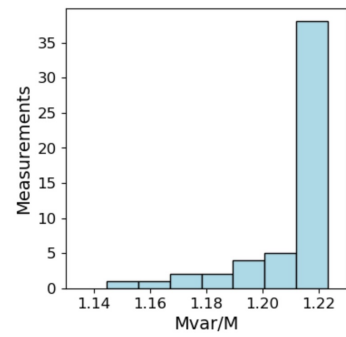
# M\*/L Gradient

**Table 1.** Parameter values for M/L gradient strength (equation 2) driven by different assumptions about the IMF (fixed or variable) used in this study.

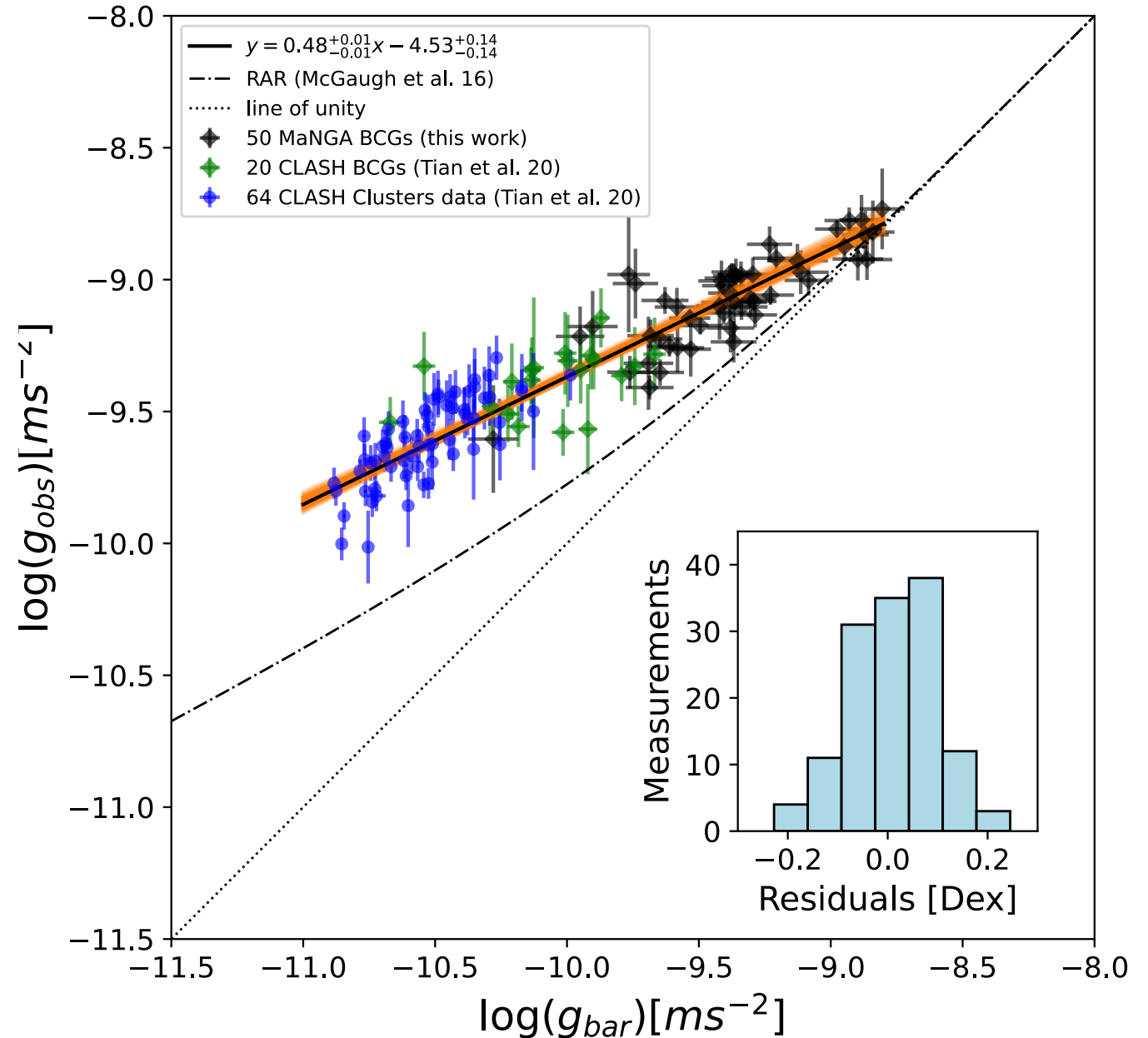
Model	IMF-var.	$\alpha$	$\beta$	Source
Fixed IMF	No	0.39	1.00	Chabrier
Salp <sup>IN</sup> -Chab <sup>OUT</sup>	Yes	1.29	3.33	Salpeter-Chabrier
vD17	Yes	2.33	6.00	van Dokkum et al.

