Stellar Radii

- To calculate *R*: $L = 4p R^2 s T^4$
- Observe:
  - . parallax *p* --> distance *d=1/p*.
  - . spectral type or colour index --> *T*
  - . apparent magnitude, e.g. *V*.

$$V - M_V = 5 \log(d / 10\,\mathrm{pc}) \qquad M_{bol} = M_V + B.C.$$

$$M_{bol} - M_{bol}(\mathrm{sun}) = -2.5 \log\left(L / L(\mathrm{sun}) \right)$$

- Not highly accurate (10-50%)

Typical Radii

- Solar radius: *R*sun=7x10^5 km

  main-sequence stars:

  R ~ 0.1 - 10 *R*sun

  giants: *R* ~ up to 100 *R*sun

  supergiants: red: *R* ~up to 1000 *R*sun

  blue: *R* ~20-50

  white dwarfs: *R* ~ 0.*01* *R*sun

Accurate Radii

- Most accurate radii (<1%) from

  . ECLIPSING BINARY STARS

  and (for nearby stars)

  . INTERFEROMETRY

  and (for a few stars)

  . LUNAR OCCULTATIONS.

- Accurate *R* improves *T* via

$$T = \left(\frac{L}{4\pi R^2 \sigma} \right)^{1/4}$$

- if distance (hence *L*) also known.

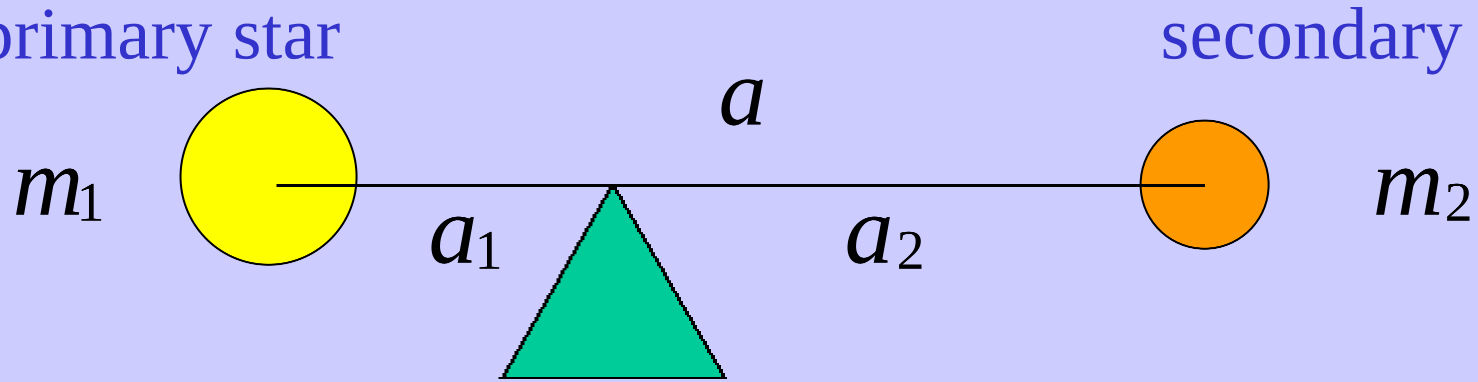
Binary Stars

-- two stars in mutual gravitational attraction, orbiting their common centre of mass

-- only source of empirical ***masses*** for stars

-- accurate sizes, shapes, temperatures, luminosities (hence distances)

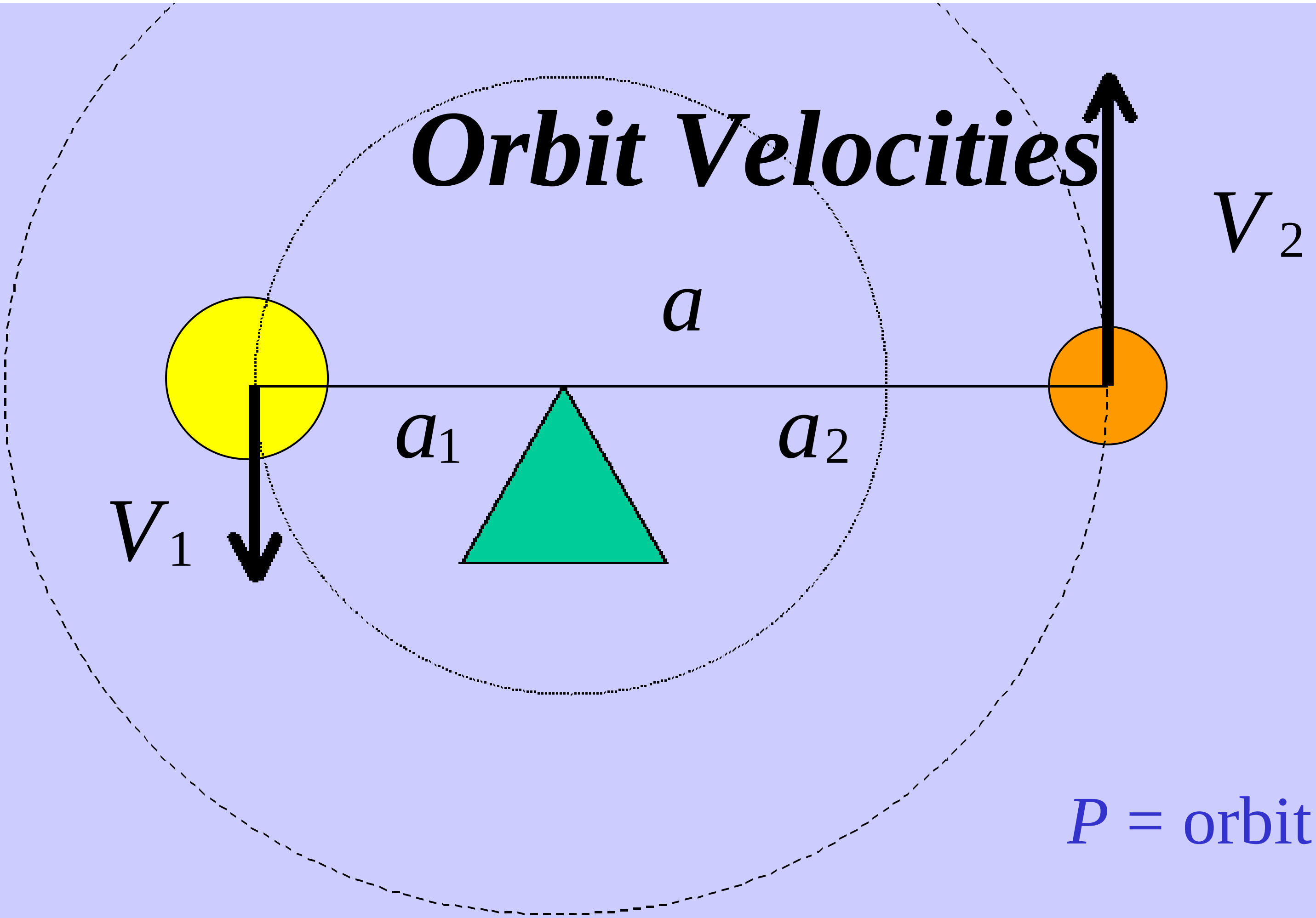
Centre of Mass



$$a_1 m_1 = a_2 m_2$$

$$\frac{a_1}{a} = \frac{m_2}{m_1 + m_2} \qquad \frac{a_2}{a} = \frac{m_1}{m_1 + m_2}$$

