The Phantom Menace:

MOND as an Alternative to Planet Nine

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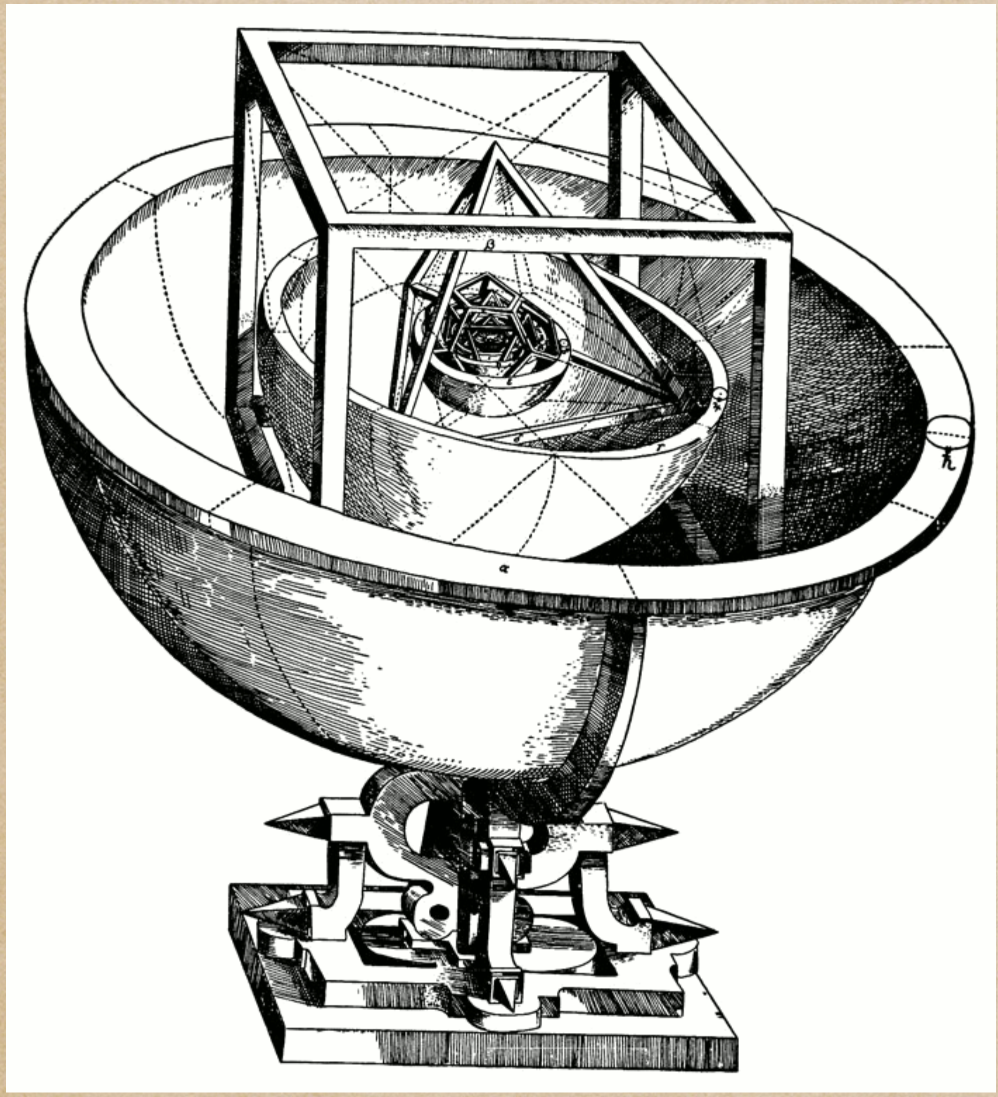
Mond40 St Andrews Jun 8, 2023

Solar System 101

Discovery of Planets



Kepler proved

there are only

six planets

Mysterium Cosmographicum (1596)

The discovery of Uranus

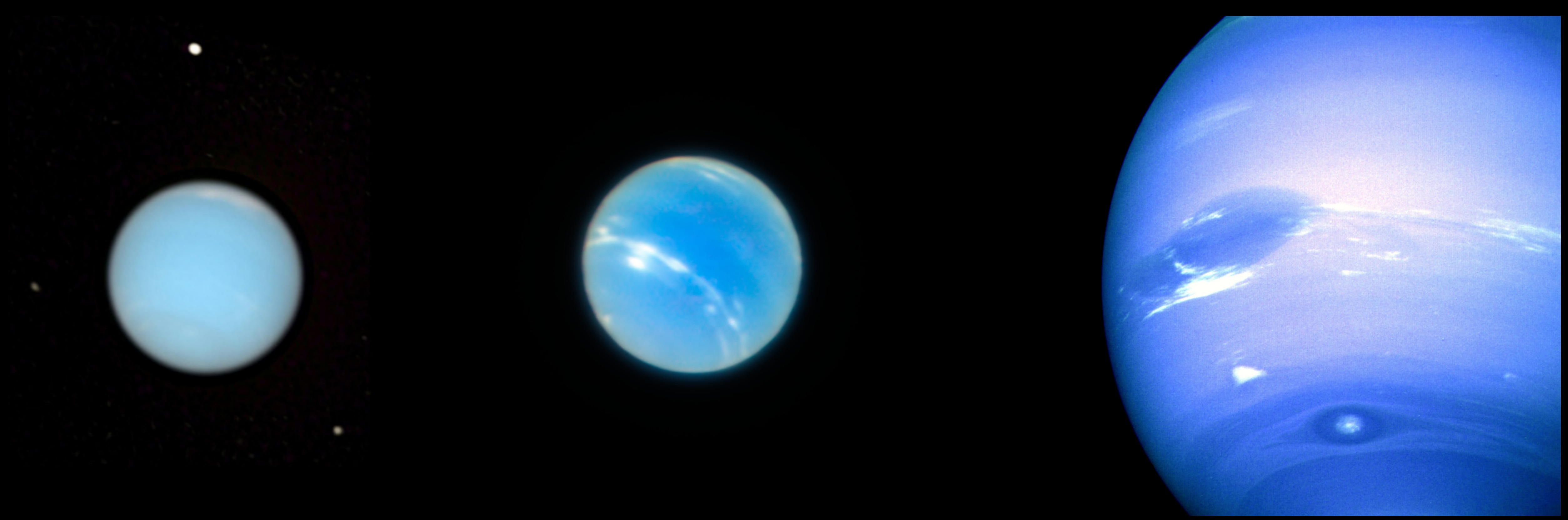
Uranus imaged by
Voyage 2 (NASA)

Planet 7

William Herschel (1781)

The prediction of Neptune

Le Verrier (1846) Adams (1846)



Three views of Neptune: Hubble Space Telescope (left); ground based Very Large Telescope (center); Voyager 2 (right)

Is there a Ninth Planet?