$$Y_*(R) = Y_{*0} (1 + \alpha - \beta R/R_e) \quad \text{if } R/R_e \leq \alpha/\beta,$$

$$M_\infty \equiv 2\pi \int_0^\infty dR R J(R) = Y_{*0} L (1 + g(n, \alpha, \beta)),$$



**The baryonic mass is increased by around 10-20%.**



# Acceleration Relation in Ellipticals

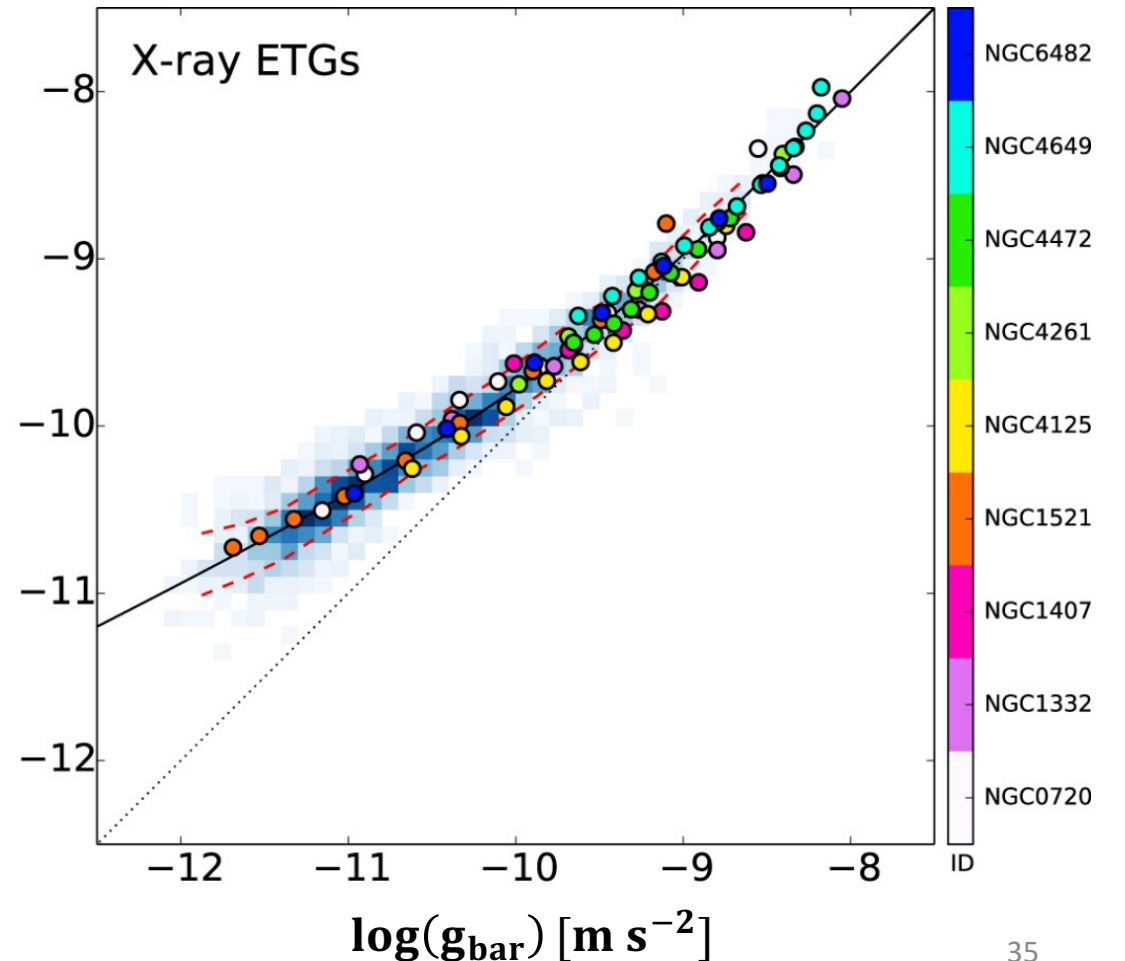
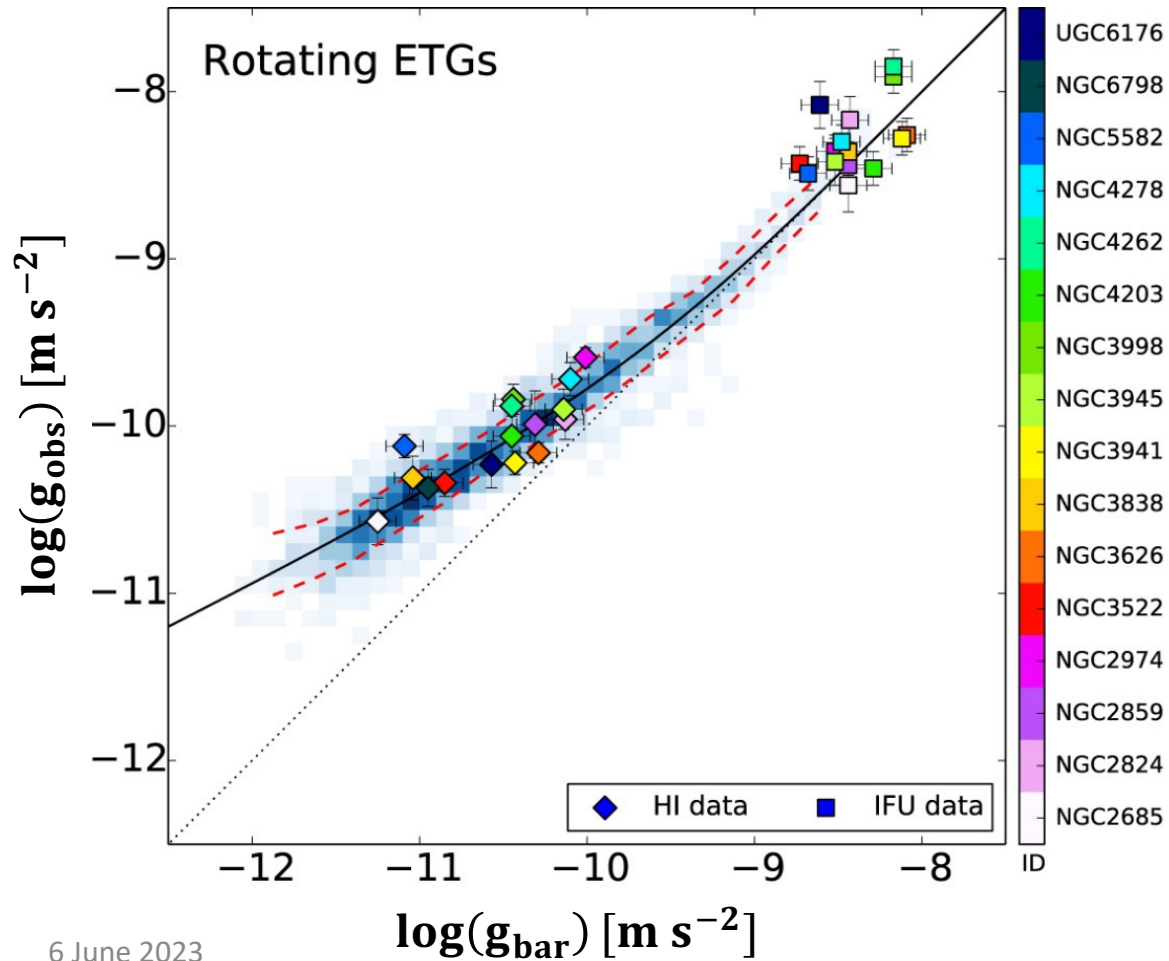