Orbit Velocities

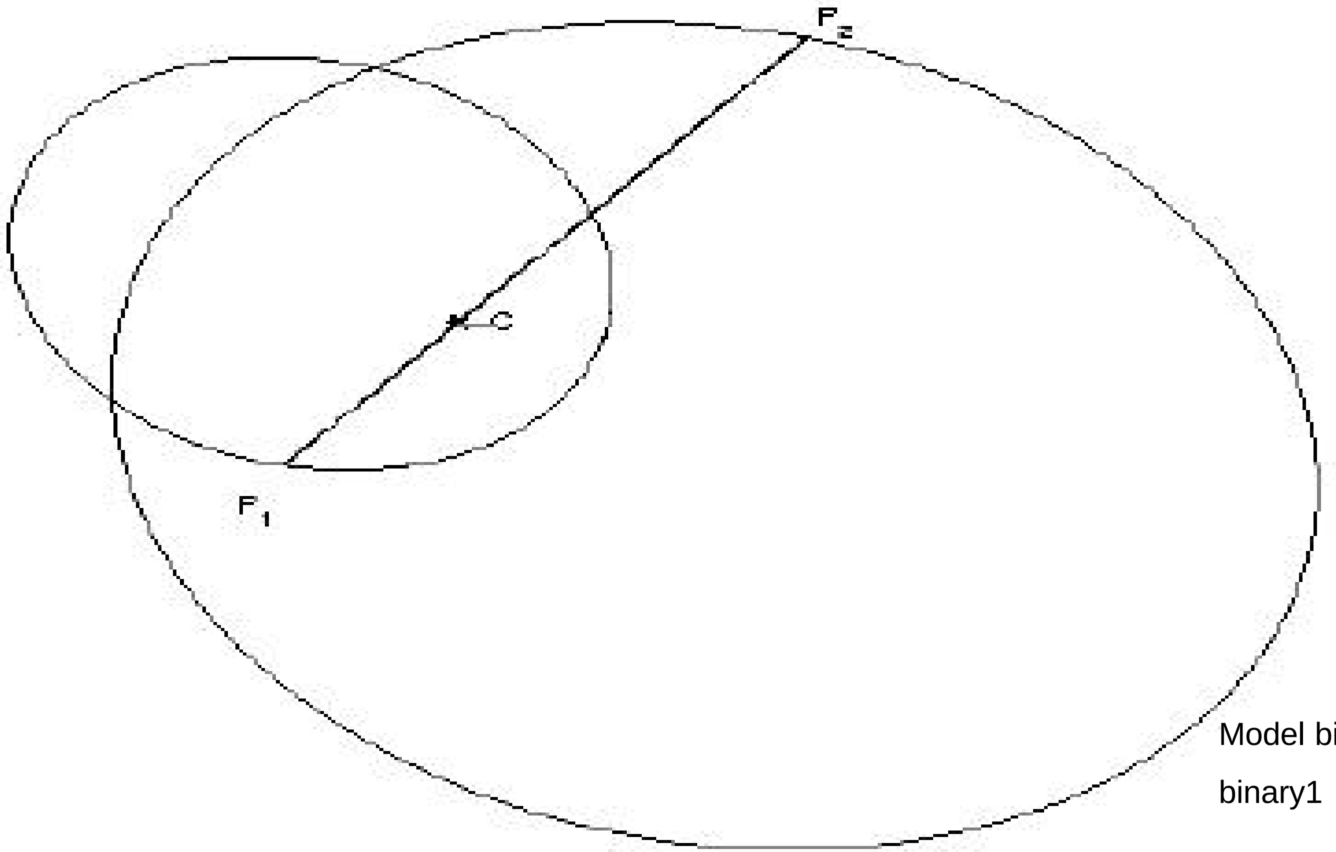


P = orbit period

$$\frac{V_1}{V_2} = \frac{a_1}{a_2} = \frac{m_2}{m_1}$$

$$V_1 + V_2 = \frac{2p\ a}{P}$$

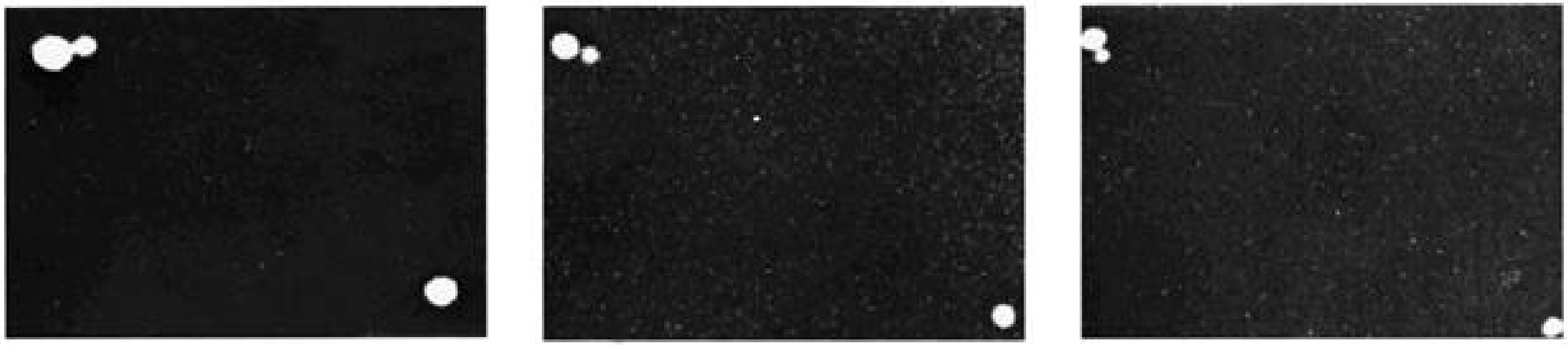
Elliptical Orbits



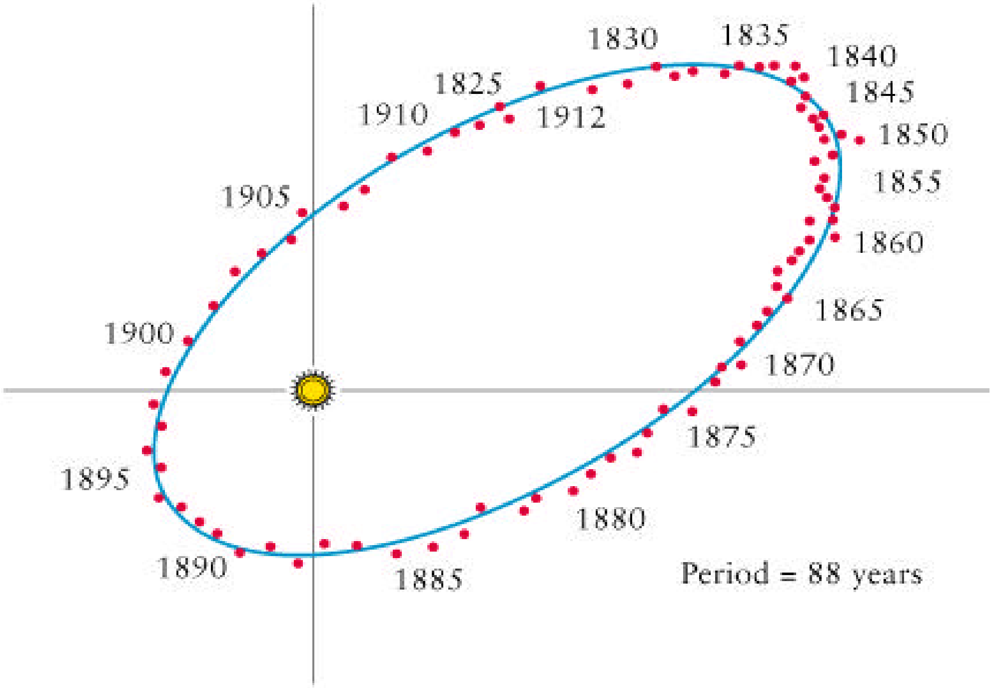
