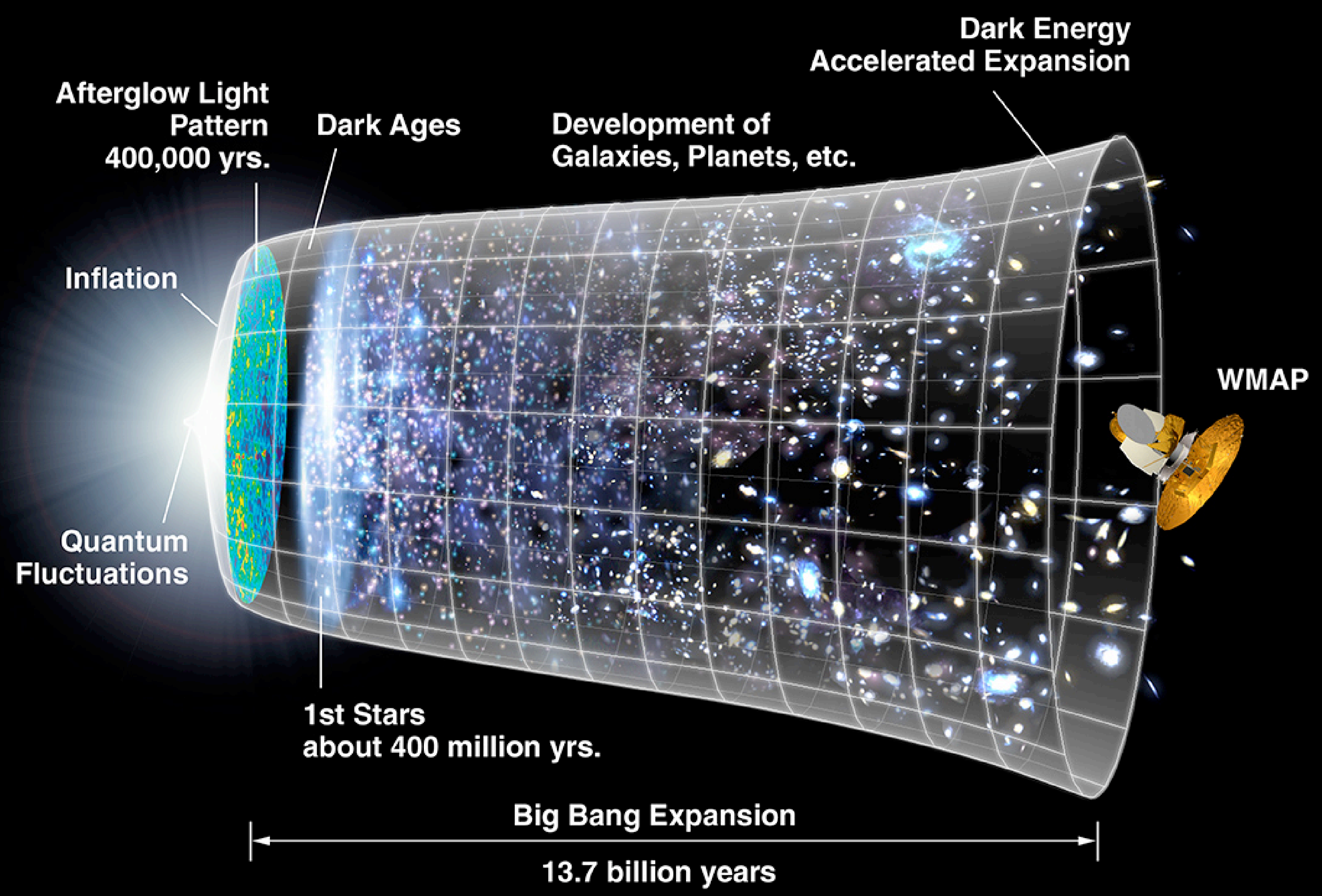
Lecture 14

Early Inflation

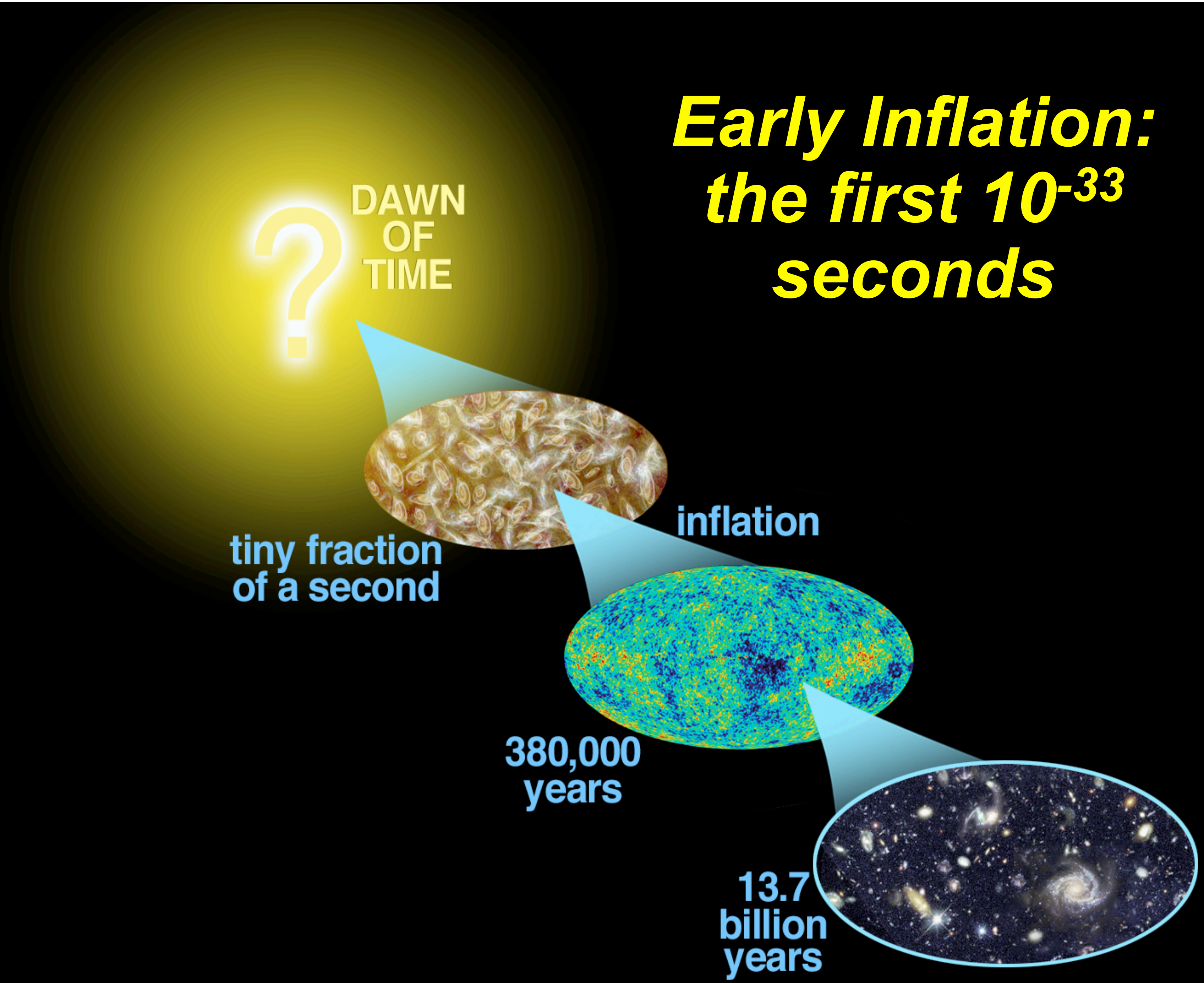
A possible solution for:

Why a hot Big Bang?
Horizon problem
Flatness problem
Monopole problem
Origin of primordial structure

Early inflation, Hot Big Bang, Decelerating Expansion, Late inflation by “Dark Energy”



Early Inflation: the first 10^{-33} seconds



1980: Inflation (Alan Guth)

- Universe born from "**nothing**" ?
- A **quantum fluctuation** produces a tiny bubble of "**False Vacuum".**
- High vacuum energy drives **exponential expansion**, also known as **"inflation."**
- Universe expands by huge factor in tiny fraction of second, as false vacuum returns to true vacuum.
- Expansion so fast that **virtual particle-antiparticle pairs** get separated to become **real particles and anti-particles**.
- Stretches out all structures, giving a **flat geometry** and uniform T and ρ, with **tiny ripples**.
