Lecture 16

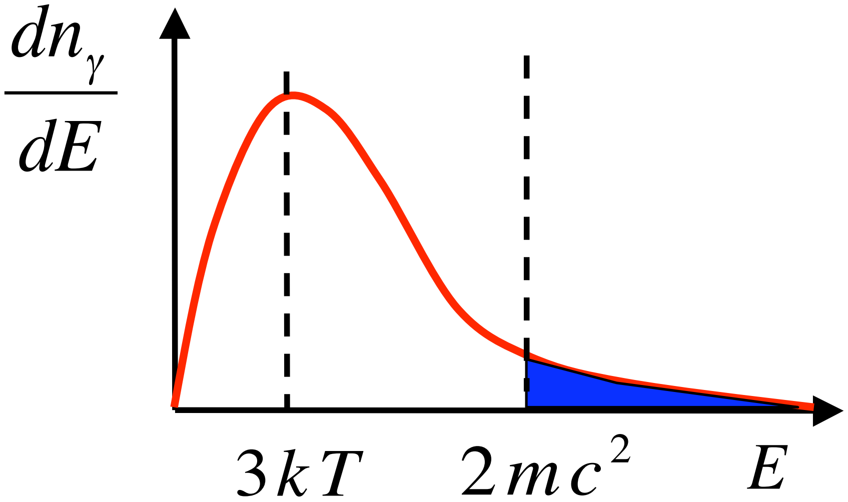
Relic Neutrinos

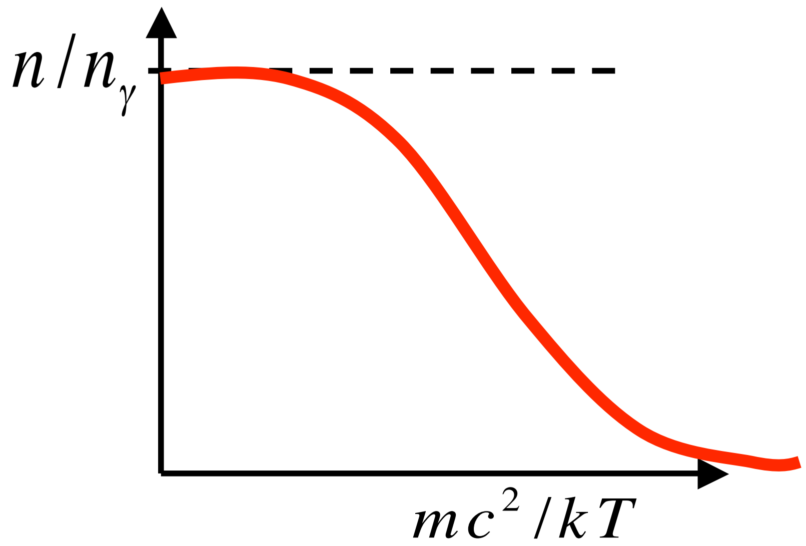
Below Threshold: k T << m c²

$$kT << mc^2$$

Photons in the tail of the blackbody photon spectrum have enough energy to create pairs.

$$\frac{n}{n_\gamma} \sim \frac{g}{2}\left(\frac{mc^2}{kT}\right)^{3/2} e^{-mc^2/kT}$$





Below Threshold: $kT \ll mc^2$

$$E = \left(p^2c^2 + m^2c^4\right)^{1/2} \quad kT \ll mc^2 \qquad \frac{E}{kT} \Rightarrow \frac{mc^2}{kT} + \frac{p^2}{2mkT} = \frac{mc^2}{kT} + y^2$$

Particle density:

$$n \Rightarrow \frac{g}{(2\pi\hbar)^3} \int \frac{4\pi p^2 dp}{\exp(E/kT)} = \frac{4\pi g}{(2\pi\hbar)^3} (2mkT)^{3/2} e^{\frac{-mc^2}{kT}} \underbrace{\int_0^\infty y^2 e^{-y^2} dy}_{\frac{\pi^{1/2}}{4}}$$