Vulcan

Problem: Anomalies in orbit of Mercury

Resolution: Einstein's theory of gravity

(1843-1916)

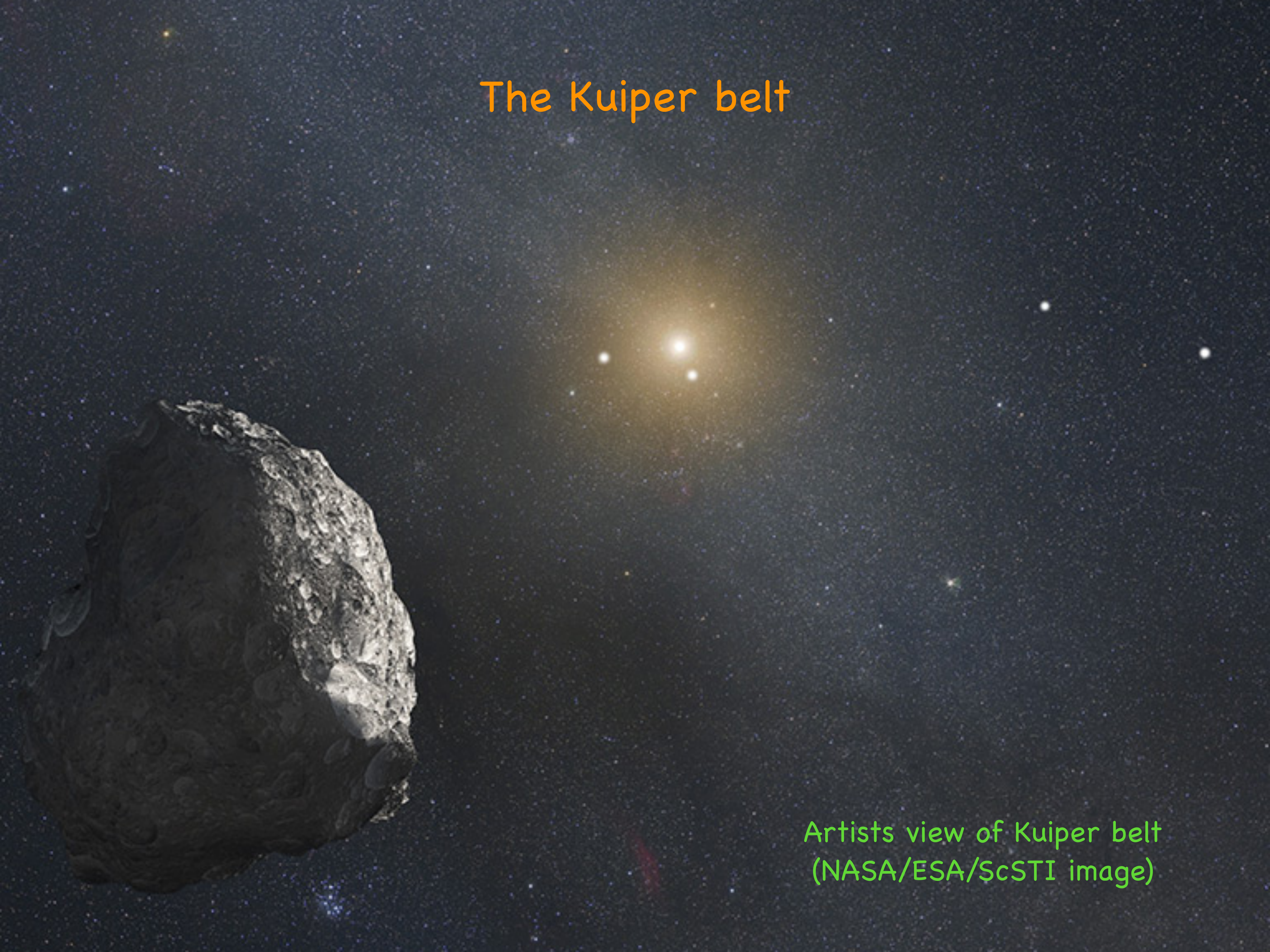
Pluto

Problem: Anomalies in the orbit of Uranus and Neptune

Resolution: Improved data shows no anomalies

(1848-1993)

The Kuiper belt



Artists view of Kuiper belt
(NASA/ESA/ScSTI image)

A brief survey of the Kuiper belt

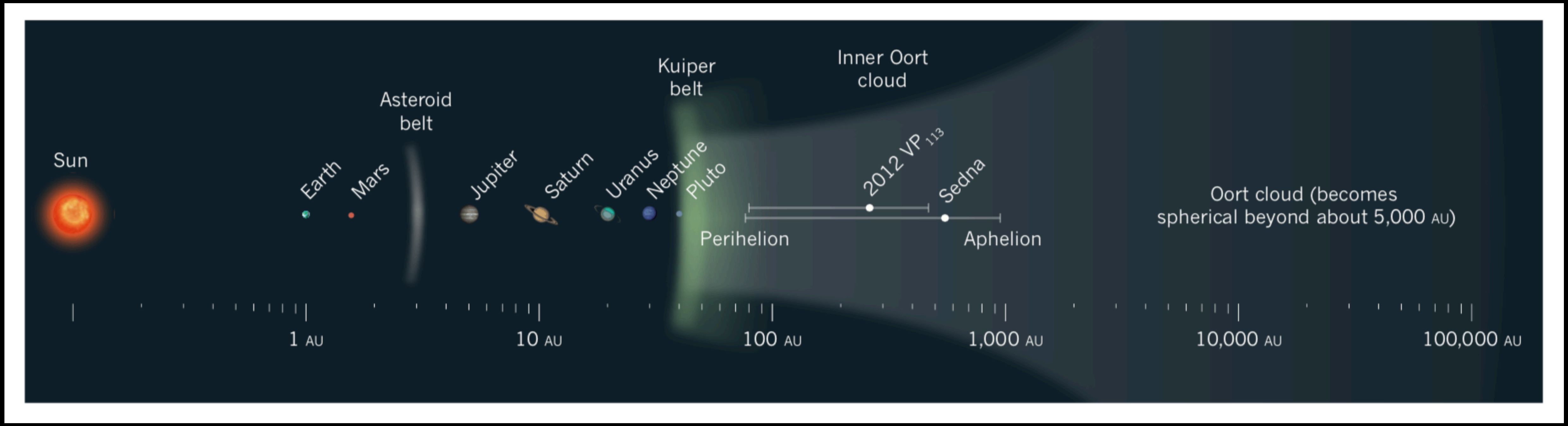
- ★ Resonant Kuiper belt
- ★ Classical Kuiper belt
- ★ Scattered disk
- ★ Centaurs