# One Law to Rule Them All: The Radial Acceleration Relation of Galaxies

Federico Lelli<sup>1,2,5</sup>, Stacy S. McGaugh<sup>1</sup>, James M. Schombert<sup>3</sup>, and Marcel S. Pawlowski<sup>1,4,6</sup>

## Radial Acceleration Relation

## Radial Acceleration Relation



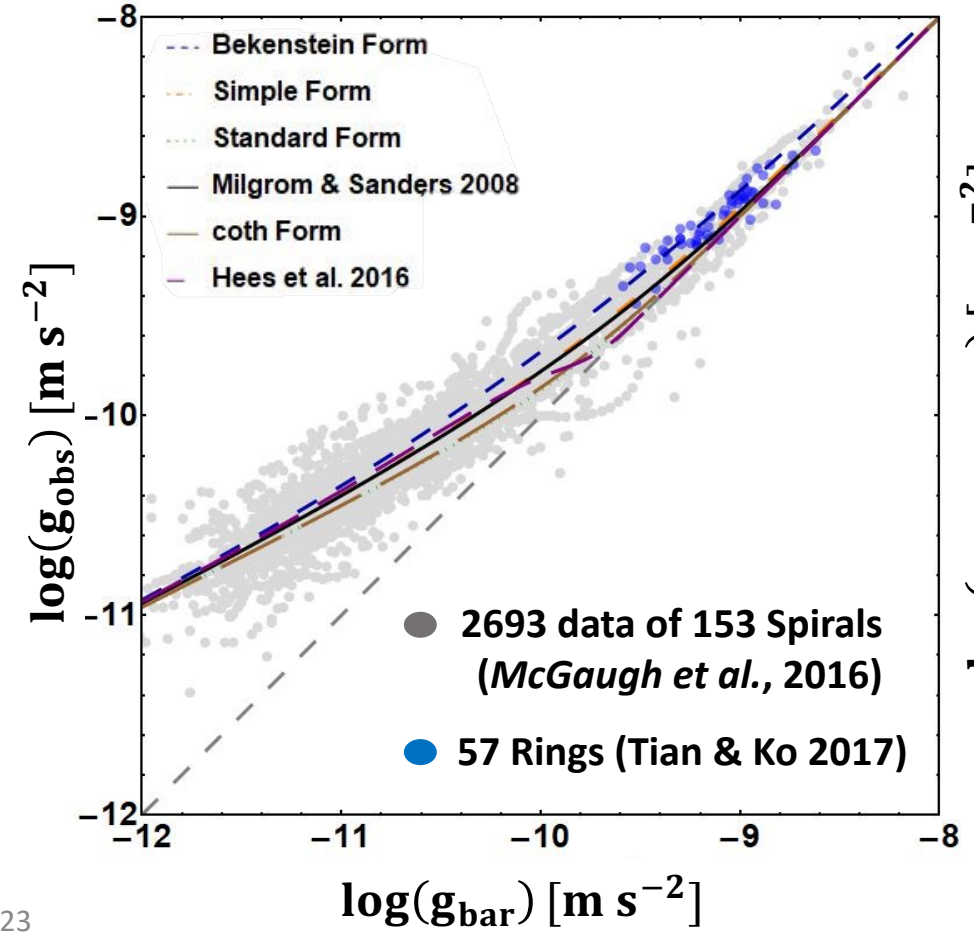
# Halo acceleration relation

Yong Tian (田雍)<sup>1★</sup> and Chung-Ming Ko (高仲明)<sup>1,2★</sup>

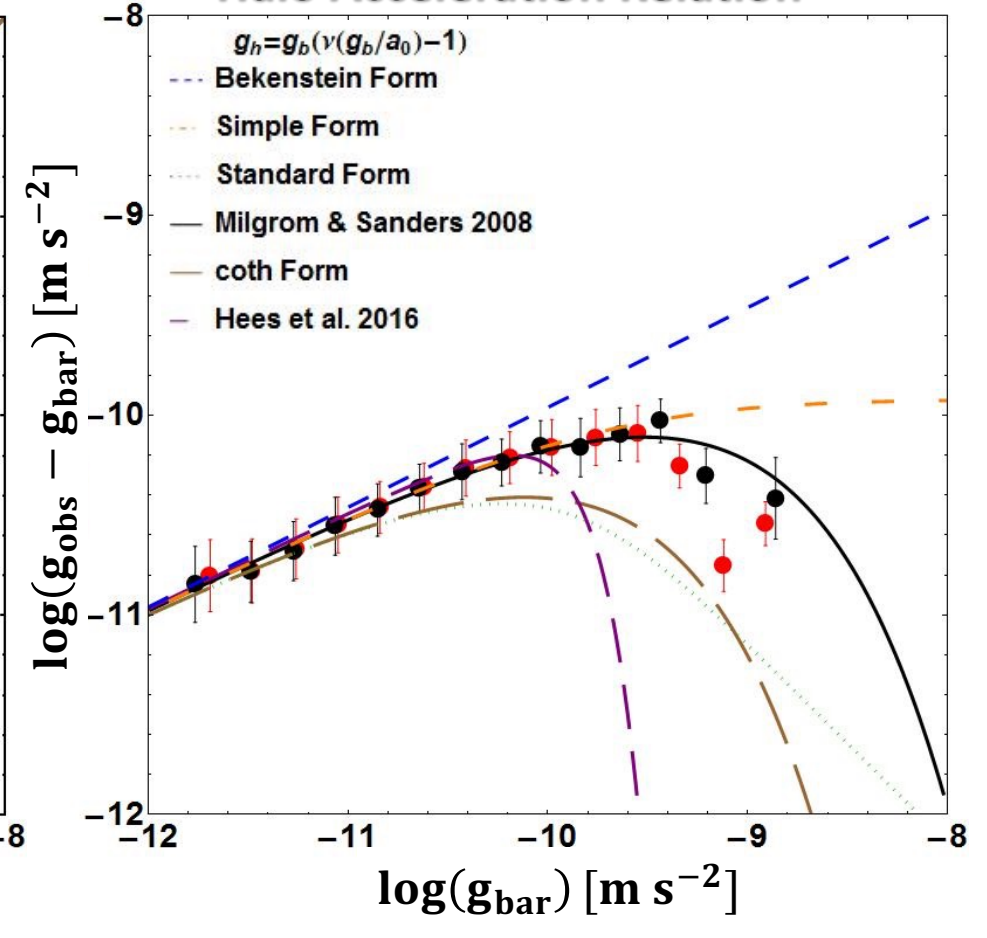
<sup>1</sup>*Institute of Astronomy, National Central University, Taoyuan City, Taiwan 32001, Republic of China*

<sup>2</sup>*Department of Physics and Centre for Complex Systems, National Central University, Taoyuan City, Taiwan 32001, Republic of China*