Energy density: $\varepsilon \Rightarrow mc^2\, n$

Pressure : $P \Rightarrow nkT \ll \varepsilon$

Entropy : $\frac{s}{k} = \frac{\varepsilon + P}{kT} \Rightarrow \left(\frac{mc^2}{kT} + 1\right) n$

Freeze-Out (Decoupling)

Particle-antiparticle pairs stay in equilibrium with photons by
Pair creations:

$$\gamma + \gamma \rightarrow A + \overline{A}$$

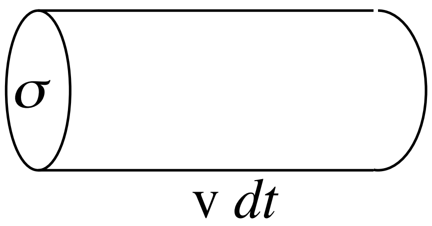
and

$$A + \overline{A} \rightarrow \gamma + \gamma$$

Annihilations:

Creation rate dies below **threshold**, when kT << mc^2.

Annihilation rate dies during **freeze-out**,
when **collision time** >> **expansion time**.



$$\frac{1}{n\langle \sigma \mathrm{v} \rangle} >> \frac{1}{H}$$

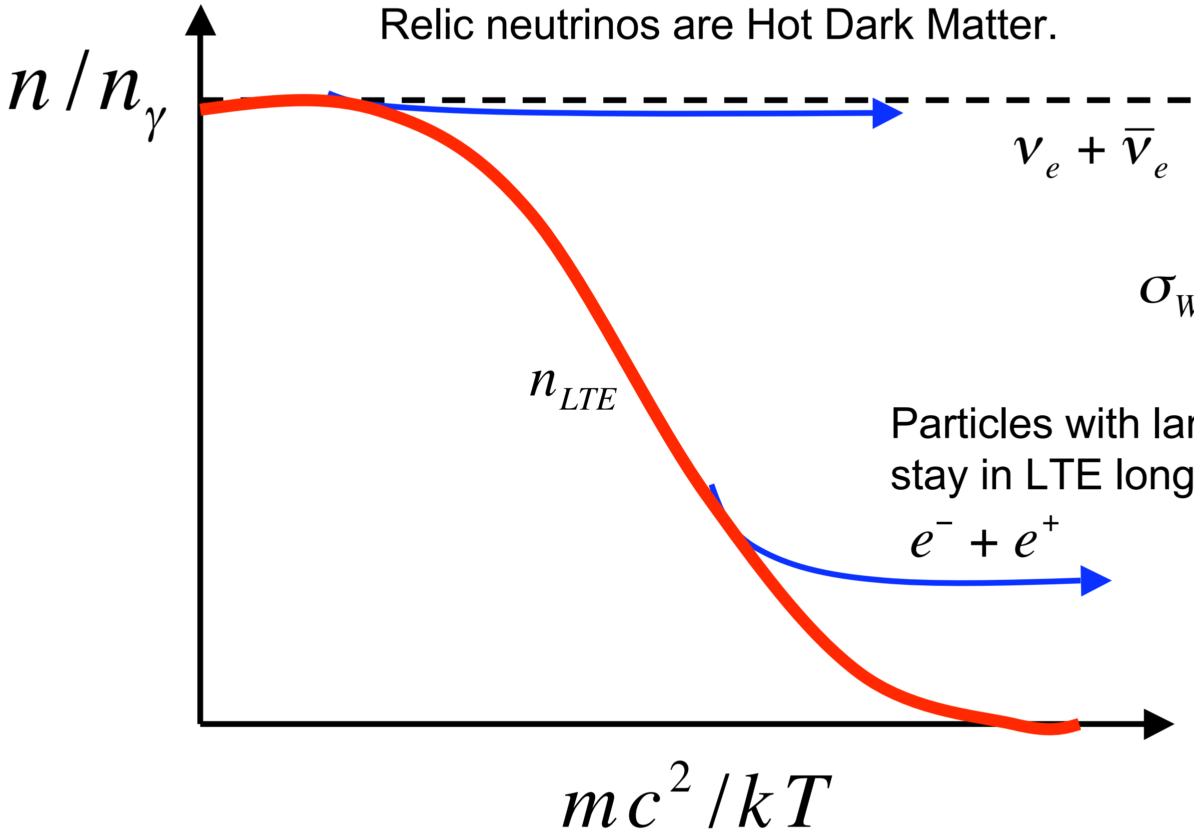
$$H = \frac{1}{2t} \sim \left(\frac{T}{10^{10} K} \right)^2 \sim \left(\frac{m c^2}{MeV} \right)^2$$