- Inflation launches the **Hot Big Bang**!

Planck Units

Planck Length: de Broglie wavelength ~ Schwarzschild radius

$$E = M c^2 = \frac{h\,c}{\lambda} \quad \Rightarrow \quad \lambda = \frac{h}{M c} \qquad\qquad R_S = \frac{2 G M}{c^2}$$

$$\left(\lambda R_S\right)^{1/2} \sim L_P \equiv \left(\frac{\hbar G}{c^3}\right)^{1/2} \sim 10^{-35}\,\mathrm{m}$$

Planck Time $$t_P \equiv \frac{L_P}{c} = \left(\frac{\hbar G}{c^5}\right)^{1/2} \sim 10^{-43}\,\mathrm{s}$$

Planck Mass $$M_P = \frac{L_P c^2}{G} = \frac{\hbar}{L_P c} = \left(\frac{\hbar c}{G}\right)^{1/2} \sim 10^{25} m_p \sim 10^{19}\,\mathrm{GeV/c^2}$$

Planck Energy $$E_P \equiv M_P c^2 = \left(\frac{\hbar c^5}{G}\right)^{1/2} \sim 10^{19}\,\mathrm{GeV}$$

Limits of Quantum Mechanics and General Relativity.
Need Quantum Gravity theory (as yet unknown)
to describe physics at these scales.