## Radial Acceleration Relation



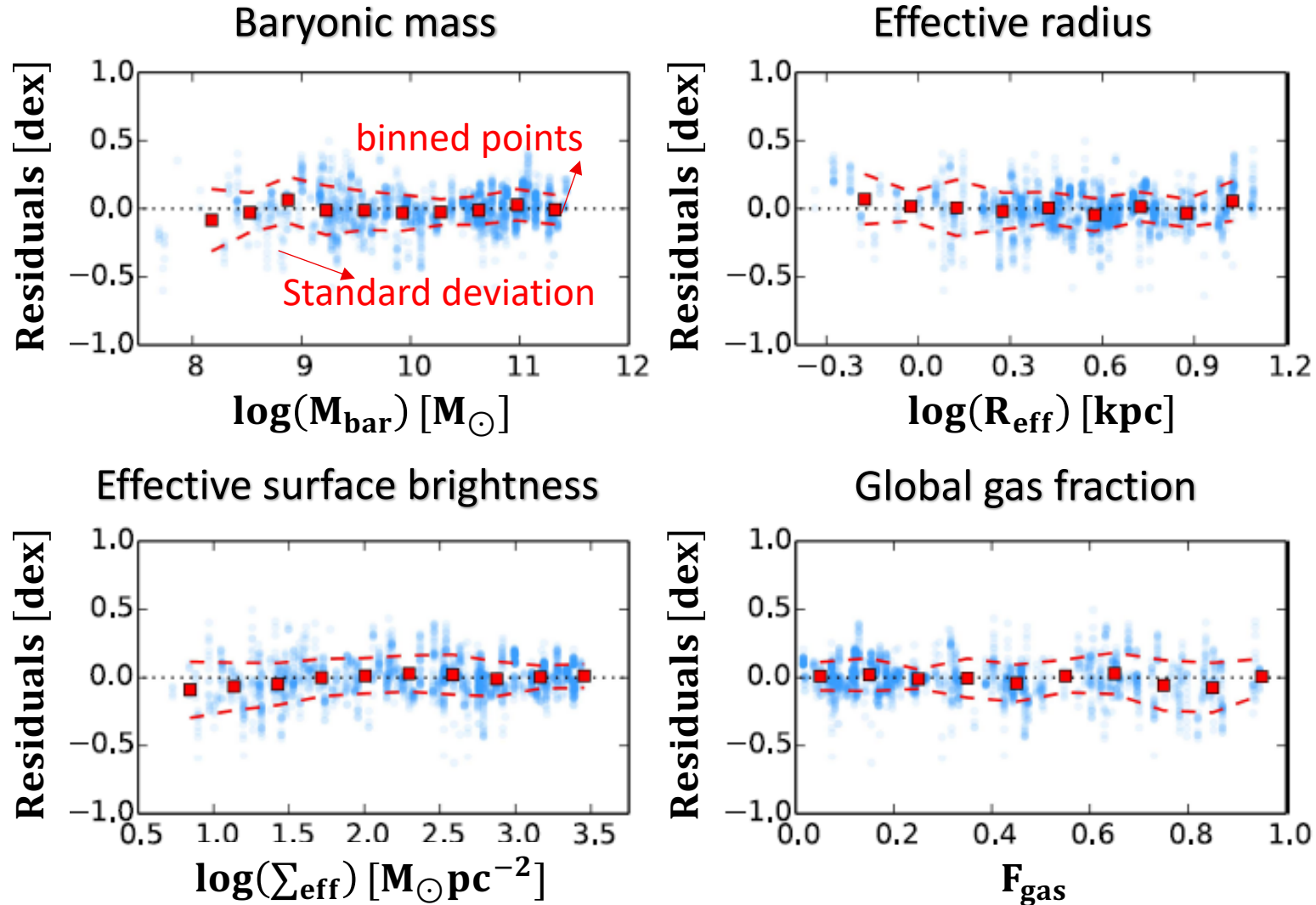
## Halo Acceleration Relation





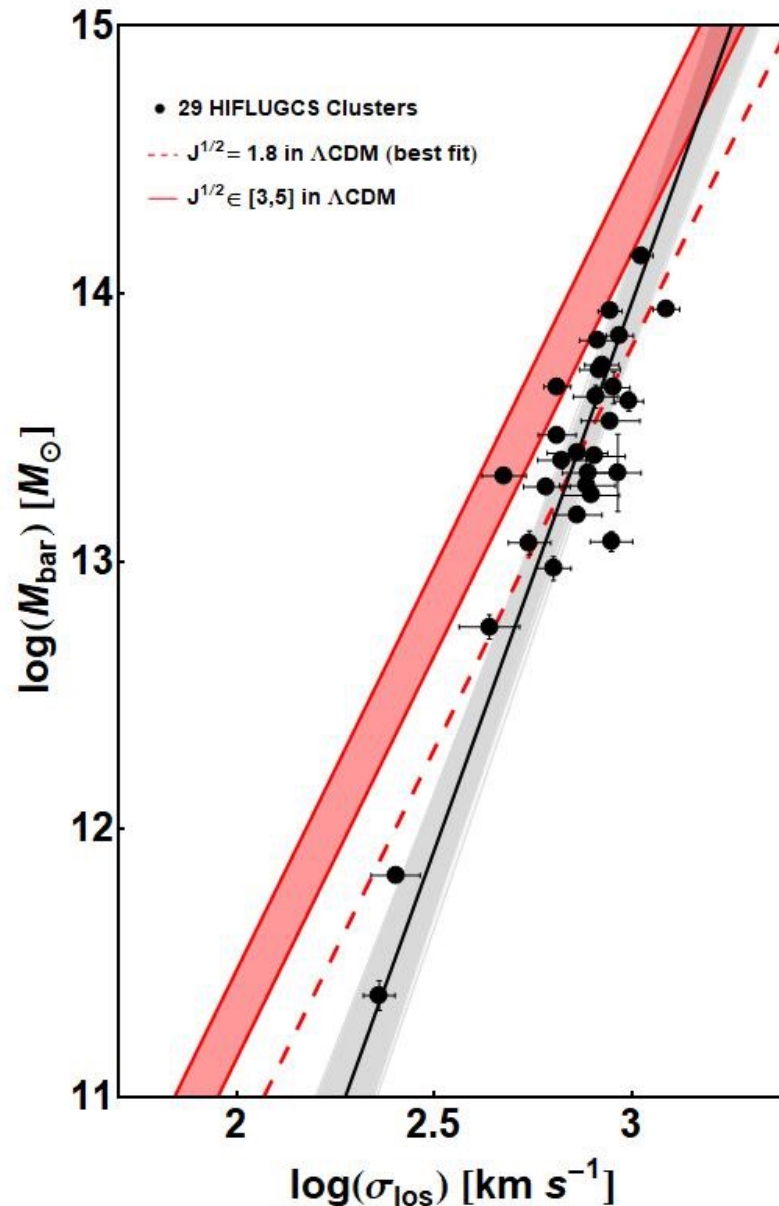
# One Law to Rule Them All: The Radial Acceleration Relation of Galaxies

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The vertical clumps of data are due to individual objects: each galaxy contributes with several points to the RAR.