Historical Highlights

Discovery of KBOs

Discovery of Sedna

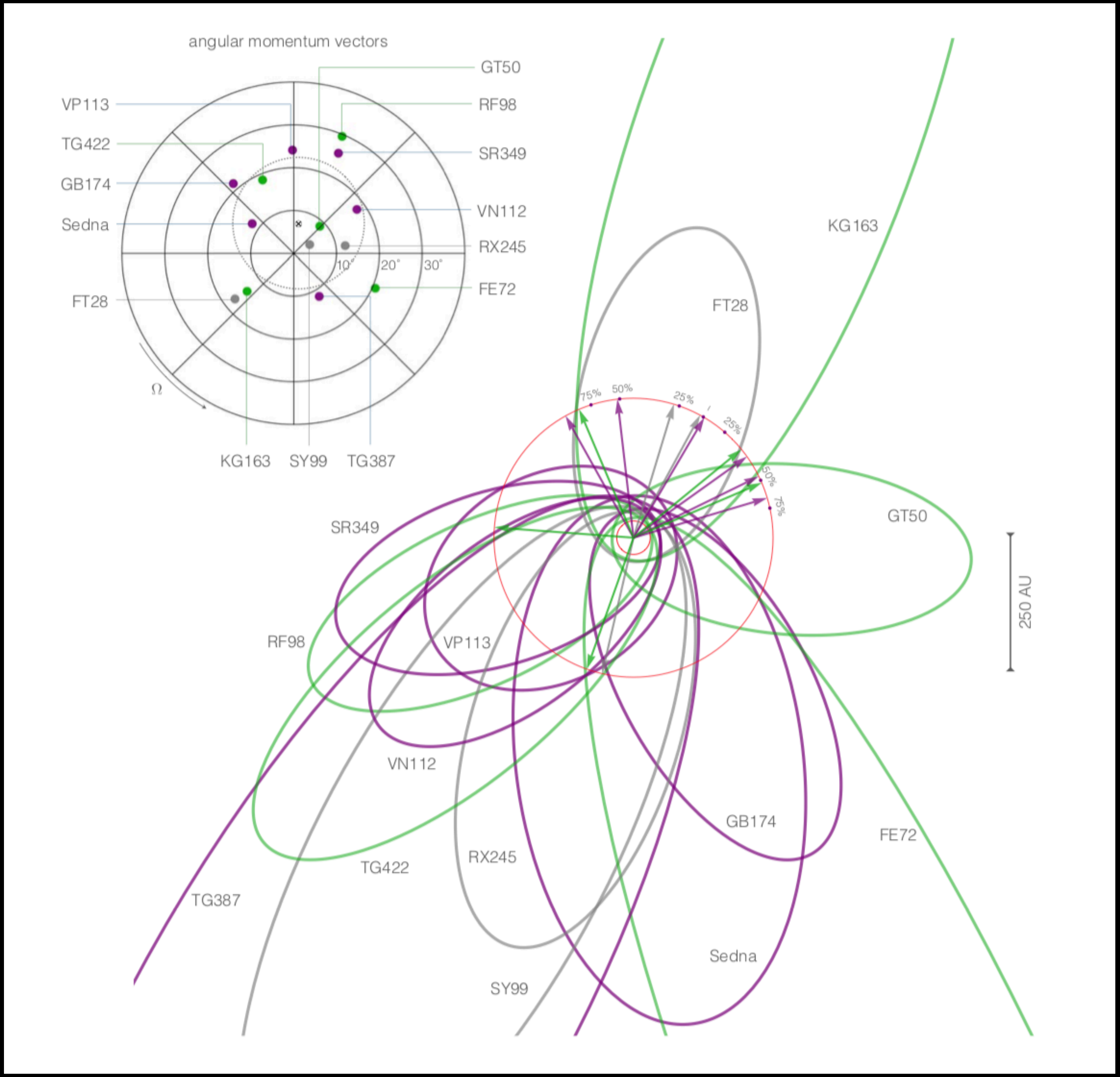
Discovery of Sedna's family



Schwamb, Nature 2014

Anomalous alignment of distant Kuiper belt objects

Batygin and Brown (2016) Batygin et al (2019)