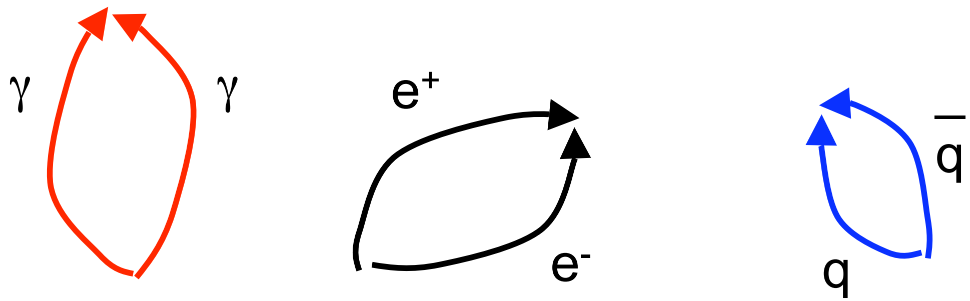
Quantum Vacuum

Heisenberg Uncertainty principle:

$$\Delta E \, \Delta t \sim \hbar$$

Can **violate energy conservation**:
"Borrow" energy from the vacuum, but only for a short time.

To create a particle-antiparticle pair, need $\Delta E > 2\, m\, c^2$.
These **"virtual" pairs** live only briefly, $\Delta t \sim h / \Delta E$.



Vacuum is not empty. Filled with all types of virtual pairs.

Quantum Vacuum

Like waves on the sea.

Quantum fields oscillate in many possible wave modes

$\phi(\mathbf{x}, t) \qquad \psi_i(\mathbf{x}, t) \qquad \mathbf{A}(\mathbf{x}, t), \quad \ldots.$

Each wave mode is a Harmonic Oscillator,

Potential energy:

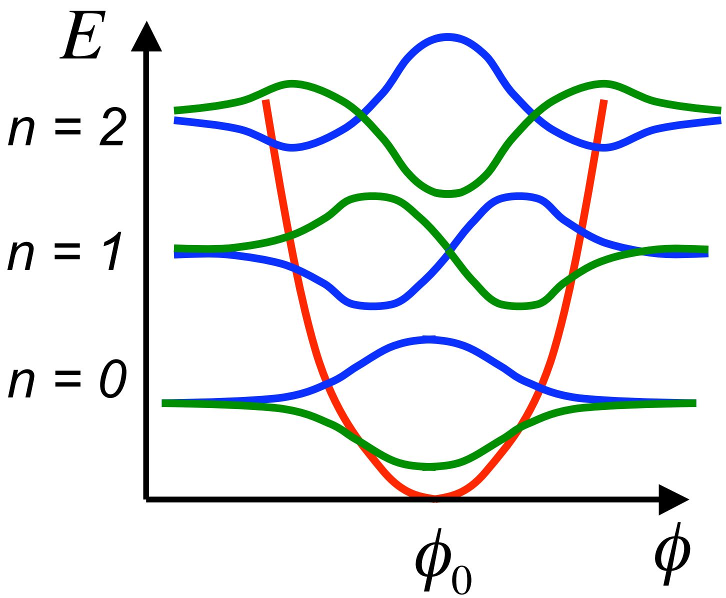
$$V(\phi, \psi_i, \mathbf{A}, \ldots) = m_\phi\,|\,\phi\,|^2 + m_\psi\,|\,\psi\,|^2 + \ldots$$

Ladder of discrete Energy States:

$$E(n) = \left(n + \frac{1}{2}\right)\hbar\omega$$

Zero-point energy:

$$E(0) = \frac{1}{2}\hbar\omega = \frac{hc}{2\lambda}$$

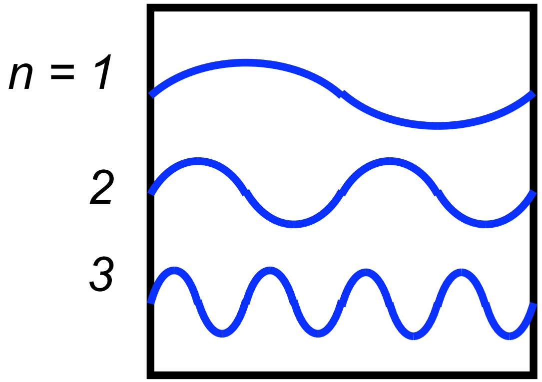


n = number of particles

Quantum vacuum = all wave modes in ground state.

