Monte Carlo Radiation Transfer

Jon Bjorkman
Ritter Observatory
3-D Radiation Transfer

• Transfer Equation

\[ \hat{n} \cdot \nabla I_\nu = -\chi_\nu \rho I_\nu + j_\nu + \sum_i n_i \sigma^i_\nu \int \left( \frac{1}{\sigma^i_\nu} \frac{d\sigma^i_\nu}{d\Omega} \right) I_\nu(\hat{n}')d\Omega' \]

– Ray-tracing (requires lambda-iteration)

– Monte Carlo (exact integration using random paths)
  • *May avoid lambda-iteration*
  • automatically an adaptive mesh method
    – Paths sampled according to their importance
Monte Carlo Radiation Transfer

- Transfer equation traces flow of energy
- Divide luminosity into equal energy packets ("photons")

\[ E_\gamma = \frac{L \Delta t}{N_\gamma} \]
- Number of physical photons

\[ n = \frac{E_\gamma}{h\nu} \]
- Packet may be partially polarized

\[ I = 1 \]
\[ Q = \frac{(E_\uparrow - E_\leftrightarrow)}{E_\gamma} \]
\[ U = \frac{(E_\rightarrow - E_\leftarrow)}{E_\gamma} \]
\[ V = \frac{(E_\circ - E_\circ)}{E_\gamma} \]
Monte Carlo Radiation Transfer

- Split luminosity between star and envelope

\[ L = L_* + L_{\text{env}} \]

**Star**

\[ \mu I_\nu = \frac{dE / dt}{dA d\nu d\Omega} \]

\[ \frac{dP}{dA d\nu d\Omega} \propto \mu I_\nu \]

\[ \frac{dP}{dA d\nu} \propto H_\nu \]

\[ \frac{dP}{dA} \propto H \]

**Envelope**

\[ j_\nu = \frac{dE / dt}{dV d\nu d\Omega} \]

\[ \frac{dP}{dV d\nu d\Omega} \propto j_\nu \]

\[ \frac{dP}{dV d\nu} \propto 4\pi j_\nu \]

\[ \frac{dP}{dV} \propto 4\pi \int_0^\infty j_\nu \, d\nu \]
Monte Carlo Radiation Transfer

- Pick random starting location, frequency, and direction
  - Sample from appropriate probability distributions:
    - Location on star:
      \[
      \frac{dP}{dA} \propto H
      \]
    - Frequency:
      \[
      \frac{dP}{d\nu} \propto H_\nu
      \]
    - Direction:
      \[
      \frac{dP}{d\Omega} \propto \mu I_\nu
      \]
Monte Carlo Radiation Transfer

- Doppler Shift photon packet as necessary
  - packet energy is frame-dependent
    \[ E_\gamma \rightarrow wE_\gamma \quad w \text{ is photon "weight"} \]
- Transport packet to random interaction location

\[
\begin{align*}
  dP &= d\tau = \chi_\nu \rho ds \quad \text{(Poisson Distribution)} \\
  dN &= -Nd\tau \\
  P &= 1 - e^{-\tau} \quad \text{(Cumulative Probability)} \\
  \tau &= -\ln \xi \quad (\xi \text{ is uniform random number}) \\
  \tau &= \int_0^s \chi_\nu \rho ds \quad \text{(find distance, } s) \quad \text{most CPU time} \\
  \mathbf{x} &= \mathbf{x}_0 + s \hat{\mathbf{n}} \quad \text{(move photon)}
\end{align*}
\]
Monte Carlo Radiation Transfer

- Randomly scatter or absorb photon packet

\[ a = \frac{\sigma_\nu}{\sigma_\nu + \kappa_\nu} \quad \text{(albedo)} \]

\[
\begin{cases} 
\xi > a & \text{(absorb)} \\
\xi < a & \text{(scatter)}
\end{cases}
\]

\[ \frac{dP}{d\Omega} = \frac{1}{\sigma_\nu} \frac{d\sigma_\nu}{d\Omega} \quad \text{(phase function)} \]

- If photon hits star, reemit it locally
- When photon escapes, place in observation bin (direction, frequency, and location)

**REPEAT 10^6-10^9 times**
Monte Carlo Maxims

• Monte Carlo is **EASY**
  – to do wrong (G.W. Collins III)
  – code must be tested *quantitatively*
  – being clever is dangerous
  – try to avoid discretization

• **The Improbable event WILL happen**
  – code must be bullet proof
  – and error tolerant
Sampling and Measurements

• **MC simulation produces random events**
  – Photon escapes
  – Photon interactions
  – Cell wall crossings
  – Photon motion

• **Events are sampled/counted**
  – Cumulative energy => measurements (flux)
  – Histogram => distribution function (spectrum)
SEDs and Images