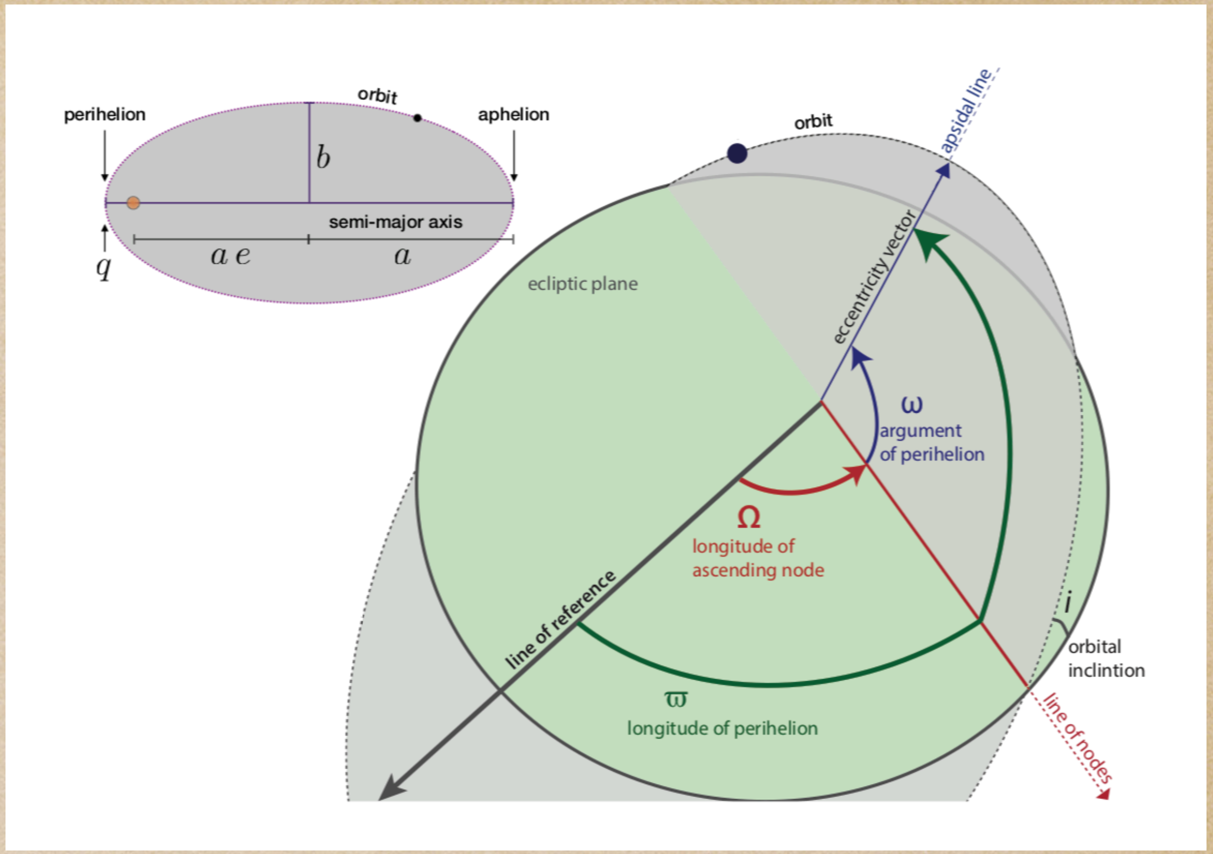


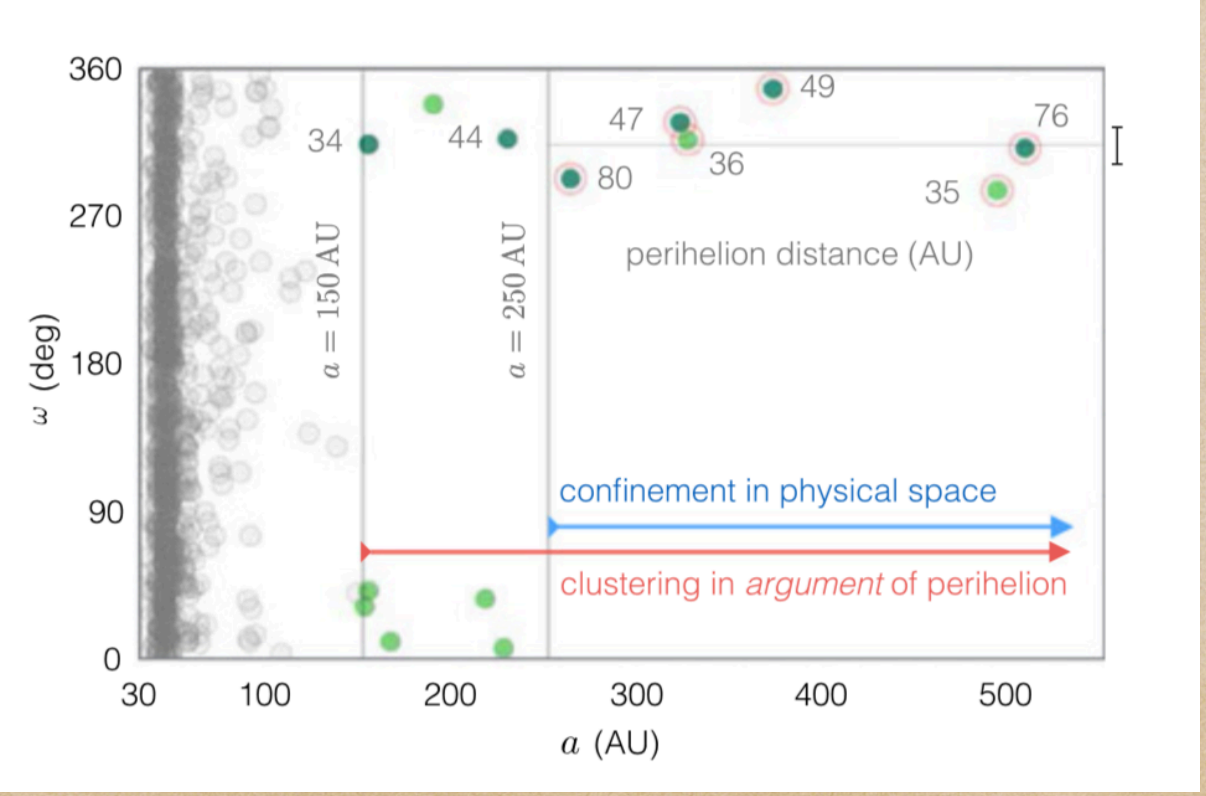
Planet Nine Hypothesis

$$m_9 = 5 - 10\, m_\oplus$$

500 AU from Sun

Batygin et al Phys Rep 2019





Batygin et al Phys Rep 2019

Six orbital elements

$$(a, e, \omega, i, \Omega, f)$$

$\Leftarrow$ Clustering in ω

The Phantom Menace

Newtonian Gravity as field theory

Field equations

$$\nabla \cdot \mathbf{g}_N = -\, 4\pi G\rho$$

$$\nabla \times \mathbf{g}_N = 0$$

Force law

$$\mathbf{F} = m\, \mathbf{g}_N$$

Quasilinear MOND

Milgrom 2010

$$\mathbf{g}_P = \mathbf{g}_N \, \nu\left(\frac{g_N}{a_0}\right)$$

$\mathbf{g}_Q$ is the curl free part of $\mathbf{g}_P$

$\mathbf{g}_Q$ is the physical gravitational field

Quasilinear MOND continued

The interpolating function ν

$$\nu(x) \rightarrow 1 \text{ as } x \rightarrow \infty$$

$$\nu(x) \rightarrow \frac{1}{\sqrt{x}} \text{ as } x \rightarrow 0$$

Not as well-constrained by data for intermediate x

$$\nu(x) = \frac{1}{1 - \exp(-\sqrt{x})}$$

Newtonian field of a point mass

$$\mathbf{g}_N = -\hat{\mathbf{r}}\frac{GM_\odot}{r^2}$$

MOND field of a point mass

$$\mathbf{g}_Q = -\hat{\mathbf{r}}\frac{GM_\odot}{r^2}\nu\left(\frac{R_M^2}{r^2}\right)$$

The MOND radius $R_M = \sqrt{\frac{GM_\odot}{a_0}}$

A first clue

MOND radius of the sun $R_M = 7000$ A.U.

$$a_0 = 1.2 \times 10^{-10} \text{ m/s}^2$$

(best fit to galaxy rotation data)

The Galactic field in the Solar System:

Newtonian Gravity

$$\mathbf{g}_N = -\,\hat{\mathbf{r}}\frac{GM_\odot}{r^2} + \boldsymbol{\gamma}_N$$

$\gamma_N =$ Uniform field of galaxy

Galactic field has no effect on relative motion

of solar system objects

Galactic field in the solar system: MOND

(Milgrom 2009)

Define $\mathbf{g}_A$, the anomalous field via

$$\mathbf{g}_Q = -\hat{\mathbf{r}}\frac{GM_\odot}{r^2} + \mathbf{g}_A + \gamma_G$$