Model binary

binary1

Visual binary



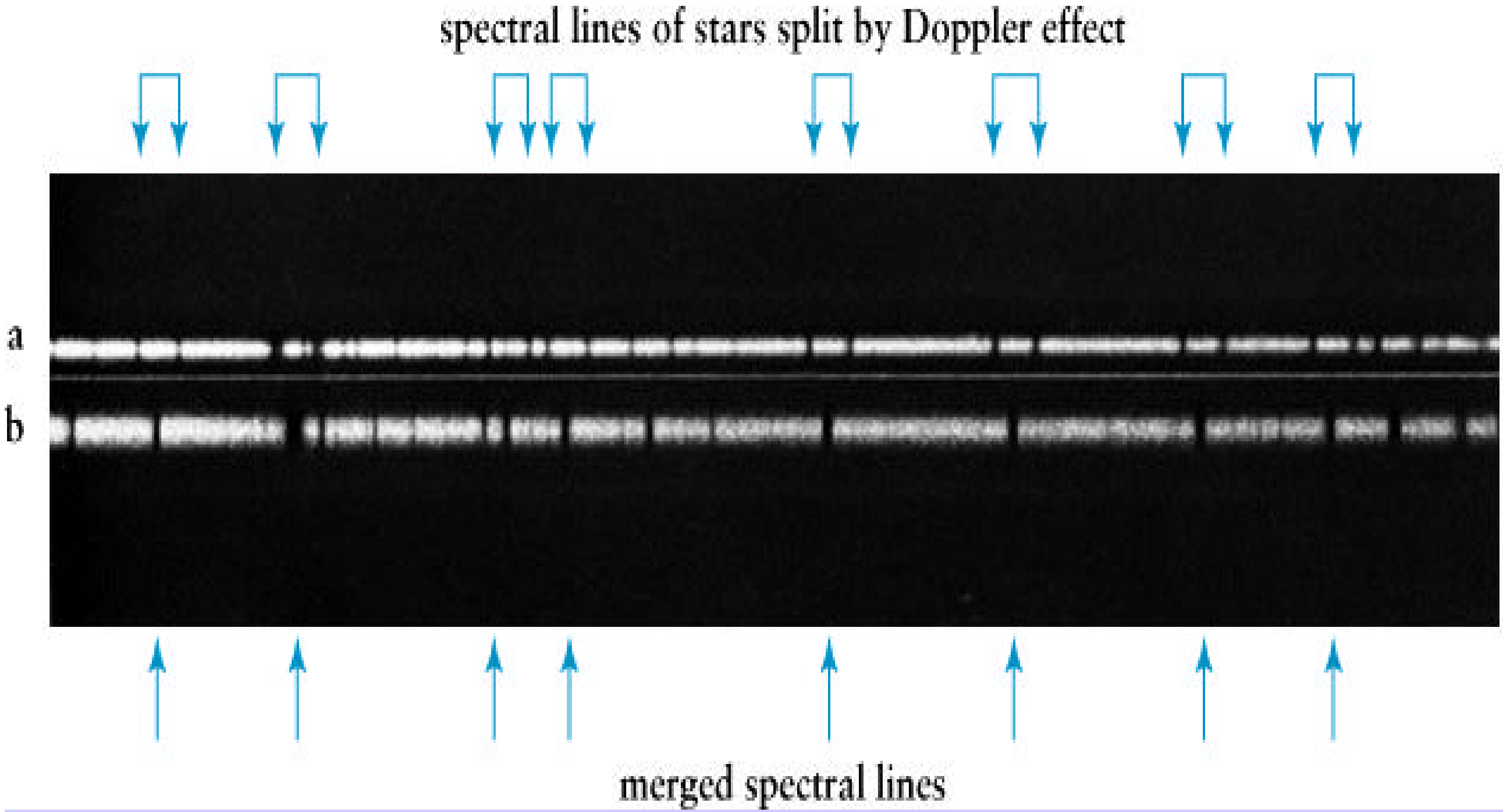
1908 1915 1920



Types of Binaries

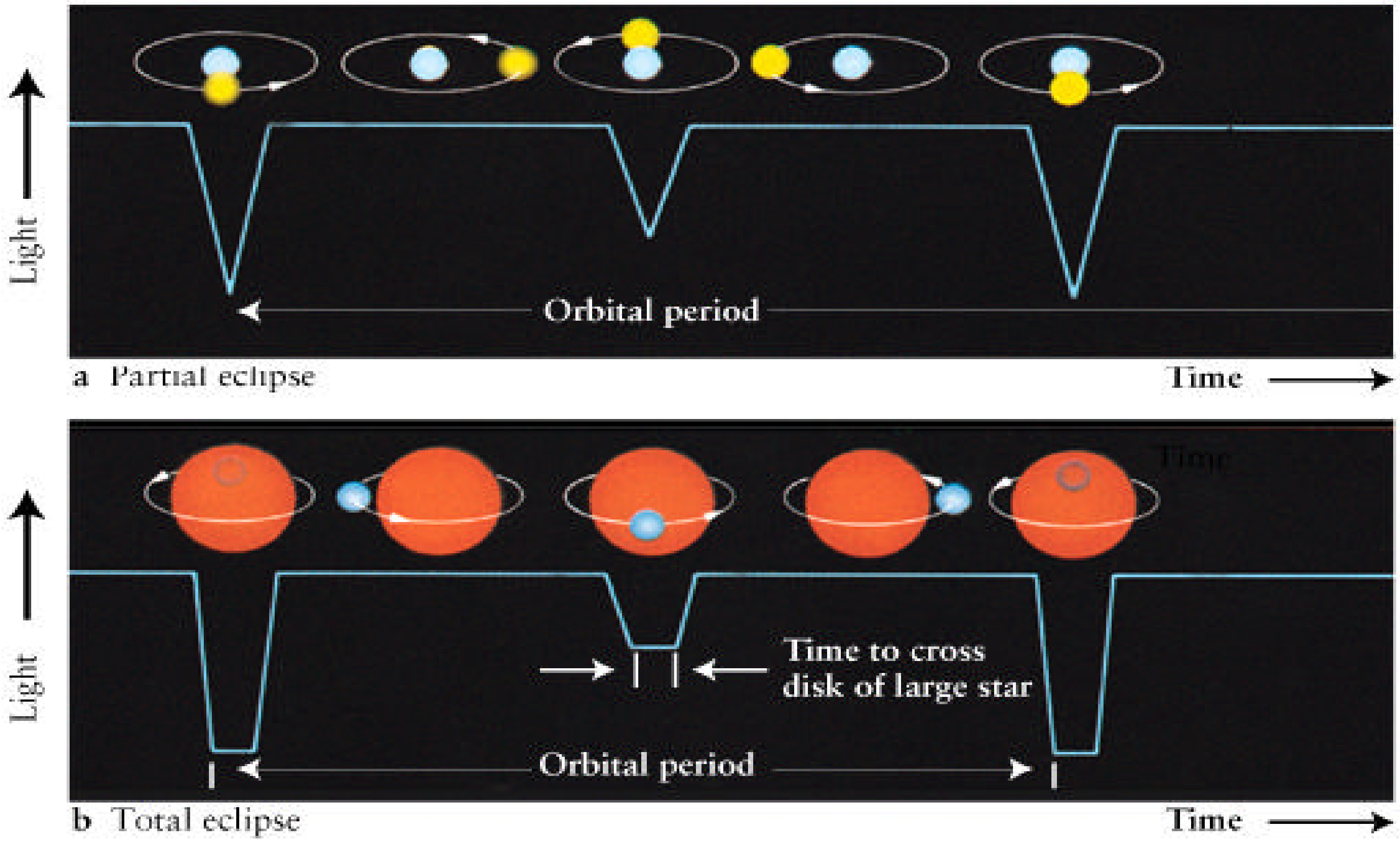
- visual binary
- spectroscopic binary
  .   SB1   SB2   lines from 1 or 2 stars
- eclipsing binary