- **Sampling Photon Escapes**

\[
\frac{F_\nu}{F_*=} = \frac{4\pi d^2}{L} \frac{dE}{dt dA d\nu} = \frac{4\pi d^2}{L} \frac{N_{ij} E_\gamma / \Delta t}{d^2 d\Omega_i \Delta \nu_j} = \frac{4\pi N_{ij}}{N_\gamma d\Omega_i \Delta \nu_j}
\]

where \( N_{ij} = \sum w_{ij} \)

\[
\frac{I_\nu}{F_*} = \frac{4\pi N_{ijkl}}{N_\gamma d\Omega_{ij} d\Omega_k \Delta \nu_l}
\]
SEDs and Images

• Advanced Sampling
  – Photon interactions: scattering, emission
    (source function sampling)

\[
dN_i = \begin{cases} 
  w \left( \frac{1}{\sigma} \frac{d\sigma}{d\Omega} \right) e^{-\tau_{esc}} \Delta\Omega_i & \text{(scattered)} \\
  \frac{w}{4\pi} e^{-\tau_{esc}} \Delta\Omega_i & \text{(emitted)} 
\end{cases}
\]

  – Photon motion (Lucy path length sampling)

\[
dN_{abs} = w d\tau_{abs} = w\kappa_\nu \rho ds \\
dN_i = w d\tau_{sc} \left( \frac{1}{\sigma} \frac{d\sigma}{d\Omega} \right) e^{-\tau_{esc}} \Delta\Omega_i
\]
Error Estimation

• **Unweighted Photons**
  – Number in bin has a binomial distribution

\[
\frac{\delta E}{E} = \sqrt{\frac{1 - N_i / N_\gamma}{N_i}} \\
\approx 1 / \sqrt{N_i}
\]

• **Weighted Photons**
  – Each photon track is statistically independent

\[
w_i = \sum_{\text{track}} w_{\text{obs}}
\]

\[
\frac{\delta E}{E} = \sqrt{\sum_i w_i^2} / \sum_i w_i
\]
Parallel Monte Carlo

- **Photon paths are independent**
  - Divide total among different CPUs
  - Each CPU independently runs its batch of photons
  - Co-add results at end
  - Embarrassingly parallel
Parallel Implementation

- **Master/Slave:**
  - Master sends messages to slaves:
    - Initialize (includes simulation parameters)
    - Run batch of N photons
    - Retrieve results
    - Reinitialize (zero all counters)
    - Die
  - Each slave reports back to master when done
  - Master gives slave new batch of photons
    - **Automatic CPU load balancing**
  - Results collected when all slaves are finished
    - **Minimizes network load**
Monte Carlo Assessment

• **Advantages**
  – Inherently 3-D
  – Microphysics easily added (little increase in CPU time)
  – Modifications do not require large recoding effort
  – Embarrassingly parallelizable

• **Disadvantages**
  – High S/N requires large number of photons
  – **Achilles heel = no photon escape paths; i.e., large optical depth**
Improving Run Time

• Photon paths are random
  – Can reorder calculation to improve efficiency
• Adaptive Monte Carlo
  – Modify execution as program runs
• High Optical Depth
  – Use analytic solutions in “interior” + MC “atmosphere”
    • Diffusion approximation (static media)
    • Sobolev approximation (for lines in expanding media)
  – Match boundary conditions
MC Radiative Equilibrium

- Sum energy absorbed by each cell
- Radiative equilibrium gives temperature

\[ E_{\text{abs}} = E_{\text{emit}} \]

\[ n_{\text{abs}} E_\gamma = 4\pi m_i\kappa_p B(T_i) \]

- When photon is absorbed, reemit at new frequency, depending on \( T \)
  - Energy conserved automatically
- Problem: Don’t know \( T \) a priori
- Solution: Change \( T \) each time a photon is absorbed and correct previous frequency distribution

avoids iteration
Temperature Correction

Frequency Distribution:

\[
\frac{dP}{dv} = j_v(T + \Delta T) - j_v(T)
\]

\[
= \kappa_v \Delta T \frac{dB_v}{dT}
\]

Bjorkman & Wood 2001
T Tauri Envelope Absorption
Disk Temperature

Bjorkman 1998
Disk Temperature

Snow Line

Water Ice
Methane Ice
Effect of Disk on Temperature

- Inner edge of disk
  - heats up to optically thin radiative equilibrium temperature
- At large radii
  - outer disk is shielded by inner disk
  - temperatures lowered at disk mid-plane
CTTS Model SED

![Graphs showing the SED spectra for different inclinations (i = 38, 80, 77, 58) with wavelength (\(\lambda_{\mu m}\)) on the x-axis and flux density on the y-axis.]
Protostar Evolutionary Sequence

Protostar Evolutionary Sequence


Planet Gap-Clearing Model

Rice et al. 2003
Protoplanetary Disks

Surface Density

\[ i = 5 \]
\[ i = 30 \]
\[ i = 75 \]
Spectral Lines

• Lines very optically thick
  – Cannot track millions of scatterings
• Use Sobolev Approximation (moving gas)
  – Sobolev length
    \[ l(\hat{n}) = \frac{v_D}{\left| \frac{dv}{dl} \right|} \]
    \[ \frac{dv}{dl} = n^i e_{ij} n^j \]
    \[ e_{ij} = (v_{i;j} + v_{j;i}) \div 2 \]
  – Sobolev optical depth
    \[ \tau_{sob} = \frac{k_L c}{\nu_0 \left| \frac{dv}{dl} \right|} \]
    \[ k_L = \frac{\pi e^2}{m_e c} g f \left( \frac{n_l}{g_l} - \frac{n_u}{g_u} \right) \]
  – Assume S, rho, etc. constant (within l)
Spectral Lines