Vacuum Energy Density

Waves in a Box:



Density of states in 6-D phase space:

$$L = n\,\lambda \qquad k \equiv \frac{2\pi}{\lambda} = \frac{2\pi\, n}{L} \qquad \Delta k = \frac{2\pi}{L}$$

$$\frac{dn}{d^3\mathbf{x}\, d^3\mathbf{k}} = \frac{g}{L_x\, L_y\, L_z\, \Delta k_x\, \Delta k_y\, \Delta k_z} = \frac{g}{(2\pi)^3}$$

Photons have *g* = 2 polarisations.
Electrons have *g* = 2 spin states.

$$\varepsilon_{vac} = \iiint \left\langle E(\mathbf{k}) \right\rangle \frac{dn(\mathbf{k})}{d^3\mathbf{x}\, d^3\mathbf{k}}\, d^3\mathbf{k}$$

$$= \int_0^\infty \left(\frac{\hbar\, c\, k}{2} \right) \left(\frac{g}{(2\pi)^3} \right) 4\,\pi\, k^2\, dk$$

$$= \frac{g\,\hbar\, c}{4\,\pi^2} \int k^3\, dk = \frac{g\,\hbar\, c}{16\,\pi^2} \left(k_{\max}{}^4 - k_{\min}{}^4 \right)$$

Short waves dominate.

Each mode is a simple harmonic oscillator.

Zero-point energy per mode:

$$\left\langle E(\mathbf{k}) \right\rangle = \frac{1}{2} \left(\frac{h\, c}{\lambda} \right) = \frac{\hbar\, c\, k}{2}$$

Cosmological Constant Problem

Range of wavelengths in the Cosmological box:

$$\lambda > L_p \sim \left(\frac{\hbar G}{c^3}\right)^{1/2} \sim 10^{-35}\,\mathrm{m} \qquad k_{\max} \sim \frac{2\pi}{L_P}$$

$$\lambda < \frac{c}{H_0} \sim 4300\ \mathrm{Mpc} \sim 10^{26}\mathrm{m} \qquad k_{\min} \sim \frac{2\pi}{c/H_0}$$

$$\varepsilon_{vac} = \frac{g\hbar c}{2\pi^2}\left(k_{\max}{}^4 - k_{\min}{}^4\right) \approx \frac{g\hbar c}{16\pi^2}\left(\frac{2\pi}{L_P}\right)^4 = \frac{\pi^2 g\hbar c}{L_P{}^4} \sim \frac{E_P}{L_P{}^3}$$

$$\Omega_{vac} \equiv \frac{\rho_{vac}}{\rho_{crit}} \sim \left(\frac{\pi^2 g\hbar}{c\,L_p{}^4}\right)\left(\frac{8\pi G}{3H_0{}^2}\right) \sim \frac{8\pi^3 g}{3}\left(\frac{c/H_0}{L_p}\right)^2 \sim 10^{120}$$

Cosmological Constant problem:

Observe $\Omega_\Lambda \sim 0.7$ Predict $\Omega_\Lambda \sim 10^{120}$
Why is Ω_Λ so small, yet not exactly zero ?
Why does quantum vacuum energy not gravitate?

Flatness Problem

Why was the initial geometry flat ?

If exactly flat, then always so.

To be **roughly** flat today, $\Omega_0 = 1 + \varepsilon$

must initially be **incredibly** close to flat:

$$\Omega(x) = \frac{8\pi G\rho}{3H^2} = \frac{\Omega_R x^4 + \Omega_M x^3 + \Omega_\Lambda}{\Omega_R x^4 + \Omega_M x^3 + \Omega_\Lambda + (1-\Omega_0)\, x^2}$$

$$\Omega(x \to \infty) - 1 \Rightarrow \frac{\varepsilon}{\Omega_R\ x^2} \sim 10^{-119} \left(\frac{\varepsilon}{0.01}\right)\left(\frac{10^{-5}}{\Omega_R}\right)\left(\frac{10^{61}}{x}\right)^2$$

$$\Omega_R \sim 10^{-5} \qquad x \sim \frac{c/H_0}{L_P} \sim 10^{61}$$

Flatness problem: Why was Ω initially so close to 1 ?
How did the Universe know precisely how fast to expand?

Horizon Problem

Why is the universe almost perfectly isotropic ?

Distant regions were never in causal contact.