Relic number density: $n \sim \dfrac{H}{\langle \sigma \mathrm{v} \rangle}$

Freeze-Out (Decoupling)

Particle density evolution : $\dot{n} + 3Hn = -\langle \sigma \mathrm{v} \rangle \left(n^2 - {n_{LTE}}^2 \right)$

Neutrino mass and cross section are small.
Neutrinos decouple while relativistic.
Relic neutrinos are Hot Dark Matter.



Particles with larger cross section
stay in LTE longer before freezing out.

Relic density:

$$n \sim \frac{3H}{\langle \sigma \mathrm{v} \rangle}$$

$$\sigma_{Weak} \sim 10^{-47}\,\mathrm{m}^2 \left(kT/1\mathrm{MeV} \right)^2$$

$$\sigma_e \sim 6 \times 10^{-29}\,\mathrm{m}^2$$

"Survival of the weakest"

WIMPS as Cold Dark Matter

WIMP = Weakly Interacting Massive Particle:

$$\sigma_{Weak} \sim 10^{-43}\,\mathrm{cm}^2 \left(kT / 1\mathrm{MeV} \right)^2 \qquad \sigma_{\mathrm{Thompson}} \sim 6 \times 10^{-25}\,\mathrm{cm}^2$$

Freeze-out when kT ~ mc²:

$$\frac{H}{\mathrm{s}^{-1}} \sim \frac{1\mathrm{s}}{t} \sim \left(\frac{kT}{\mathrm{MeV}} \right)^2 \sim \left(\frac{mc^2}{\mathrm{MeV}} \right)^2$$

Number density at freeze-out :

$$n \sim \frac{H}{\langle \sigma \mathrm{v} \rangle} \sim \frac{1}{\langle \sigma \mathrm{v} \rangle t}$$

Number density today :

$$n_0 = \frac{n}{(1+z)^3} = n \left(\frac{T_0}{T} \right)^3$$

$$\Omega = \frac{m\, n_0}{\rho_c} = \frac{mc^2}{\rho_c c^2 \langle \sigma \mathrm{v} \rangle \times 1s} \left(\frac{kT}{\mathrm{MeV}} \right)^2 \left(\frac{kT_0}{kT} \right)^3$$

$$= \frac{\left(2 \times 10^{-4}\,\mathrm{eV} \right)^3}{\left(5200\,\mathrm{eV\,cm}^{-3} \right) \langle \sigma \mathrm{v}/c \rangle \left(3 \times 10^{10}\,\mathrm{cm} \right) \left(10^6\,\mathrm{eV} \right)^2}$$

$$= \left(\frac{5 \times 10^{-38}\,\mathrm{cm}^2}{\langle \sigma \mathrm{v}/c \rangle} \right)$$

Neutrino Decoupling

Annihilation of pairs releases energy.
Interactions share this out among remaining (lighter) particles.

Neutrino interactions are weak (no EM charge).
Decouple from LTE just above the corresponding lepton threshold:

$$T_\tau = 10^{13.3} K \quad T_\mu = 10^{12.1} K \quad T_e = 10^{9.7} K$$

Neutrinos take no share of the $e^+ e^-$ annihilation energy.