External field effect = existence of $\mathbf{g}_A$

Galactic counterpart observed by Chae et al

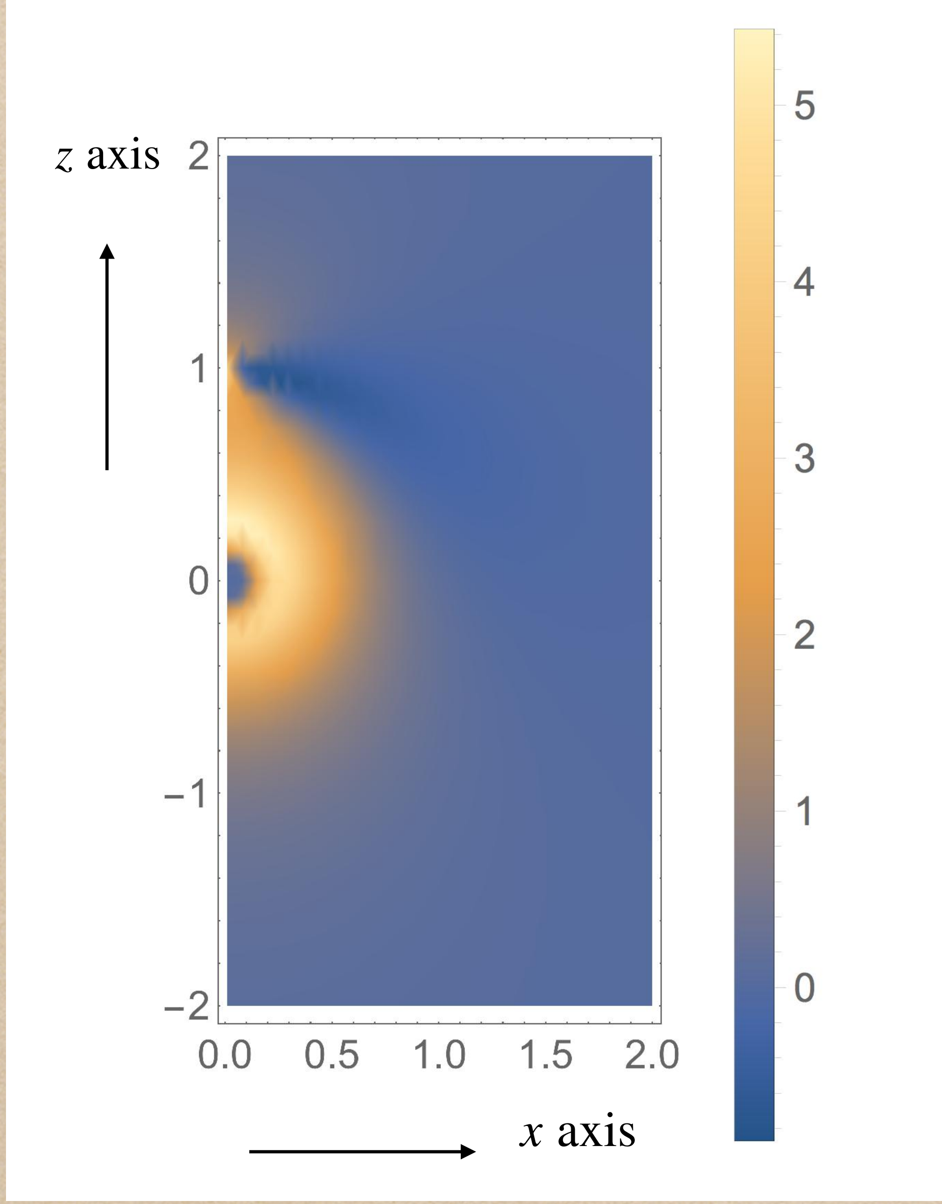
The Phantom Mass

Can interpret $\mathbf{g}_A$ = Newtonian field of ρ_{ph}

$$\rho_{\mathrm{ph}} = \text{phantom mass density}$$

$$= -\frac{1}{4\pi G}\mathbf{g}_N \cdot \nabla \nu\left(\frac{g_n}{a_0}\right)$$

The Phantom Mass Distribution



Sun is at origin;
Galatic center is
along pos z-axis

Rotationally symmetric about z-axis; Localized at $\sim R_M$ from Sun

Multipole Expansion

Quadrupole field of remote mass distribution

$$\psi = f \frac{GM}{R^3} r^2 P_2(\cos\theta)$$

Phantom Mass $\quad M \to M_\odot \,, R \to R_M \,, f = 0.0986\ldots$

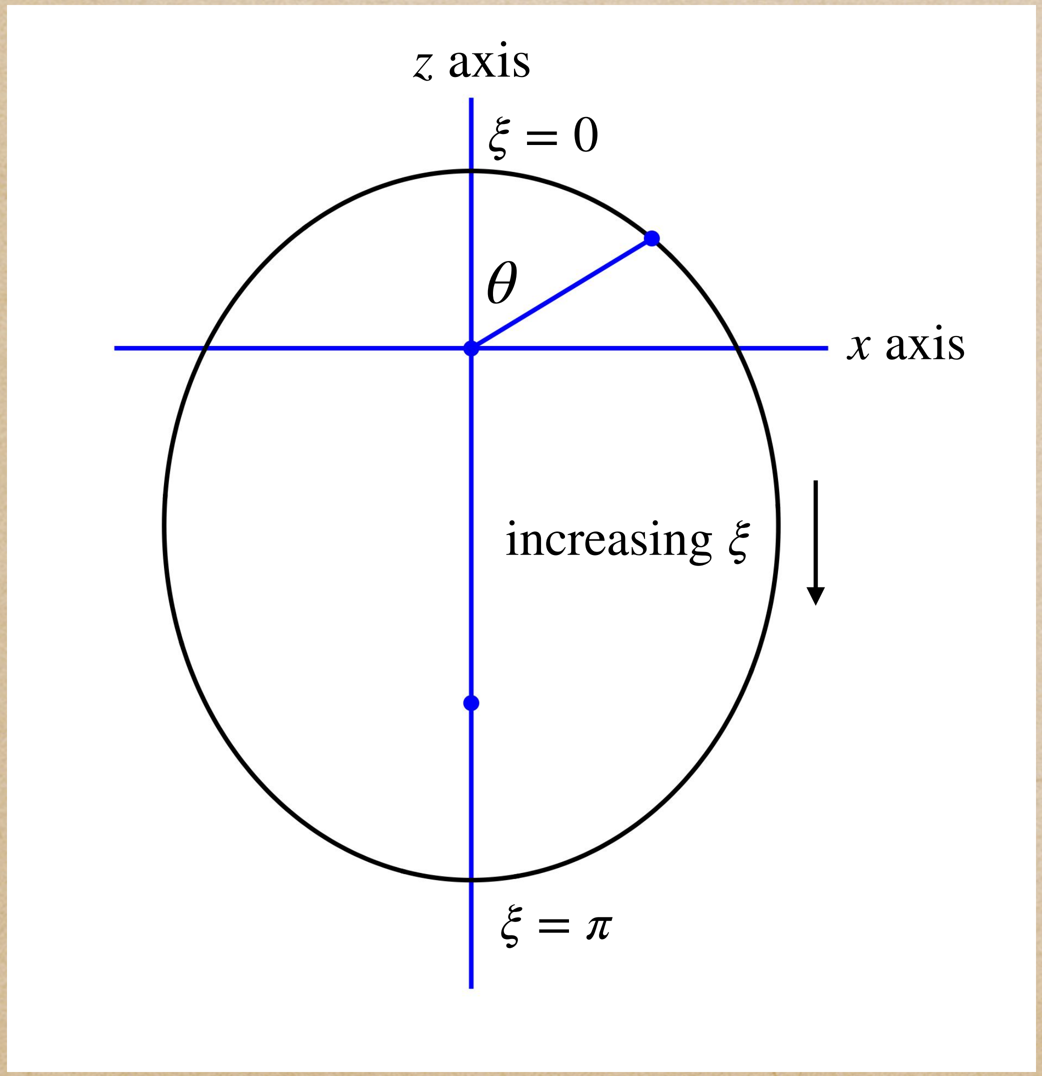
Planet Nine $\quad M \to m_9 = 5\, m_\oplus \,, R \to a_9 \,, f = -\frac{1}{(1 - e_9^2)^{3/2}}$