Yet we see:

Same CMB temperature to 1 part in $10^{5.}$

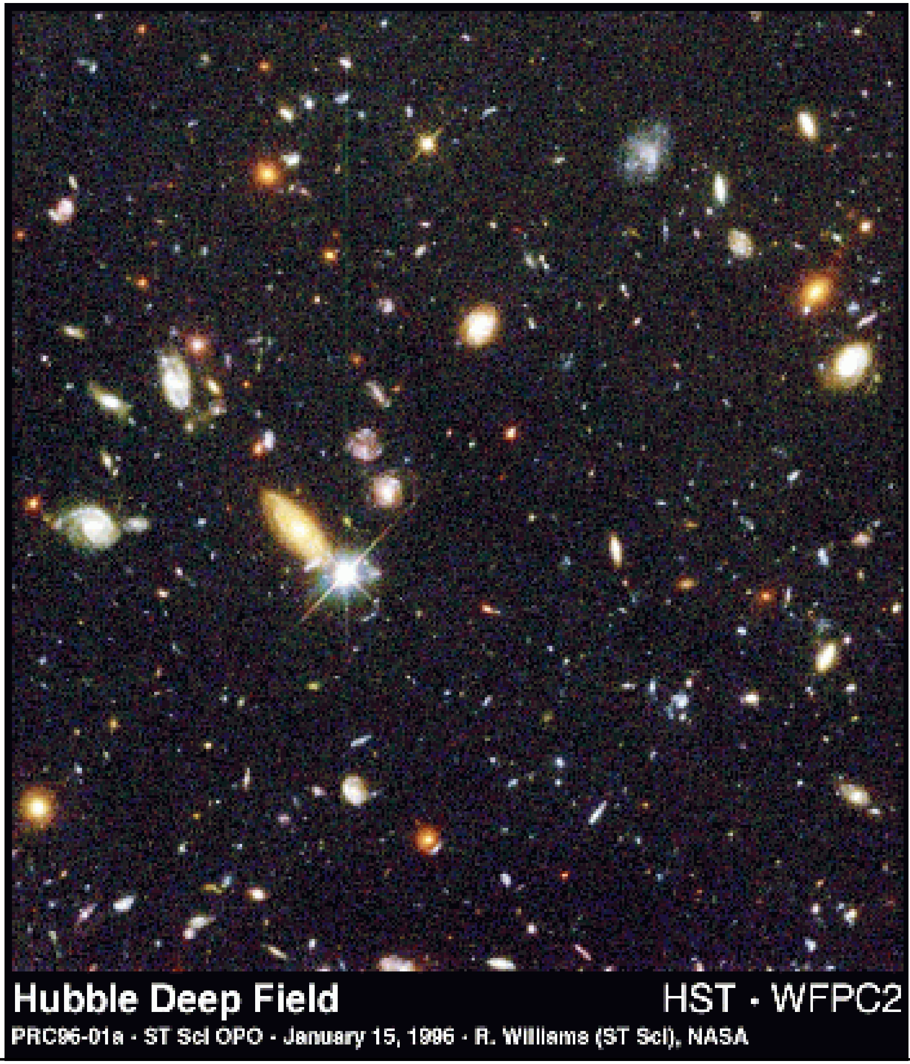
Similar galaxy distributions.

The Hubble Deep Fields

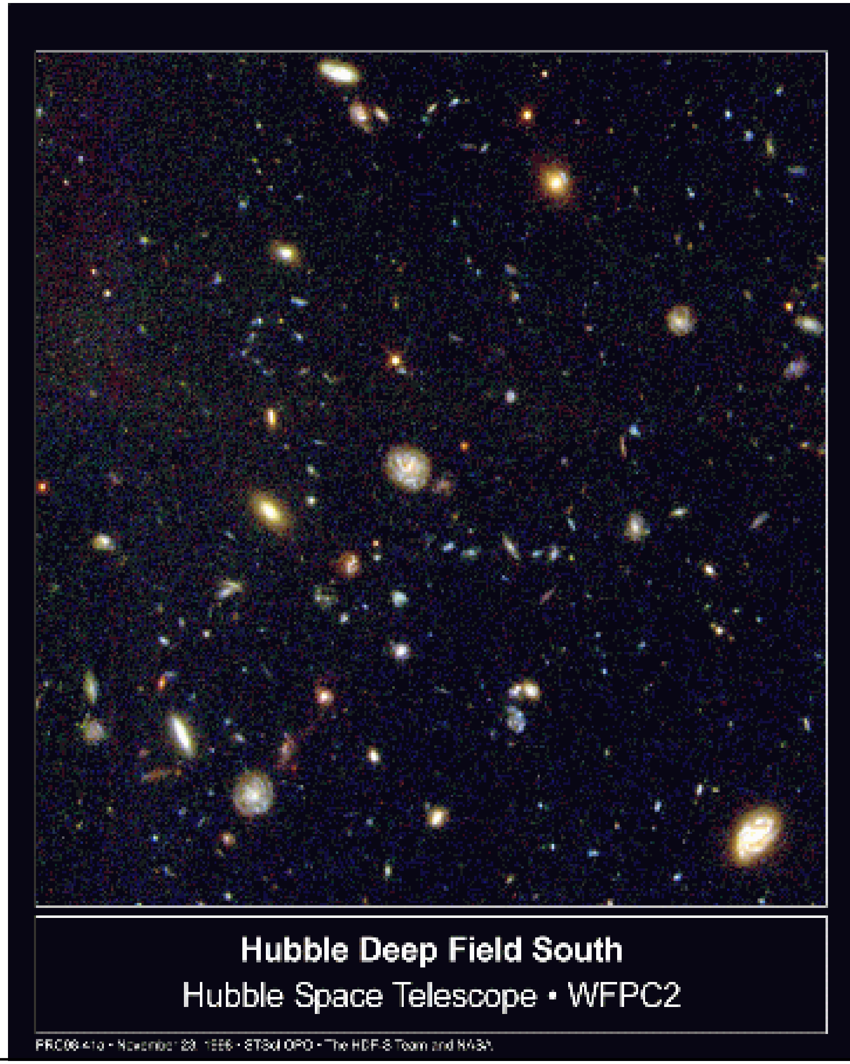
Thousands of galaxies, just a few stars.