# Implication in $\Lambda$ CDM Model



The enclosed total mass within the radius in  $\Lambda$ CDM model is given by (Mo et al. 1998)

$$M_{\Delta} = \frac{4\pi}{3} (\Delta\rho_{crit}) r_{\Delta}^3$$

Where  $\rho_{crit} = 3H_0^2/8\pi G$  is the critical density of the universe.

To relate the DM mass with velocity dispersion, we consider the gravitational potential traced by the velocity dispersion as

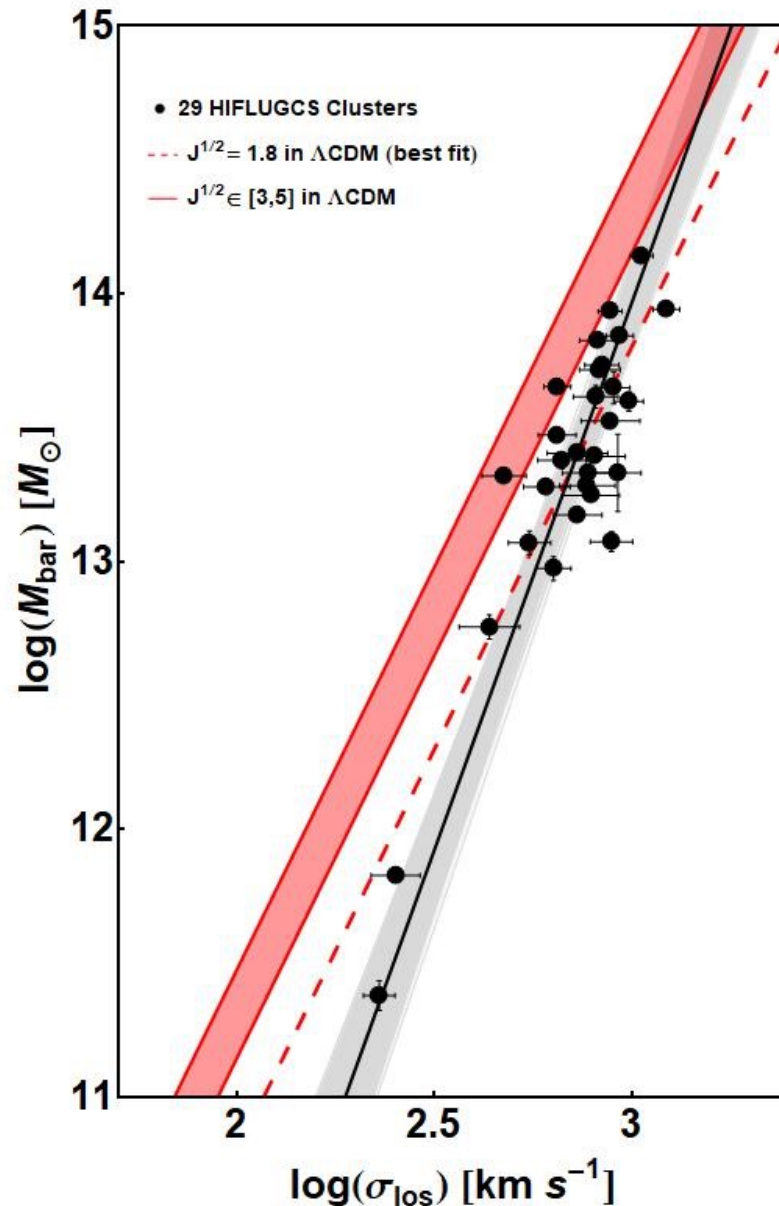
$$\frac{GM_{\Delta}}{r_{\Delta}^2} = \frac{J^{1/2} \sigma^2}{r_{\Delta}}$$