Spectroscopic binary

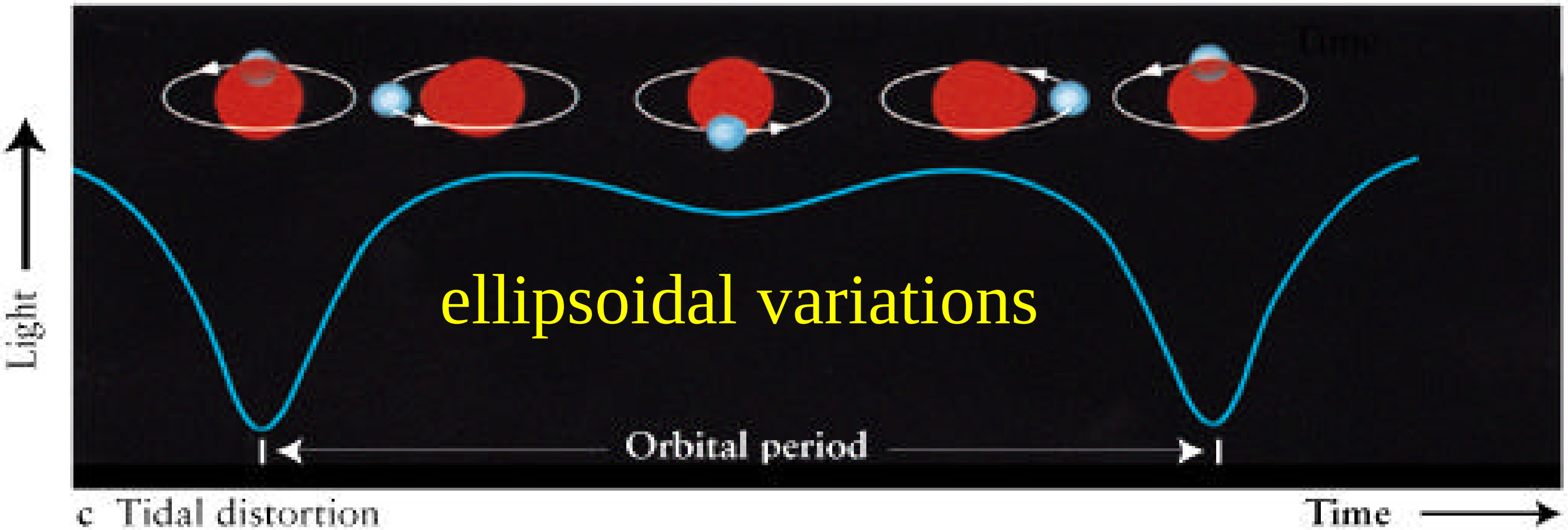


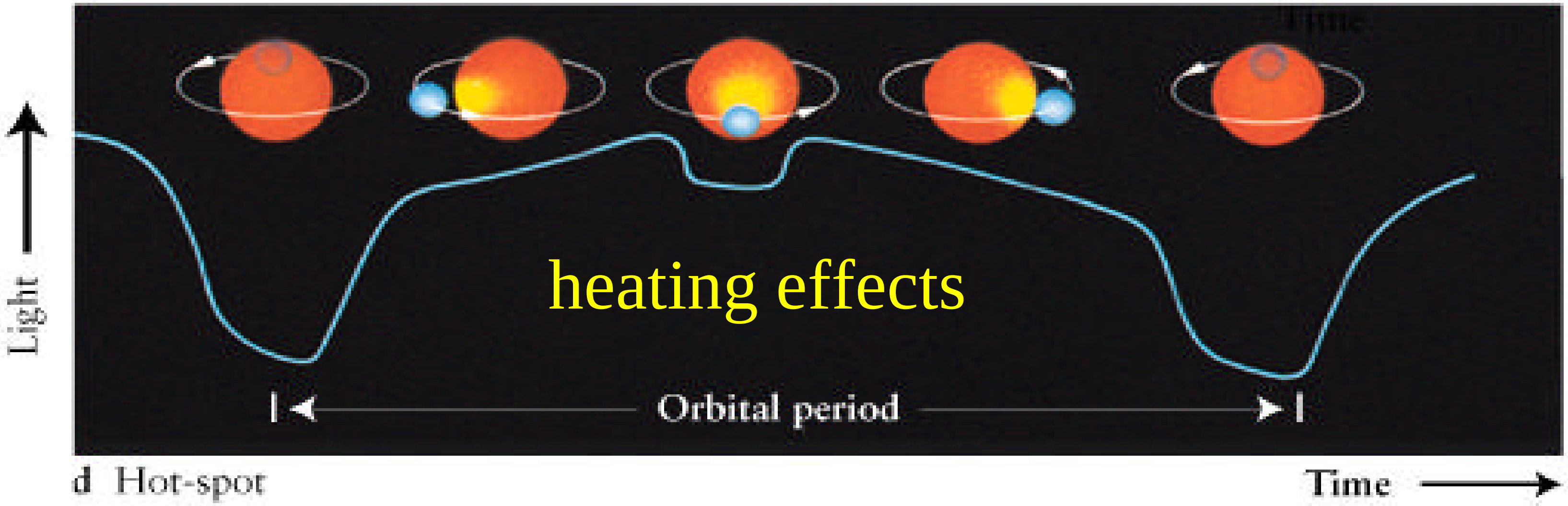
Eclipsing binary

Star sizes from timing



Proximity Effects



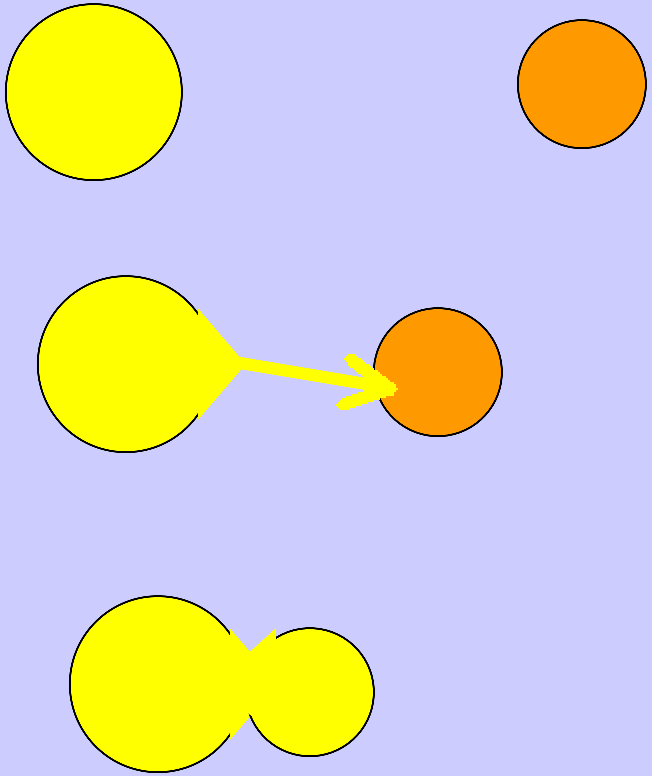


Types of Binaries

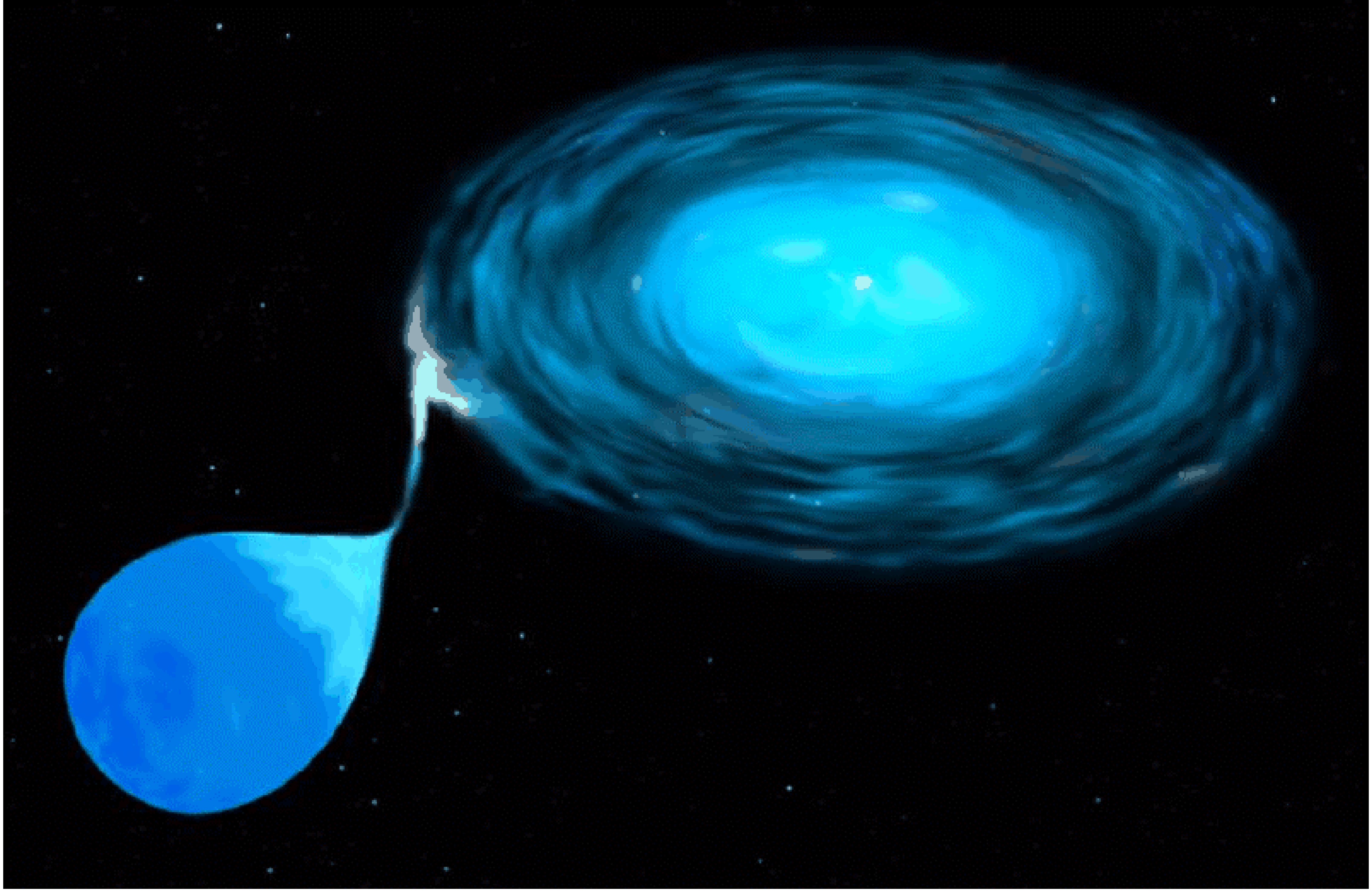
detached

semi-detached

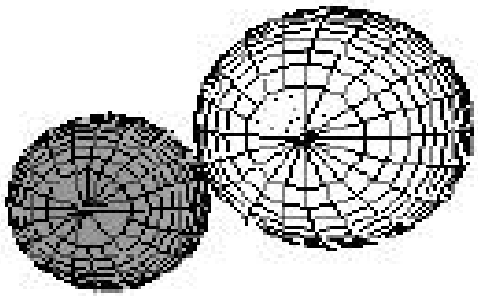
contact

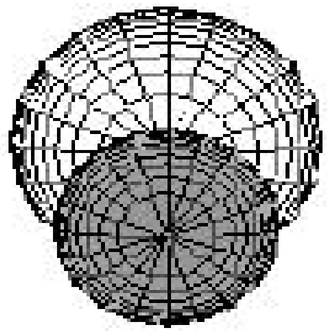


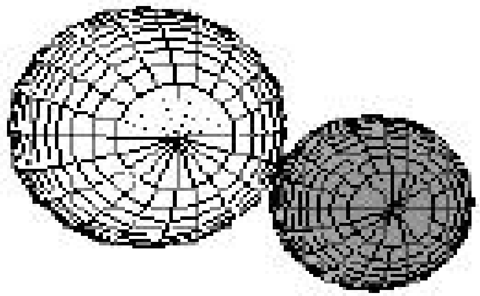
Binary Star with Accretion Disc

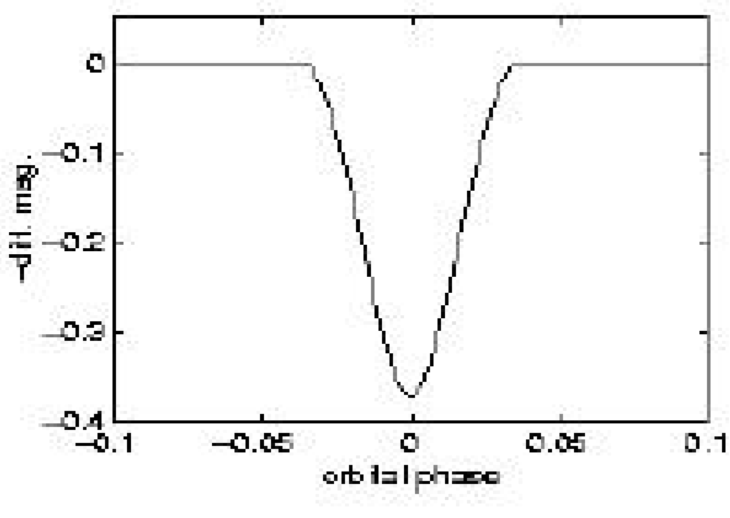


Computer Models of Eclipsing Binary Stars

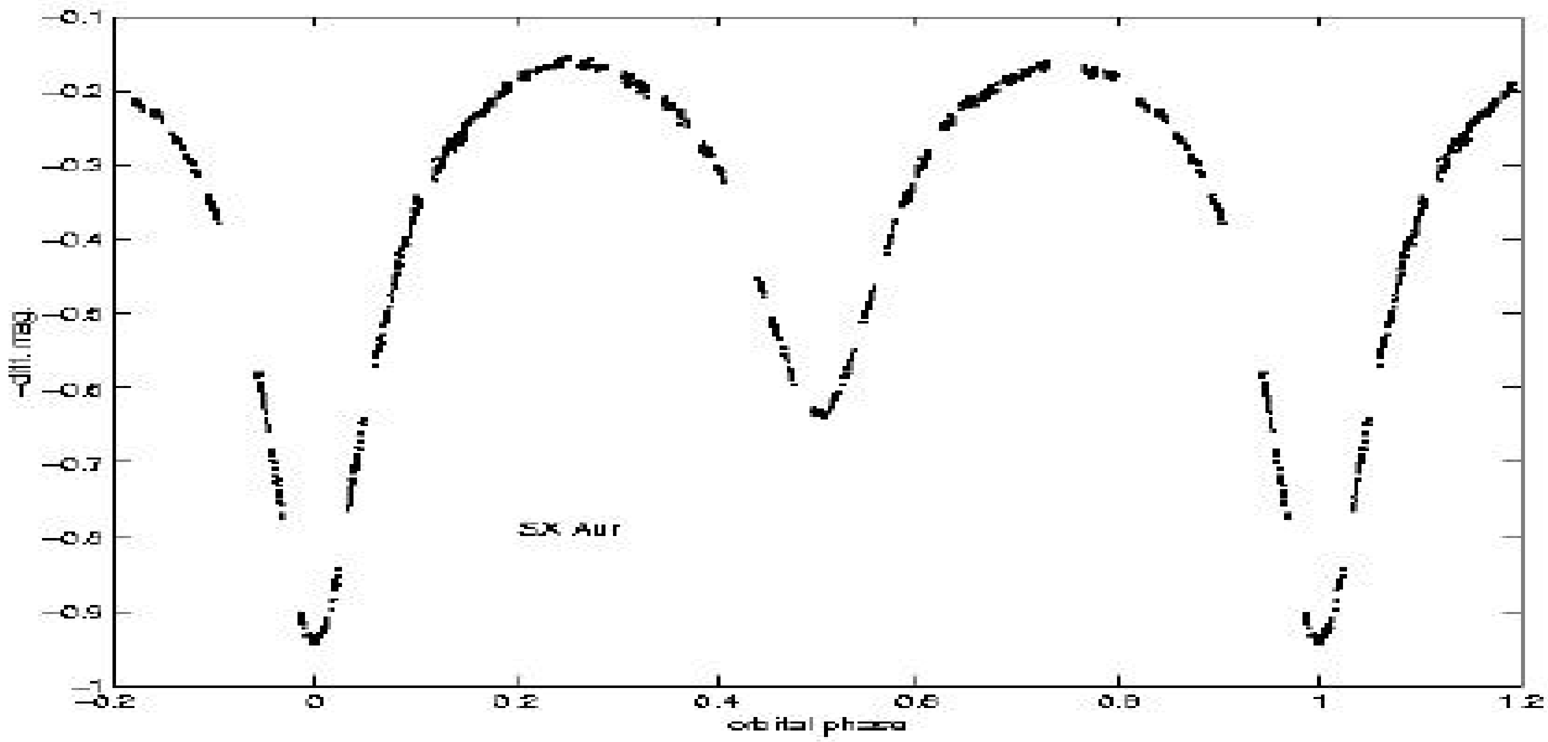




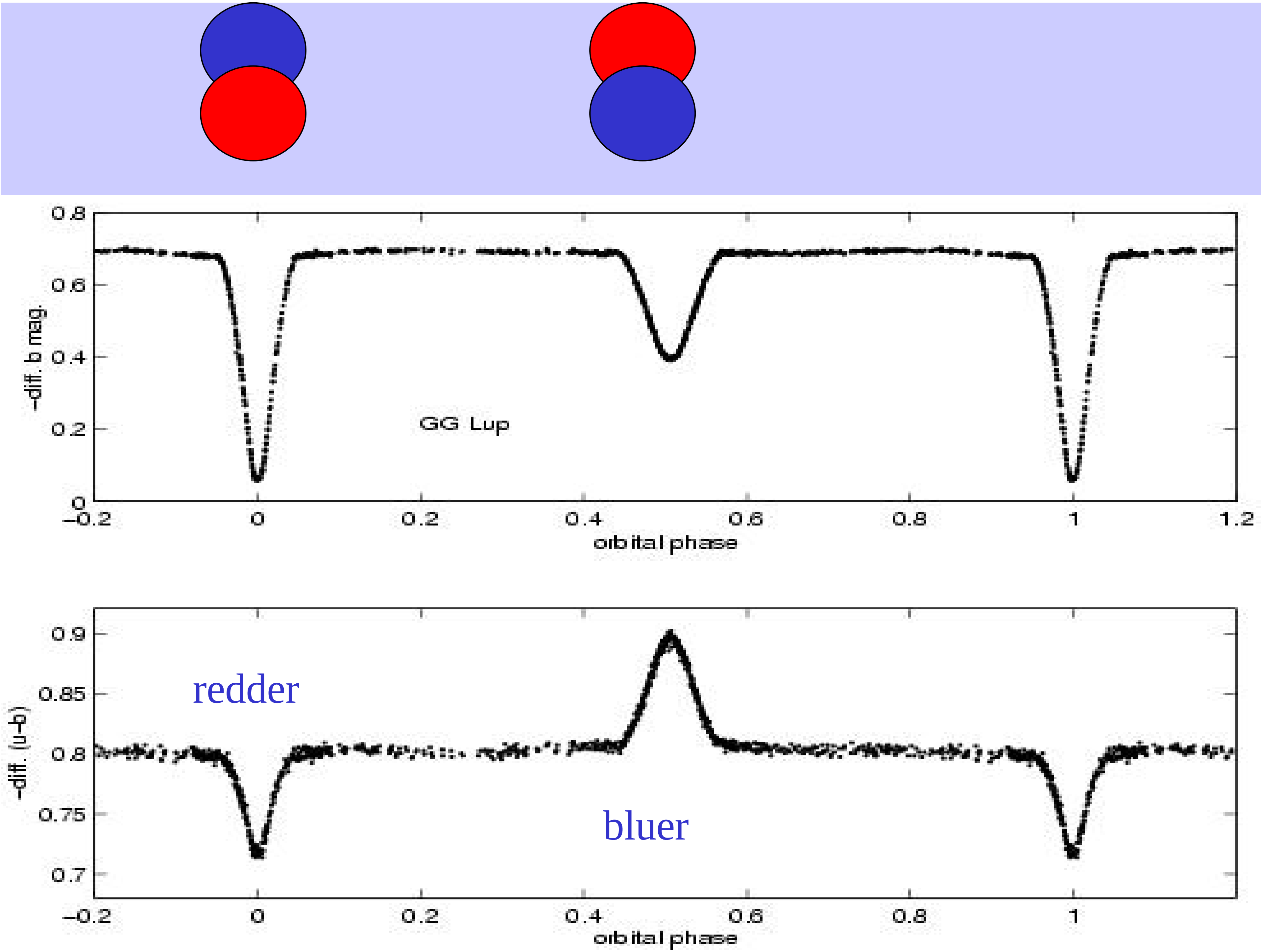