Second clue: Quadrupoles are comparable; note signs

Planet Nine: Orbit averaged multipole moments

Mass of planet is assumed distributed along its orbit

Mass of an arc proportional to time spent on that arc



Kepler's anomaly equation

$$\frac{t}{2\pi T} = \xi - e \sin \xi$$

$$\Rightarrow dm = \frac{dt}{T} = \frac{m_9}{2\pi}(1 - e \cos \xi)$$

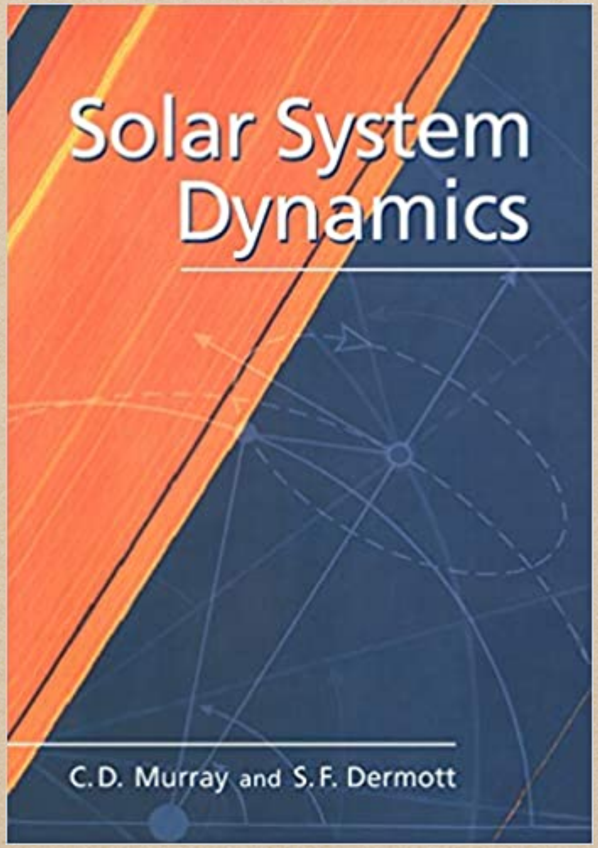
Multipole moments

Rotate like rank ℓ tensors

Advertisement

- New method for calculating multipole moments of disturbing function to arbitrary orders using quantum theory of angular momentum
- Useful for perturbations of KBOs by phantom mass as well as by other planets

Orbital Dynamics



Orbital Dynamics: Physical Idea

Under action of sun alone Kuiper object pursues Kepler orbit with orbital elements

$$a_K, e_K, \omega_K, i_K, \Omega_K, \xi_K$$

Under action of Planet Nine or MOND orbit slowly evolves

Orbital Dynamics: Mathematical Implementation

Dynamical variables: work directly with orbital elements as dynamical variables

Contrast Lagrangian mechanics:

General coordinates and velocities are dynamical variables

Contrast Hamiltonian mechanics:

General coordinates and conjugate momenta are dynamical variables

Orbital Dynamics in Secular Approximation

Useful for long time behavior

Simplifications: (1) a does not change (2) ξ is irrelevant

Dynamics is generated by "disturbing function" $\mathscr{R}$

$\mathscr{R}$ is analogous to Hamiltonian

$\mathscr{R} =$ orbit averaged potential energy of KBO in MOND field or orbit averaged field of Planet 9

MOND Disturbing Function

Use Multipole Expansion; to quadrupole order

$$\mathscr{R}_Q = \frac{G m_K M_\odot}{R_M} \left(\frac{a_K}{R_M}\right)^2 \frac{f}{8} \mathscr{S}_Q(e_K, \omega_K, i_K)$$

$$\mathscr{S}_Q = -2 - 3e_K^2 + 15e_K^2 \cos(2\omega_K) + 6\cos^2 i_K + 9e_K^2 \cos^2 i_k - 15e_K^2 \cos(2\omega_K)\cos^2 i_K$$

Exact expression!

Independence of Ω_K due to rotational invariance

Coordinate System

Origin = Sun; Galactic center along pos z axis

Standard orientation: KBO orbit lies in x-y plane;

perihelion along pos x axis

Orientation $(\omega_K, i_K, \Omega_K)$:

Twist by ω_K about z; i_K about x; Ω_K about z

Conservation laws

- Angular momentum along symmetry axis z

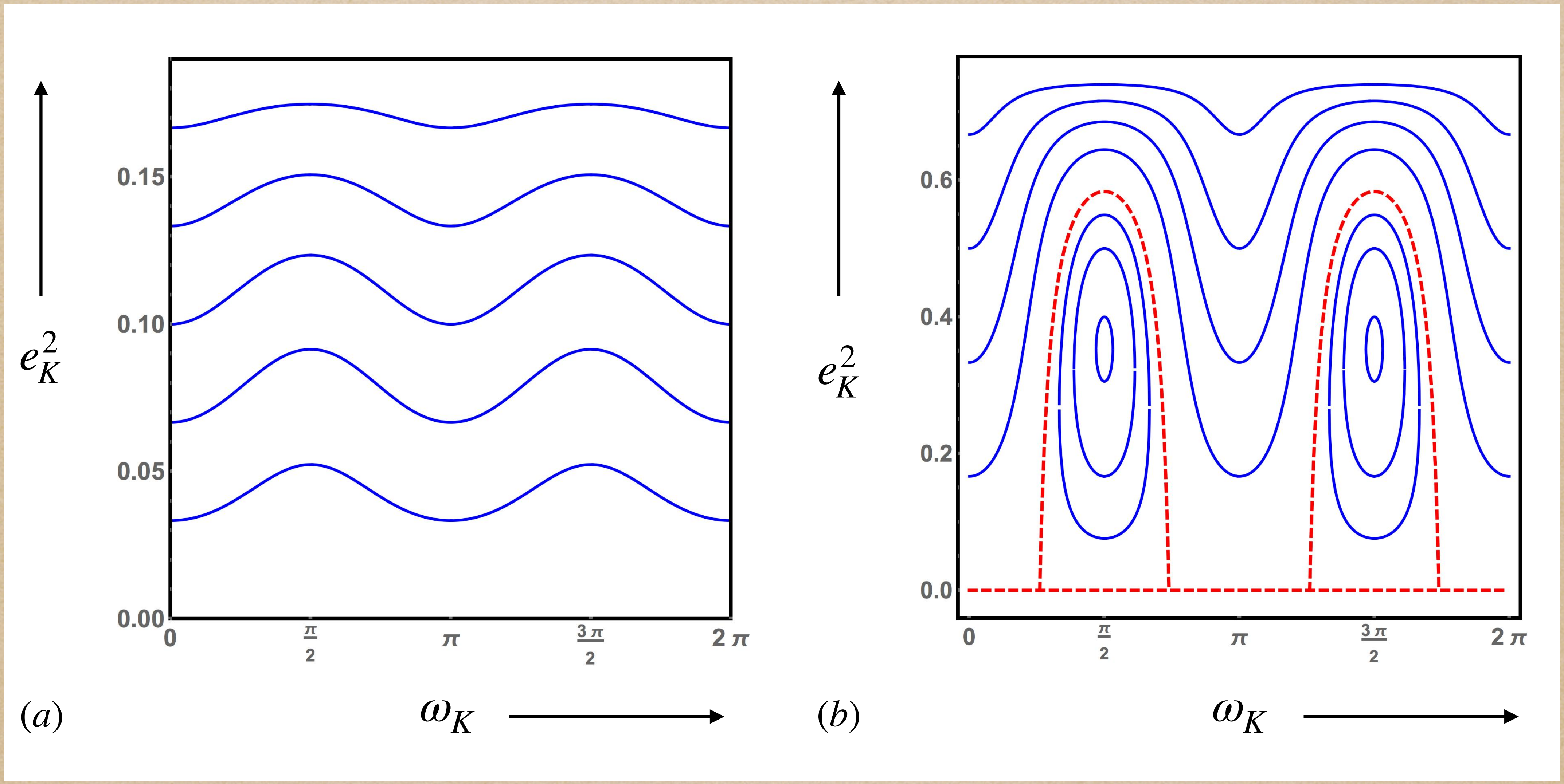
$$h = \sqrt{1 - e_K^2} \cos i_K$$

- $\mathscr{R}$ itself is conserved

Integrable model:

4 degrees of freedom 2 conservation laws

MOND Dynamics in Quadrupole Approximation



$$\frac{3}{5} \leq h^2 \leq 1$$

$$0 \leq h^2 < \frac{3}{5}$$

Precession

Note fixed points!