Similar galaxy distributions, supporting the Cosmological Principle.

HDF North



HDF South

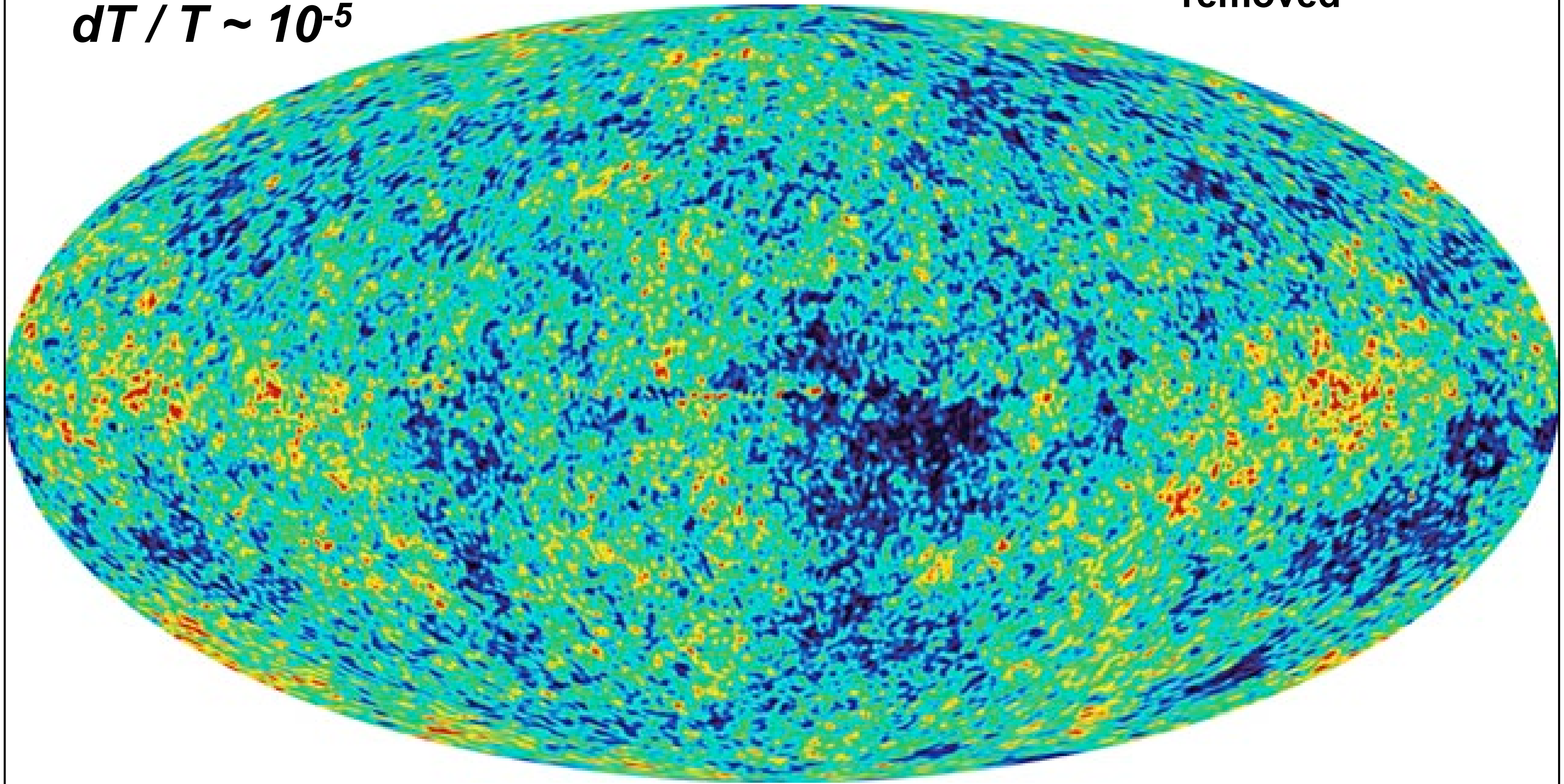


2003 WMAP all-sky

T = 2.73 K

$dT / T \sim 10^{-5}$

Dipole and foreground galaxy removed



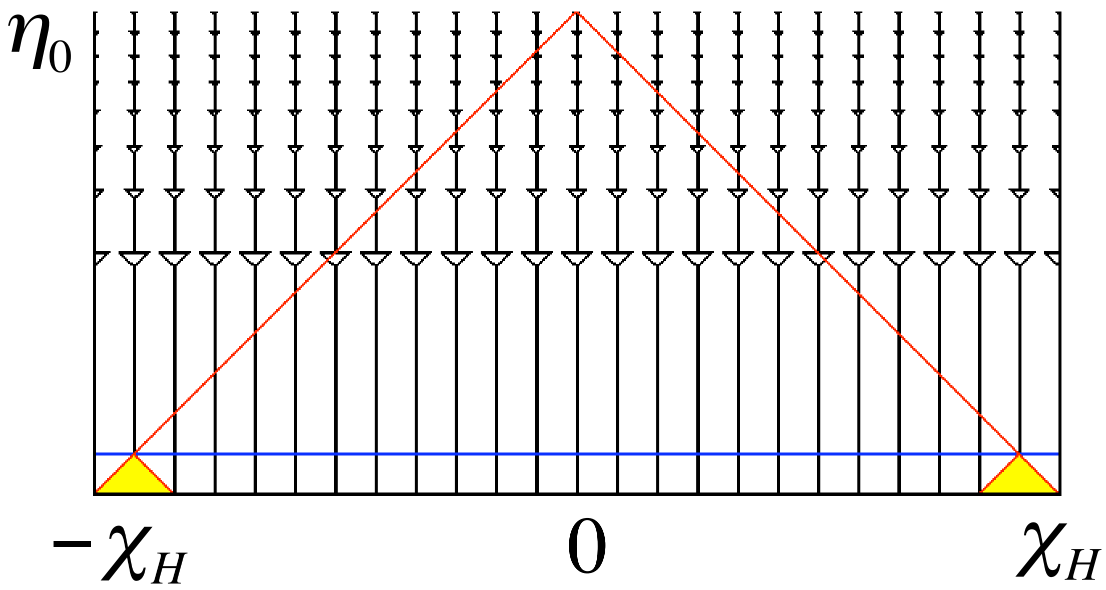
Snapshot at z=1100 of quantum fluctuations stretched by inflation.

Dark matter potential wells that seed later galaxy formation.

The Horizon: How far can we see?

suppose $R(t) = R_0 \left(t/t_0 \right)^{\alpha}$

$$\chi_H = \eta_0 = \int_{t_1}^{t_0} \frac{c\,dt}{R(t)} = \frac{c\,t_0^{\ \alpha}}{R_0} \int_{t_1}^{t_0} \frac{dt}{t^{\alpha}} = \left(\frac{c\,t_0}{R_0} \right) \times \begin{cases} \dfrac{1-\left(t_1/t_0\right)^{1-\alpha}}{1-\alpha} & \text{if} \quad \alpha < 1 \\ \ln\left(t_0/t_1\right) & \text{if} \quad \alpha = 1 \\ \dfrac{\left(t_0/t_1\right)^{\alpha-1} - 1}{\alpha - 1} & \text{if} \quad \alpha > 1 \end{cases}$$



As t_1 => 0,

Finite for α < 1,
Infinite for α =1, >1.

matter - dominated :

$\alpha = 2/3$

radiation - dominated :

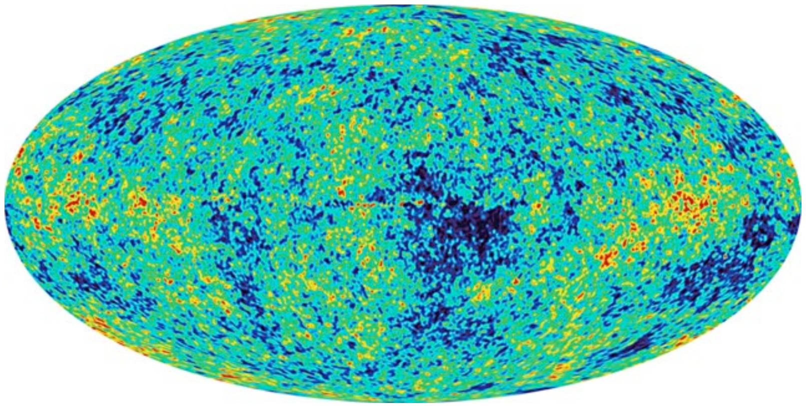
$\alpha = 1/2$

Particle Horizon at z = 1100

$$L_H(t_R) = \int_0^{t_R} \frac{R(t_R)\,c\,dt}{R(t)} = \frac{c}{(1+z)} \int_{1+z}^{\infty} \frac{dx}{H(x)} \approx \frac{c}{(1+z)\,H_0} \int_{1+z}^{\infty} \frac{dx}{\sqrt{x^4\,\Omega_R + x^3\,\Omega_M}}$$