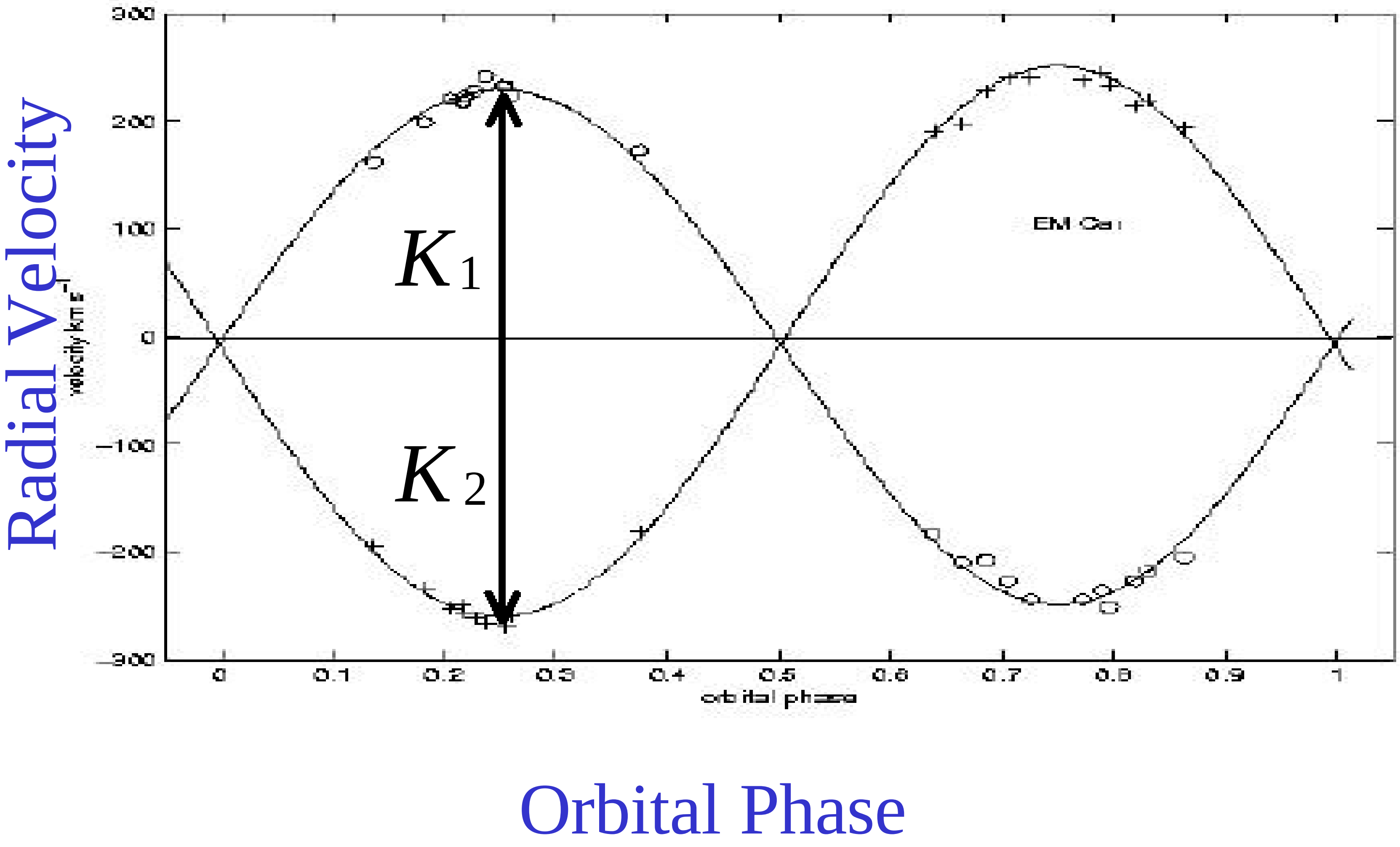
Light curve of Contact Binary



Orbital Phase


0.8
0.6
0.4
0.2
0
-diff. b mag.
GG Lup
-0.2
0
0.2
0.4
0.6
0.8
1
1.2
orbital phase
0.9
0.85
0.8
0.75
0.7
-diff. (u-b)
redder
bluer
-0.2
0
0.2
0.4
0.6
0.8
1
orbital phase

Velocity curve



Orbit inclination

$i = 0$ for face-on orbit

$i = 90$ degrees for edge-on orbit

Doppler shifts measure

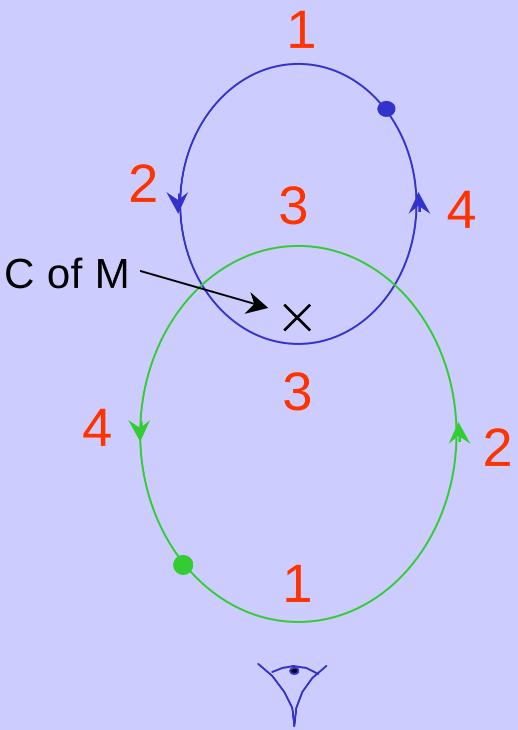
.

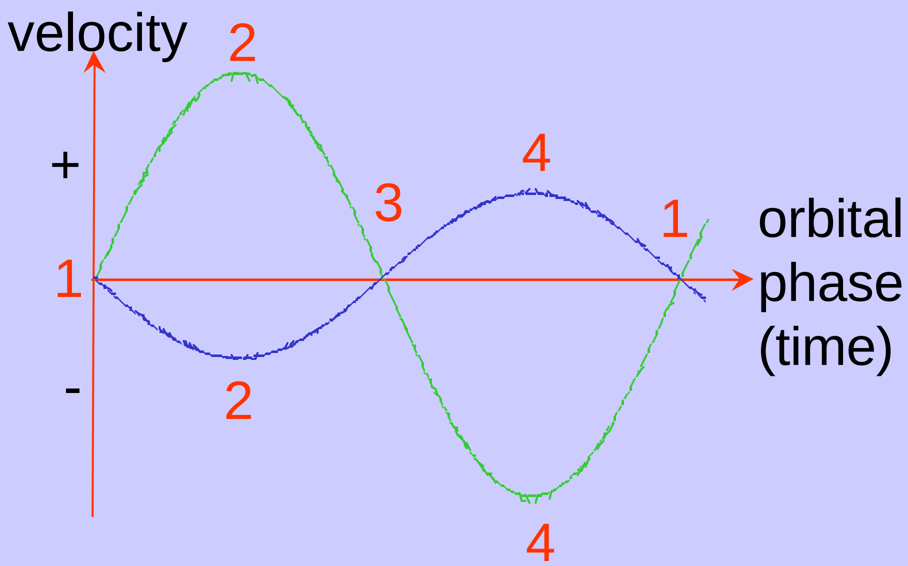
$$K = V \sin i$$

To measure masses:

– radial velocity and light variations

- visual
- spectroscopic
- eclipsing





radial velocity curves

Masses

Observe:

$$K_1 = V_1 \sin i$$