- **Split Mean Intensity**
  \[ J = J_{\text{local}} + J_{\text{diffuse}} \]

- **Solve analytically for** \( J_{\text{local}} \)

- **Effective Rate Equations**

  \[
  \beta_e n_u A_{ul} + \beta_p n_u B_{ul} \bar{J}_{\text{diff}} - \beta_p n_l B_{lu} \bar{J}_{\text{diff}} + \ldots = 0
  \]

  \[
  \beta_e = \frac{1}{4\pi} \int \frac{1 - e^{-\tau_{\text{sob}}}}{\tau_{\text{sob}}} \, d\Omega
  \]

  (escape probability)

  \[
  \beta_p = \frac{1}{4\pi} \int \frac{1 - e^{-\tau_{\text{sob}}}}{\tau_{\text{sob}}} \frac{\bar{J}_{\text{diff}}}{\bar{J}_{\text{diff}}} \, d\Omega
  \]

  (penetration probability)

  \[
  \frac{dP}{d\Omega} \propto J_{\text{esc}} = \frac{h\nu n_u A_{ul}}{4\pi} \left( \frac{1 - e^{-\tau_{\text{sob}}}}{\tau_{\text{sob}}} \right)
  \]

  (effective line emissivity)
Resonance Line Approximation

- Two-level atom => pure scattering
- Find resonance location
  \[ \nu_0 = \nu(1 - v \cdot \hat{n} / c) \]
- If photon interacts
  - Reemit according to escape probability
    \[ \frac{dP}{d\Omega} \propto \frac{1 - e^{-\tau_{\text{sob}}}}{\tau_{\text{sob}}} \]
  - Doppler shift photon; adjust weight
Wind Line Profiles

(b) Pole-On View (0°)  Edge-On View (90°)

Bjorkman 1998
**NLTE Monte Carlo RT**

- **Gas opacity depends on:**
  - temperature
  - degree of ionization
  - level populations

- **During Monte Carlo simulation:**
  - sample radiative rates

- **Radiative Equilibrium**
  - Whenever photon is absorbed, re-emit it

- **After Monte Carlo simulation:**
  - solve rate equations
  - update level populations and gas temperature
  - update disk density (integrate HSEQ)
What are Be stars?

B stars (T ~ 20000 K) with hydrogen emission lines

Detected by:
- Spectral lines in emission (frequently double peaked)
- IR excess (ff+bf emission)
- Linear polarization (scattering in the disk)

Rapidly rotating (non-supergiants) with a circumstellar disk

Disk is geometrically thin
Viscous Decretion Disk

- Lee, Saio, Osaki 1991
Be Star Disk Temperature

Carciofi & Bjorkman 2004
SED and Polarization

$F_\lambda$ (ergs cm$^{-2}$ s$^{-1}$ Å$^{-1}$)

- $\zeta$ Tau
- Kurucz (19000 K)
- $i = 70$ degrees

$P$ (%)

$\lambda$ (µm)

Carciofi & Bjorkman 2004
Emission Line Formation: Iso-Velocity Contours
Emission Line Formation

V = -175 km/s

V = -75 km/s

V = -25 km/s

Continuum

V = 0 km/s

V = 25 km/s

V = 75 km/s

V = 175 km/s
V/R Variations
Global Disk Oscillations

- Elliptical Orbits in Disk
  - Periastron
    speed high => low density
  - Apastron
    speed low => high density
- Orbits can precess (Papaloizou, Savonije & Henrichs 1992)
  - Density wave rotates at precession period
Precessing Density Wave
Zeta Tau: Precessing Density Wave

Amber/VLTI Brg Observations
(Stefl et al. 2009)

Carciofi et al. (2009)
Acknowledgments

• Rotating winds and bipolar nebulae
  – NASA NAGW-3248
• Ionization and temperature structure
  – NSF AST-9819928
  – NSF AST-0307686
• Geometry and evolution of low mass star formation
  – NASA NAG5-8794
• UT Students: B. Abbott, I. Mihaylov, J. Thomas
• REU Students: A. Moorhead, A. Gault
Acknowledgments

• University of Toledo Sabbatical Leave
• FAPESP Grant (Processo nº 2010/16037-2)
• Collaborators
  – Alex Carciofi, Xavier Haubois (São Paulo)
  – Stan Stefl, Thomas Rivinius, Dietrich Baade (ESO)
  – Rene Oudmaijer, Hugh Wheelwright (Leeds)
  – Atsuo Okazaki (Hokkai-Gakuen, Sapporo)
  – Karen Bjorkman, Stephanie Rety (Toledo)