Thus, we have

$$M_{\Delta} = \frac{\sqrt{2} J^{3/4}}{\sqrt{\Delta} G H_0} \sigma^3$$

[Tian et al. \(2021a\), ApJ, 910, 56](#)

# Implication in $\Lambda$ CDM Model



The  $\Lambda$ CDM model in pressure supported systems gives the dynamical MVDR as

$$M_{500} \propto \sigma_{los}^3$$

By adopting the cosmic baryon fraction, we have

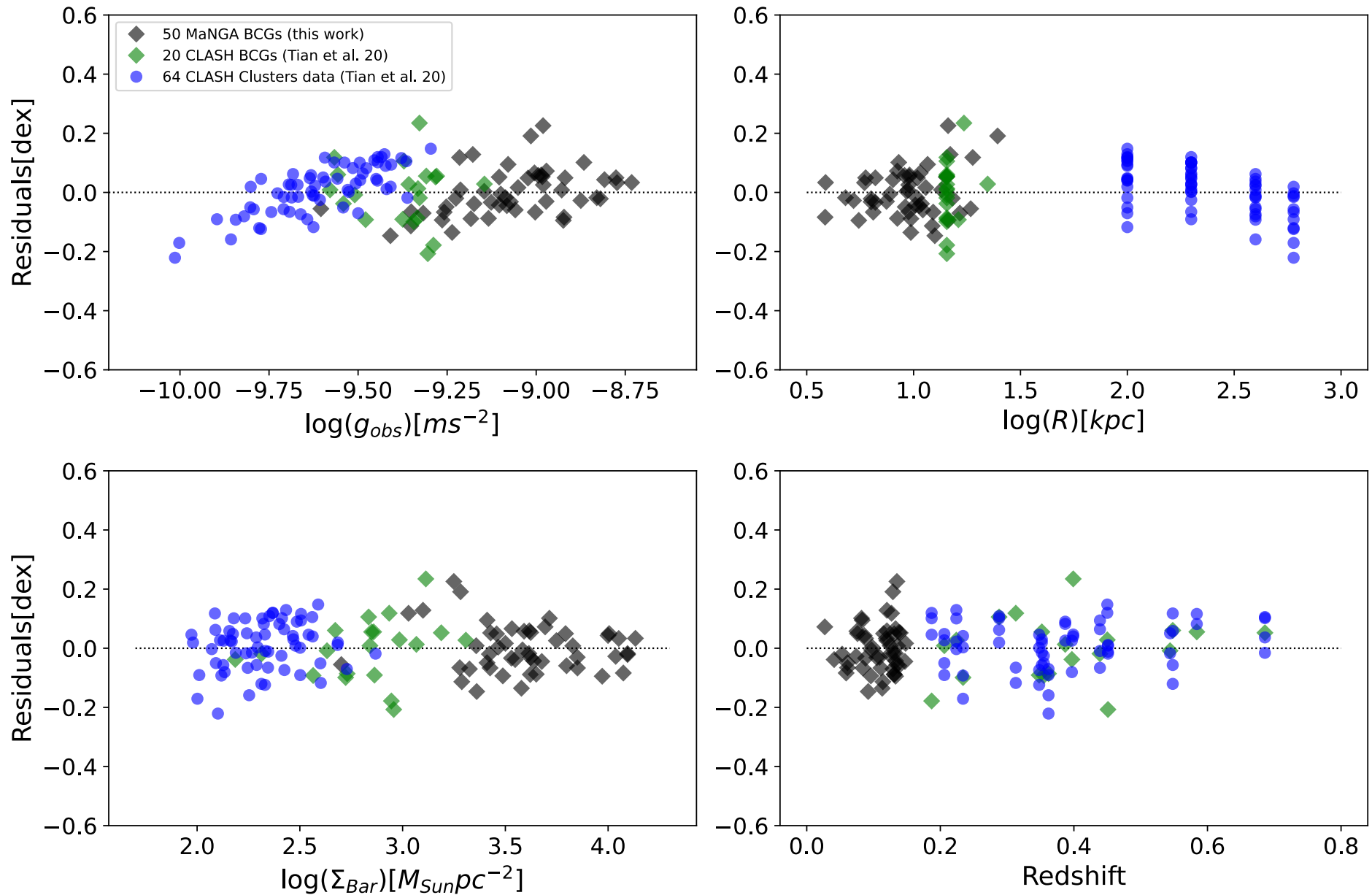
$$f_{bar} = M_{bar} / M_{500}$$

Thus,  $\Lambda$ CDM model predicts different slope comparing with the empirical MVDR