$$K_2 = V_2 \sin i$$

$$P$$

Calculate masses:

$$\frac{m_1}{m_2} = \frac{K_2}{K_1}$$

$$2p\ a \sin i = (K_1 + K_2)\, P$$

Kepler's Law:

$$\left(\frac{m_1 + m_2}{M_{\mathrm{sun}}}\right)\left(\frac{P}{\mathrm{yr}}\right)^2 = \left(\frac{a}{\mathrm{AU}}\right)^3$$

- Analysis of RV curves gives "minimum masses"

$$\left(M_1 \sin^3 i\right), \left(M_2 \sin^3 i\right)$$

and projected sizes of orbits

$$\left(a_1 \sin i\right), \left(a_2 \sin i\right)$$

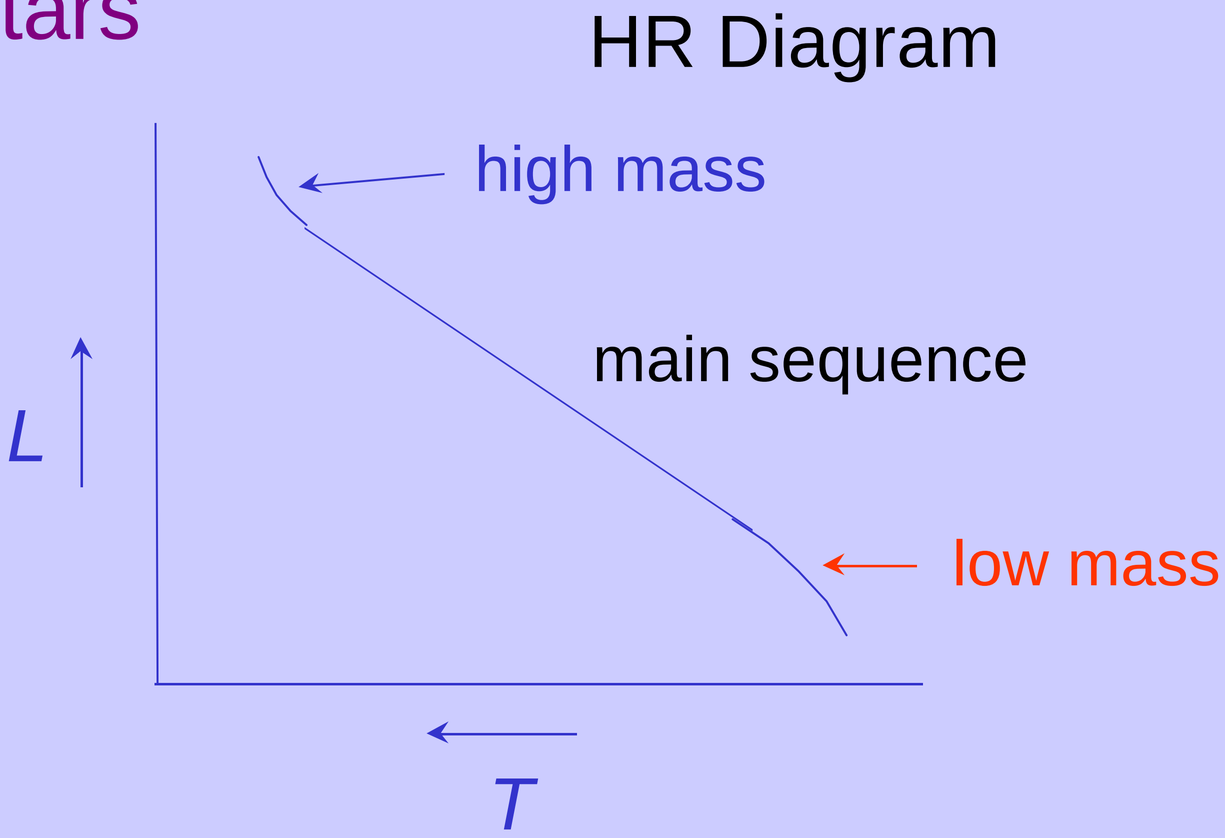
- Analysis of light curves of eclipsing binaries gives
  - orbital inclination *i*
  - radii of both stars, relative to the size of the orbit