Fixed Point Analysis

Location of fixed points $(e_K, \omega_K) = (e_C, \pm \pi/2)$

$$e_C^2 = 1 - \sqrt{5/3}\,|h|$$

Effect of octupole term: destabilizes + relative to -

Additional Perturbations: Giant planets, non-secular terms, motion of sun around galaxy

Their effect: chaotic flow except near stable fixed point

Key Predictions

Expect a population of KBOs with orbits

close to stable fixed point $(e_C, -\pi/2)$

For small h find $i_K \approx \pi/2$

- Orbits with $i_K \approx \pi/2$ and $\omega_K = -\pi/2$

  have $\hat{\boldsymbol{\alpha}}_K$ antiparallel to $\hat{\mathbf{n}}_G$

$\hat{\boldsymbol{\alpha}}_K =$ unit vector from sun to KBO perihelion
$\hat{\mathbf{n}}_G =$ unit vector from sun to galactic center

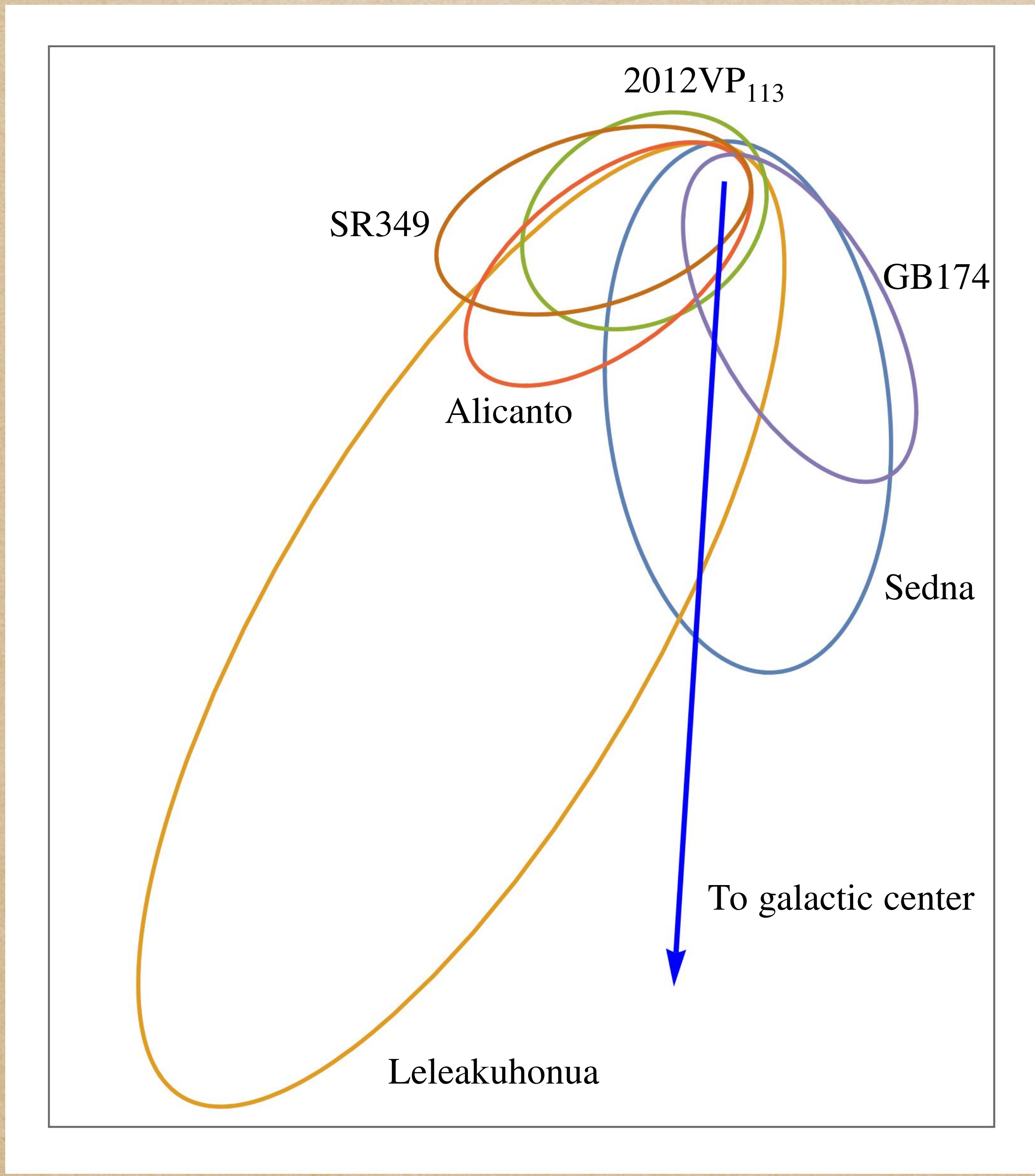
Data: Sedna Family Orbital Parameters

Object	ω	Ω	i	e	q (AU)	a (AU)
Sedna	311.1°	144.2°	11.9°	0.85	76.34	499.70
TG387[a]	118.0°	300.8°	11.7°	0.94	65.04	1031.49
2012 VP_{113}	293.5°	90.7°	24.1°	0.69	80.39	258.27
VN112[b]	326.8°	66.0°	25.6°	0.85	47.30	318.97
GB174	347.0°	130.9°	21.6°	0.86	48.61	336.67
SR349	340.0°	34.8°	18.0°	0.84	47.69	302.23

TABLE I. Orbital elements of six KBOs of the Sedna family. The data are from the Minor Planet Database of the International Astronomical Union. [a] TG387 is named Leleakuhonua. [b] VN112 is named Alicanto.