$$M_{bar} \propto \sigma_{los}^3$$

$$\log(M_{bar}) = 3 \log(\sigma_{los}) + \log\left(\frac{\sqrt{2} J^3 / 4 f_{bar}}{\sqrt{500} G H_0}\right)$$

[Tian et al. \(2021a\), ApJ, 910, 56](#)







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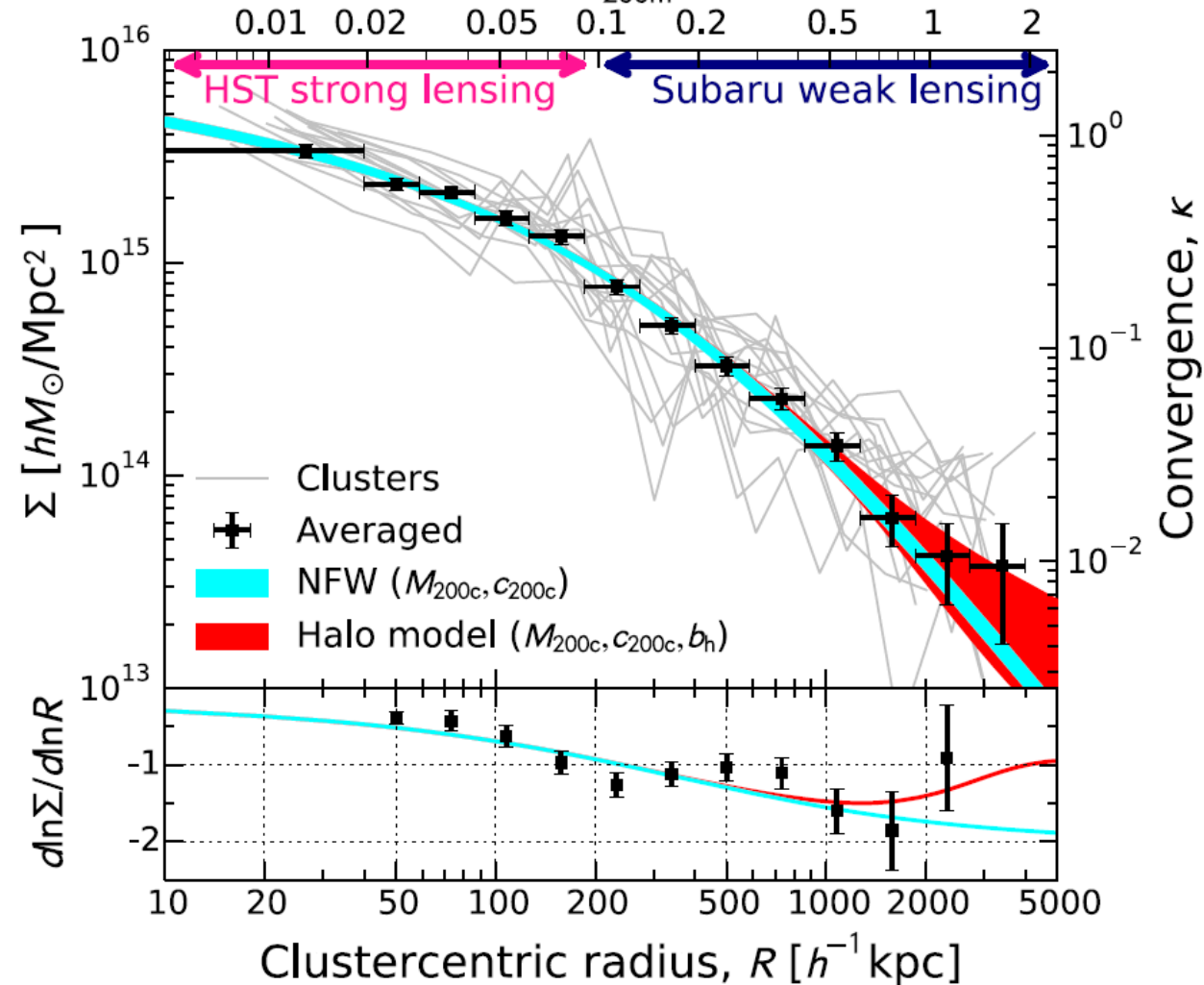
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