$$\left(\frac{r_1}{a}\right),\left(\frac{r_2}{a}\right)$$

- Hence, for eclipsing, spectroscopic binaries, we obtain:

  - masses M_1 and M_2
  - radii R_1 and R_2
  - luminosities L_1 and L_2
    - (if T_1 or T_2 known)

– used as tests of theoretical models of stars



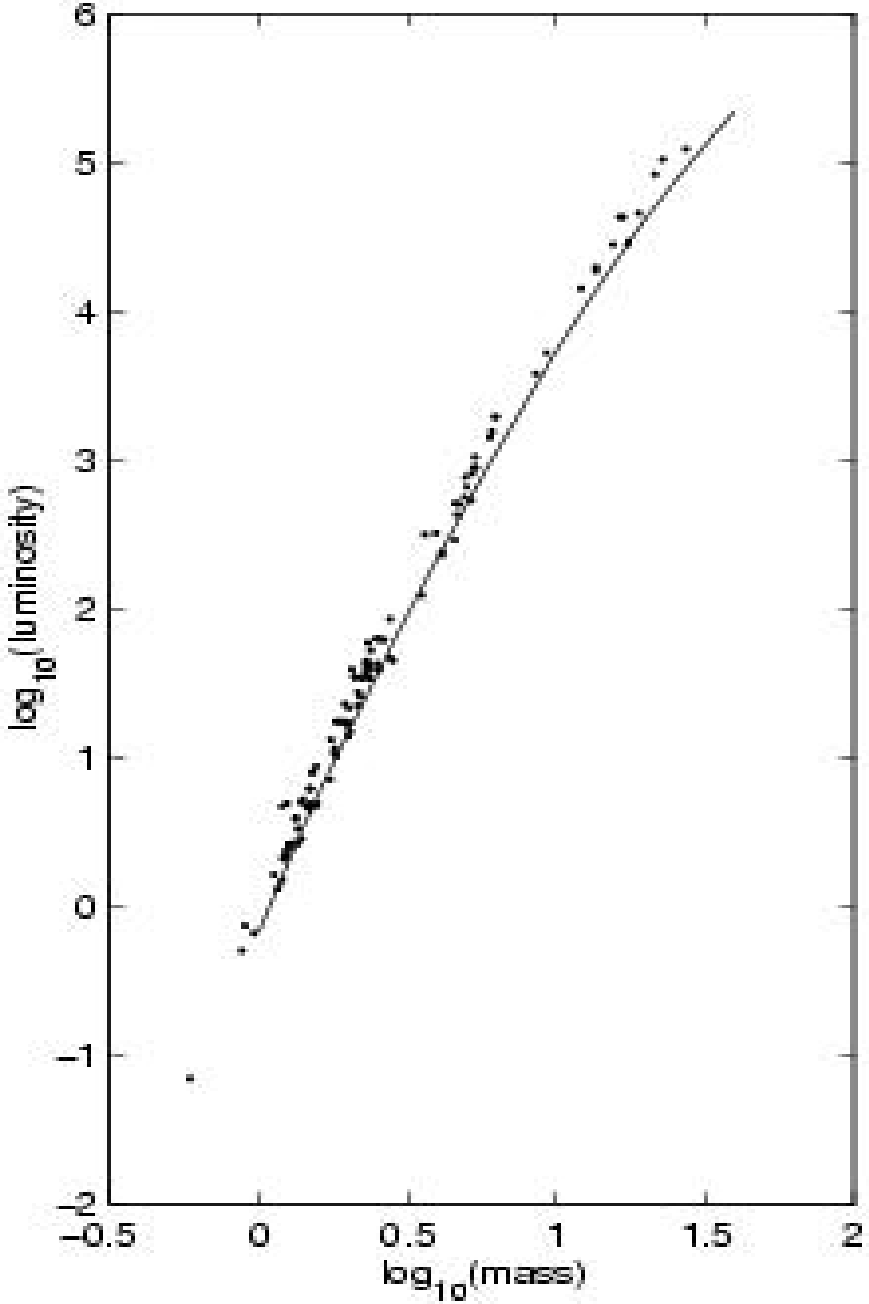
- Empirical MASS-LUMINOSITY relationship for main-sequence stars:

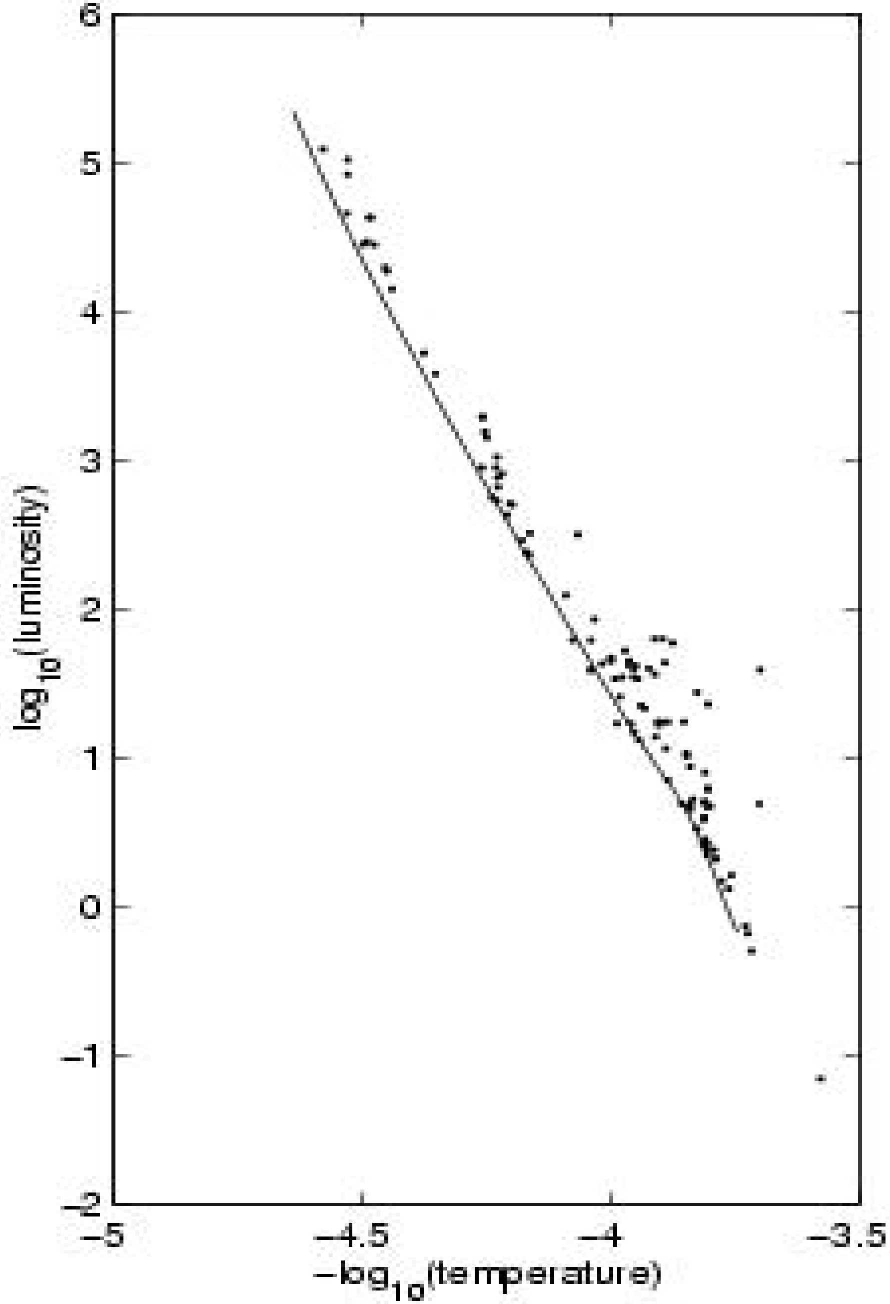
. $L \propto M^4$ i.e.