Apsidal Alignment

Caveat: Observational bias



Mean value of

$\hat{\boldsymbol{\alpha}}_K \cdot \hat{\mathbf{n}}_G$

is 3σ away from

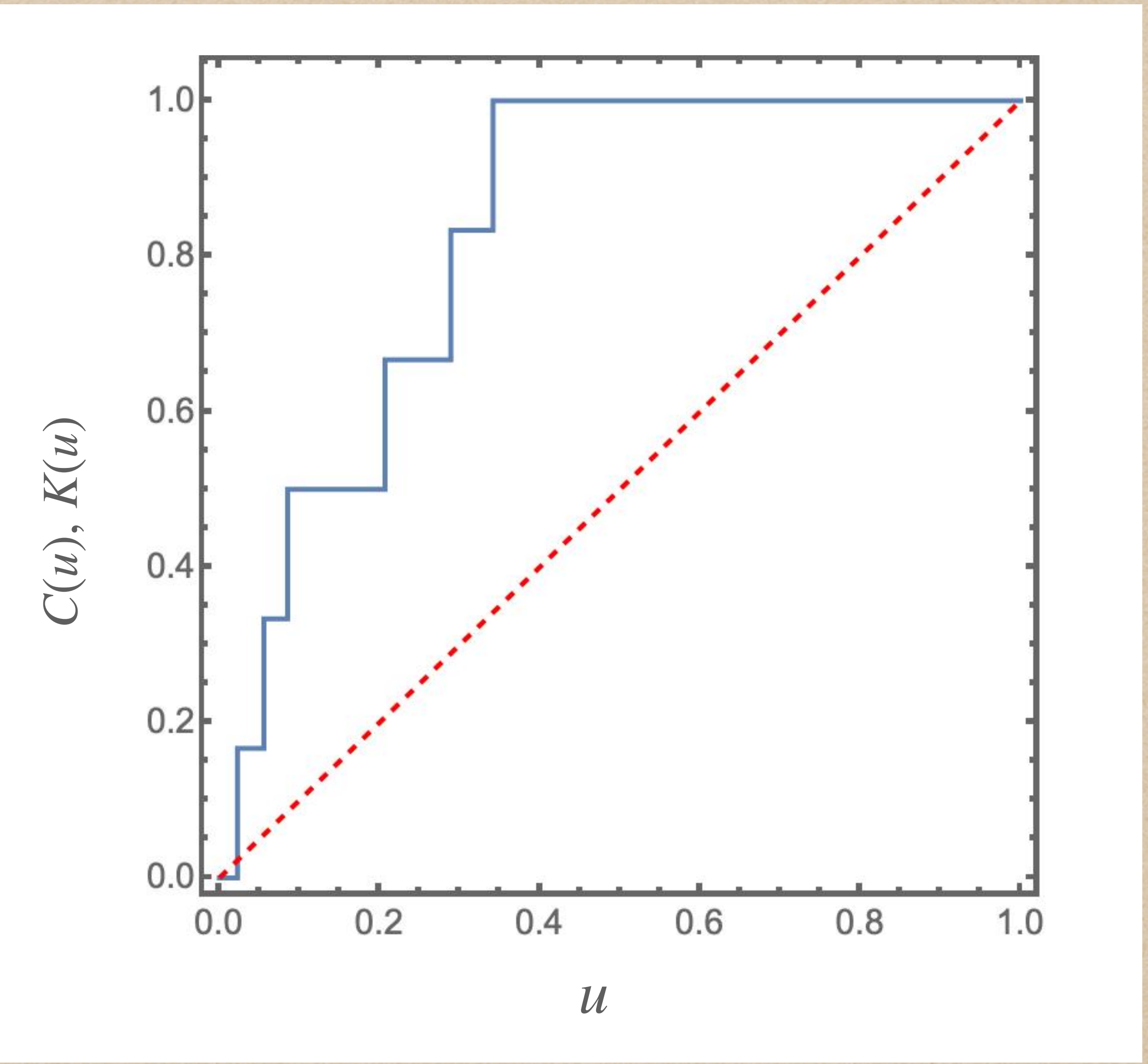
purely random $\hat{\boldsymbol{\alpha}}_K$

1 in 200 odds

relative to

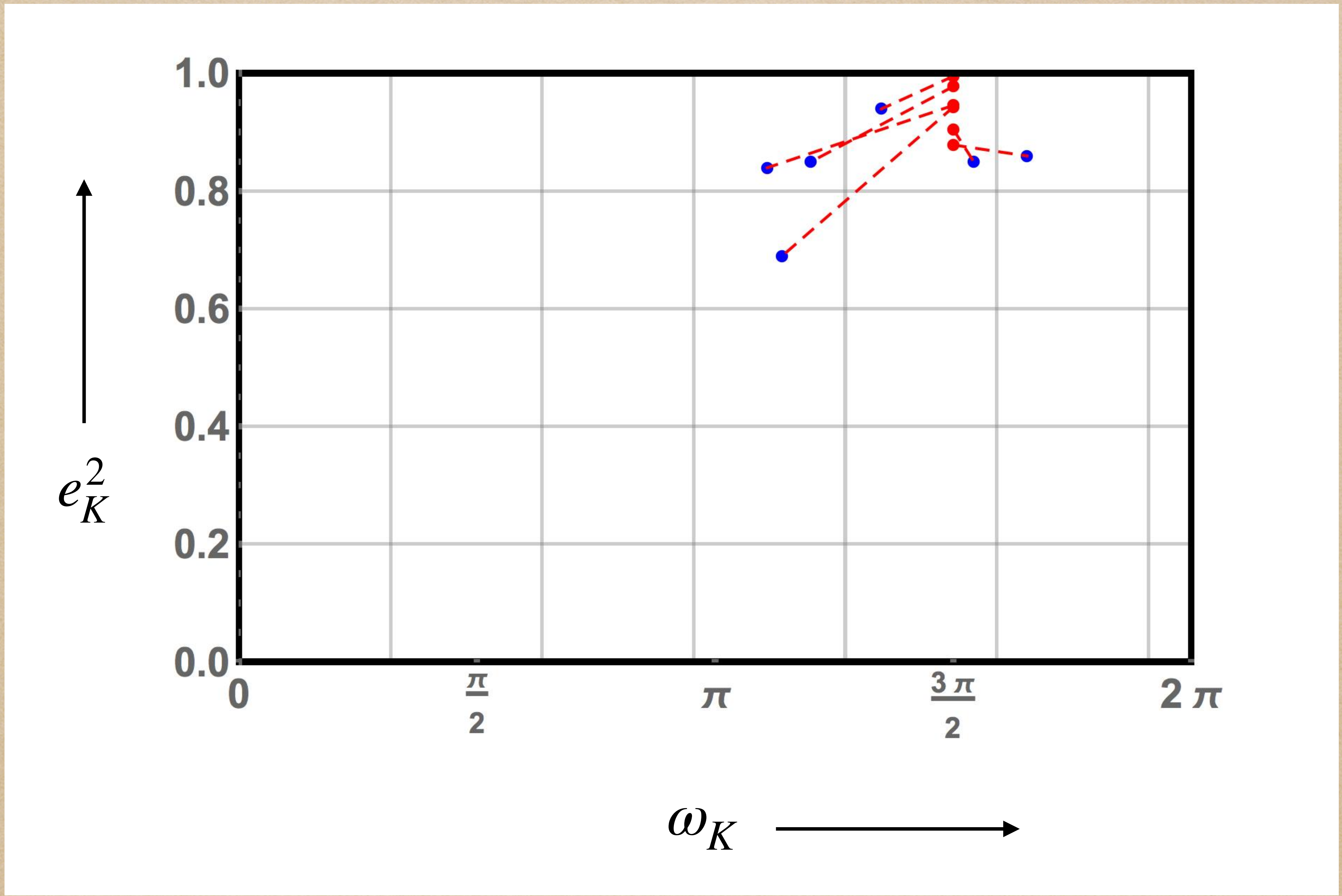
pure random $\hat{\mathbf{a}}_K$

Kolmogorov-Smirnoff test



$u_i = \frac{1}{2}(1 + \boldsymbol{\alpha}_{Ki} \cdot \boldsymbol{n}_G)$ Null hypothesis = u_i is uniformly distributed

Phase Space Clustering



Conclusion and Discussion

Future: Compare a larger sample of Sedna family objects to dynamical simulations to test MOND hypothesis

Future and ongoing surveys:

Deep ecliptic survey

Transiting Exoplanet Survey Satellite (TESS)

Outer Solar System Origins Survey (OSSOS)

Vera Rubin Telescope

CMB-S4 ...

Clustering - Spurious, Planet Nine or MOND?

Historically gravitational anomalies in solar system were often spurious but also led to discovery of Neptune and General Relativity

Kuiper belt is a laboratory for studying fundamental physics