Entropy conserved: $s \propto g_{\rm eff} T^3$ $\quad \dfrac{T_\gamma(\text{before})}{T_\gamma(\text{after})} = \left(\dfrac{g(\text{after})}{g(\text{before})}\right)^{1/3} = \left(\dfrac{4}{11}\right)^{1/3}$

$$\frac{g(\gamma)}{g(\gamma + e^+ + e^-)} = \frac{2}{2 + \frac{7}{8}(2+2)} = \frac{4}{11}$$

Relic neutrino temp lower than photon temp:

$$\frac{T_\nu}{T_\gamma} = \left(\frac{4}{11}\right)^{1/3} = \frac{1.945\ K}{2.725\ K}$$

Relic Neutrinos

Today: Neutrino temp lower than photon temp:

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma = 1.95\ K$$

3 neutrino types (e μ τ) and anti-neutrinos.

Neutrinos are left-handed fermions: g(ν) = 1 x (7/8) .

Neutrino contribution to radiation energy density today:

$$\frac{\varepsilon(3(\nu+\bar{\nu}))}{\varepsilon(\gamma)} = 6 \times \frac{g(\nu)}{g(\gamma)} \left(\frac{T(\nu)}{T(\gamma)}\right)^4 = 6 \times \frac{1}{2} \times \frac{7}{8} \left(\frac{4}{11}\right)^{4/3} = 0.68$$

Although not detected, the 1.95K neutrino background makes a significant contribution to the radiation energy density today.

$$\Omega_\gamma = 5 \times 10^{-5} \qquad z_{M\,\gamma} = \Omega_M / \Omega_\gamma \approx 6000$$

$$\Omega_\nu = 3 \times 10^{-5}$$

$$\Omega_R = 8 \times 10^{-5} \qquad z_{M\,R} = \Omega_M / \Omega_R \approx 3500$$

Relic Neutrinos as Dark Matter

Number density of relic neutrinos :

$$n\left(\nu+\bar{\nu}\right)=\frac{3}{4}\left(\frac{T_\nu}{T_\gamma}\right)n_\gamma=\frac{3}{4}\times\frac{4}{11}\times\frac{411}{\mathrm{cm}^3}=\frac{113}{\mathrm{cm}^3}$$

If neutrino mass is $m_\upsilon > k\,T_0 \sim 10^{-3.6}$ eV
then non-relativistic (Cold) Dark Matter today.

Neutrino mass needed to account for Dark Matter :

$$\Omega_\nu\, h^2=\frac{\sum m_i}{93.5\ \mathrm{eV}}=\left(\frac{\sum m_i}{11.9\ \mathrm{eV}}\right)\left(\frac{\Omega_\nu}{0.26}\right)\left(\frac{h}{0.7}\right)^2$$

Experimental limits on neutrino masses:

$$\nu_e \le 2.2\ \mathrm{eV}$$

$$\nu_\mu \le 0.17\ \mathrm{MeV}$$

$$\nu_\tau \le 15\ \mathrm{MeV}$$

Neutrino Masses

Neutrino oscillations => neutrino mass:

each type (e μ τ) is a mix of 3 mass states (m_1 m_2 m_3)

travel time depends on mass

interference,oscillation between types

Solar neutrino problem solved:

2/3 of solar neutrinos change type enroute to Earth.

Neutrinos from cosmic ray showers

change type enroute thru the Earth.

Oscillation wavelength depends on energy difference:

$$E = \left(p^2c^2 + m^2c^4\right)^{1/2} = pc\left(1 + \frac{m^2c^2}{p^2}\right)^{1/2} \approx pc\left(1 + \frac{m^2c^2}{2p^2}\right) = pc + \frac{m^2c^3}{2p}$$

$$E_1 - E_2 \propto m_1^2 - m_2^2 \equiv \Delta(m_{12})^2$$

Experimental limits:

$$\Delta(m_{12})^2 = 8.0\times10^{-5}\mathrm{eV}^2$$

$$\Delta(m_{23})^2 = 2.5\times10^{-3}\mathrm{eV}^2$$