$$\frac{L}{L_{sun}} = \left[\frac{M}{M_{sun}}\right]^4 \quad \text{for } 0.4\,M_{sun} < M < 10\,M_{sun}$$

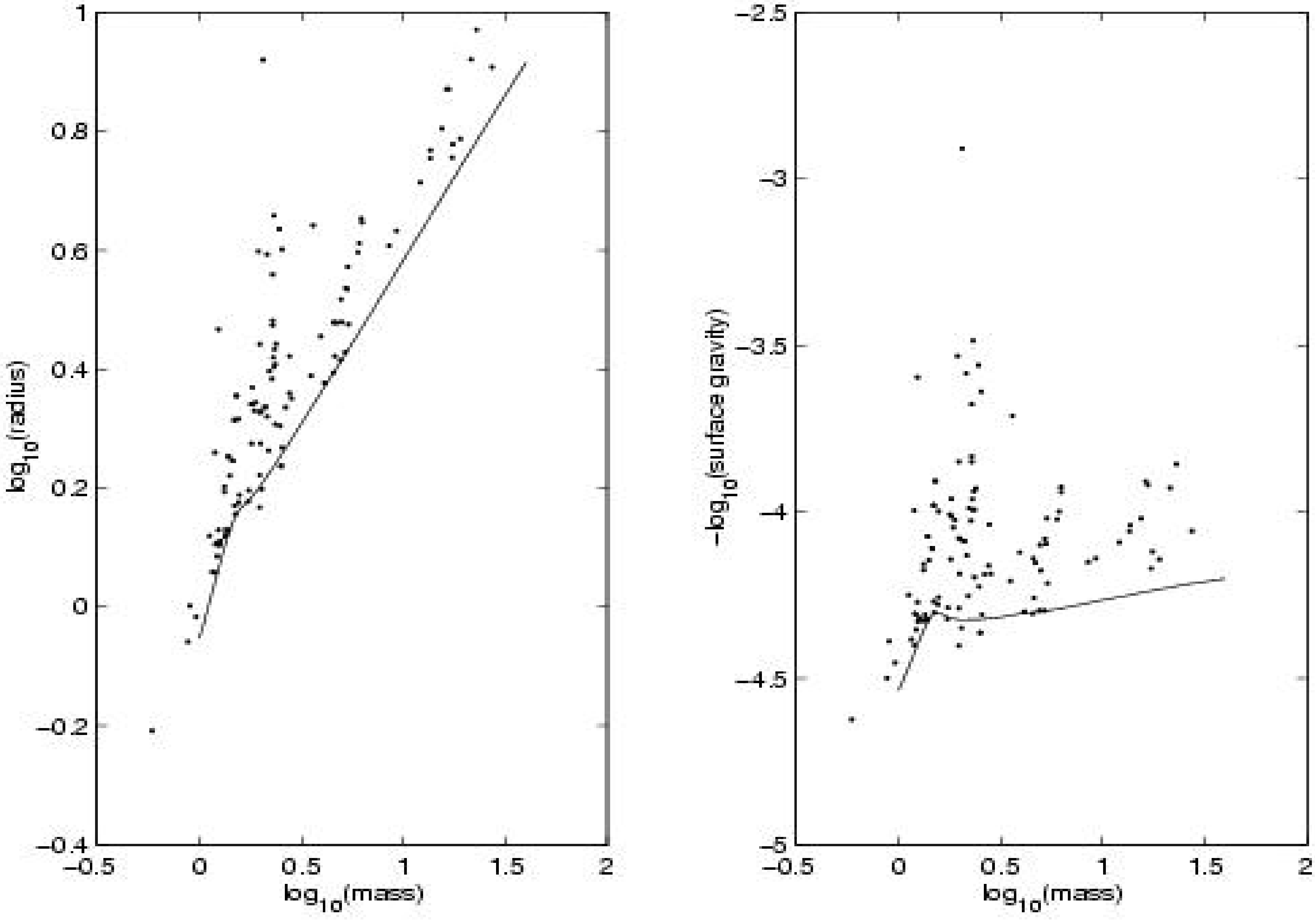
see $\log M$ *vs.* $\log L$ plots (handout)

Mass-luminosity and HR diagram (T-L) for detached eclipsing binaries

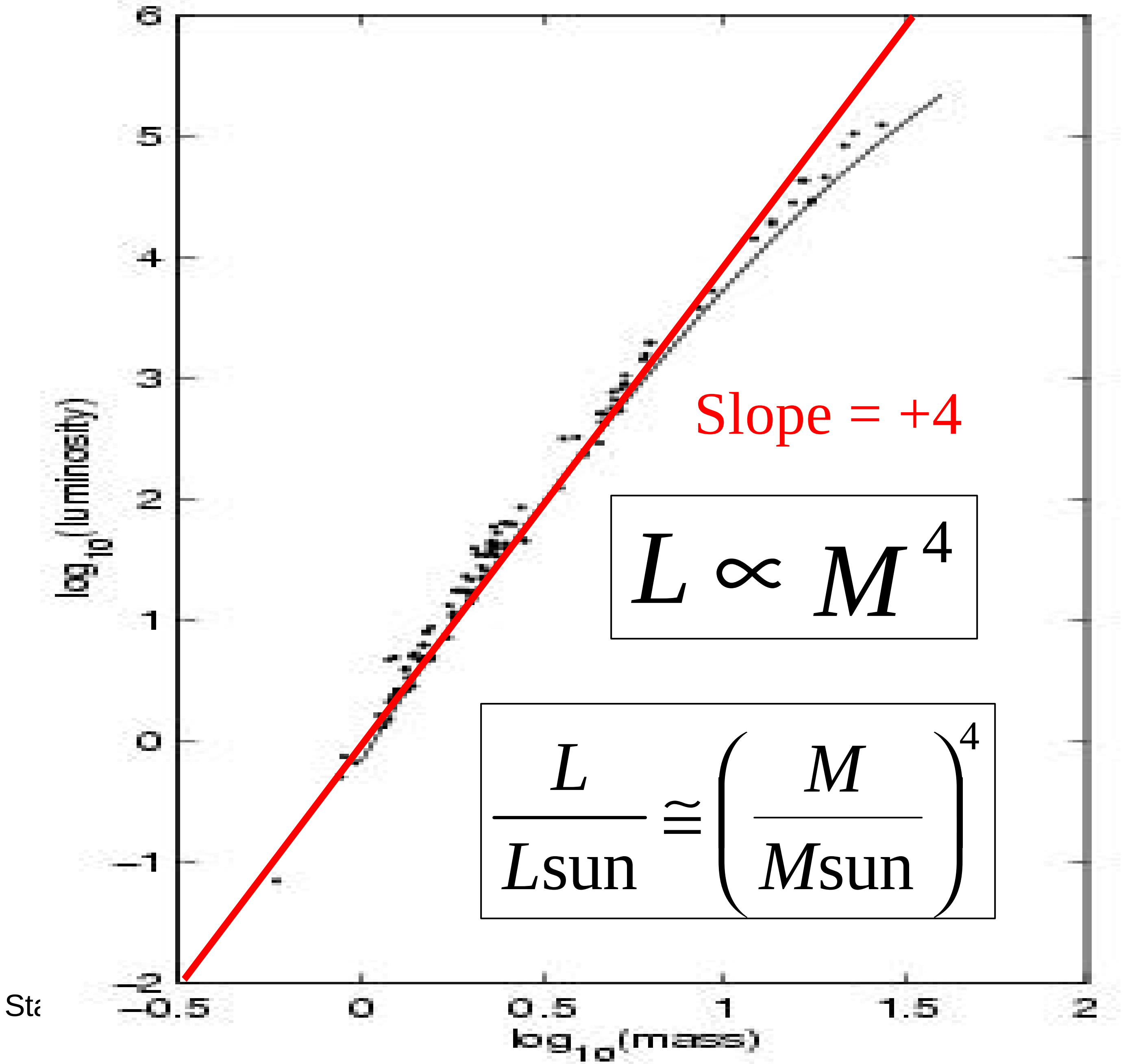




Mass-radius plot for detached eclipsing binaries



Mass-Luminosity for Main Sequence



Star Lifetimes

- Energy supply: $E = \Delta M \; c^2$ (Joules)
- Rate of burning: $L \propto M^4$ (W = Joule/s)
- Lifetime:

$$t \sim \frac{E}{L} = \frac{\Delta M \; c^2}{L} = \frac{\Delta M}{M} \, \frac{M \; c^2}{L}$$

$$\sim 10^{10} \text{ yr} \left(\frac{\Delta M / M}{0.0015} \right) \left(\frac{M}{M_{\text{sun}}} \right)^{-3}$$

- Stars burn for a long time.
- Big stars burn out faster.