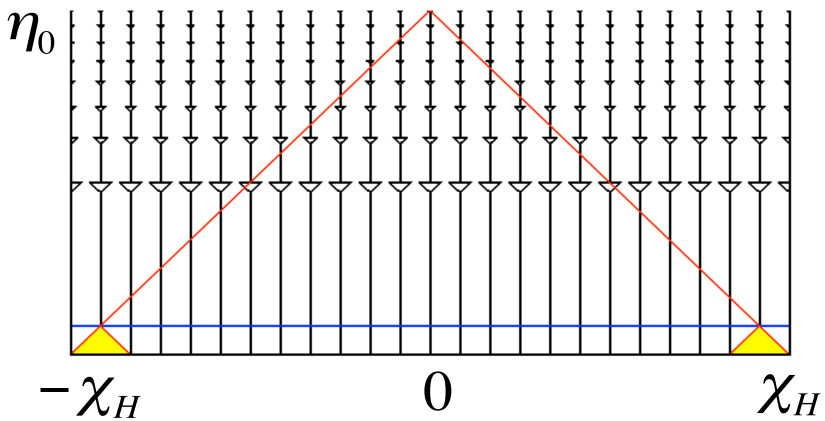
$$= \frac{2c}{(1+z)\,H_0\,\sqrt{\Omega_M\,x_0}} \left(\sqrt{1+\frac{x_0}{1+z}} - 1 \right) \qquad x_0 \equiv \frac{\Omega_M}{\Omega_R} \approx 3500 \left(\frac{\Omega_M}{0.3} \right)$$

$$= \frac{c}{H_0} \frac{2\left(\sqrt{4.6}-1\right)}{1100\sqrt{0.3\times 3500}} = 3.4\times 10^{-5}\,\frac{c}{H_0} \approx 190 \left(\frac{0.7}{h}\right) \left(\frac{0.3}{\Omega_M}\right)^{1/2} \text{kpc}$$



Expands by factor
1 + z = 1100
to ~200 Mpc today.

How did these 20,000 causally disconnected regions know what temperature to be, to 1 part in 10^5 ?



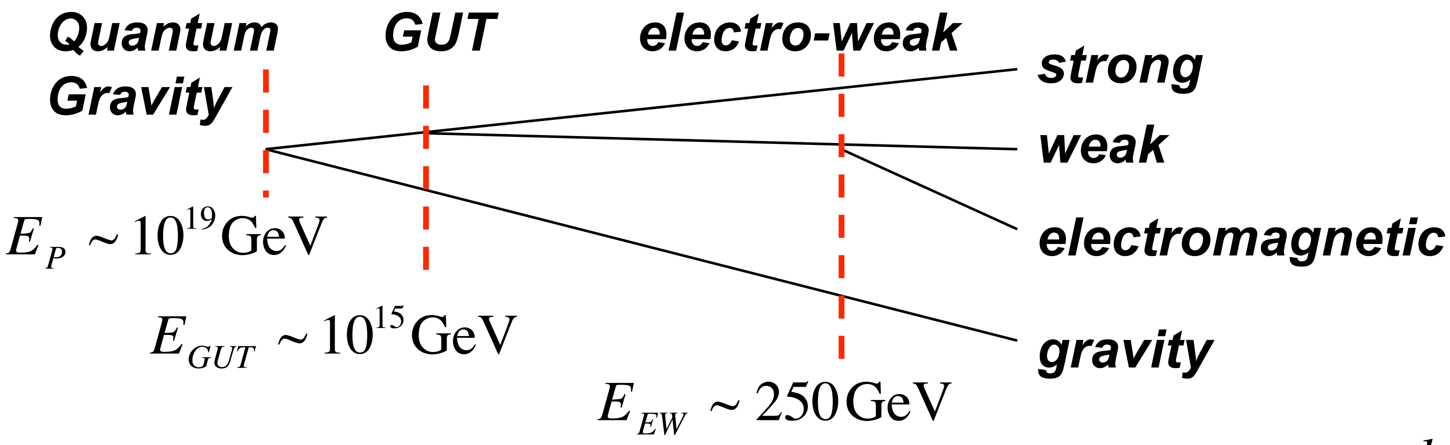
Magnetic Monopole Problem

The GUT (Grand Unified Theory) phase transition should produce “topological defects” that look like magnetic monopoles.

We don’t see any.

Why is the universe devoid of magnetic monopoles ?

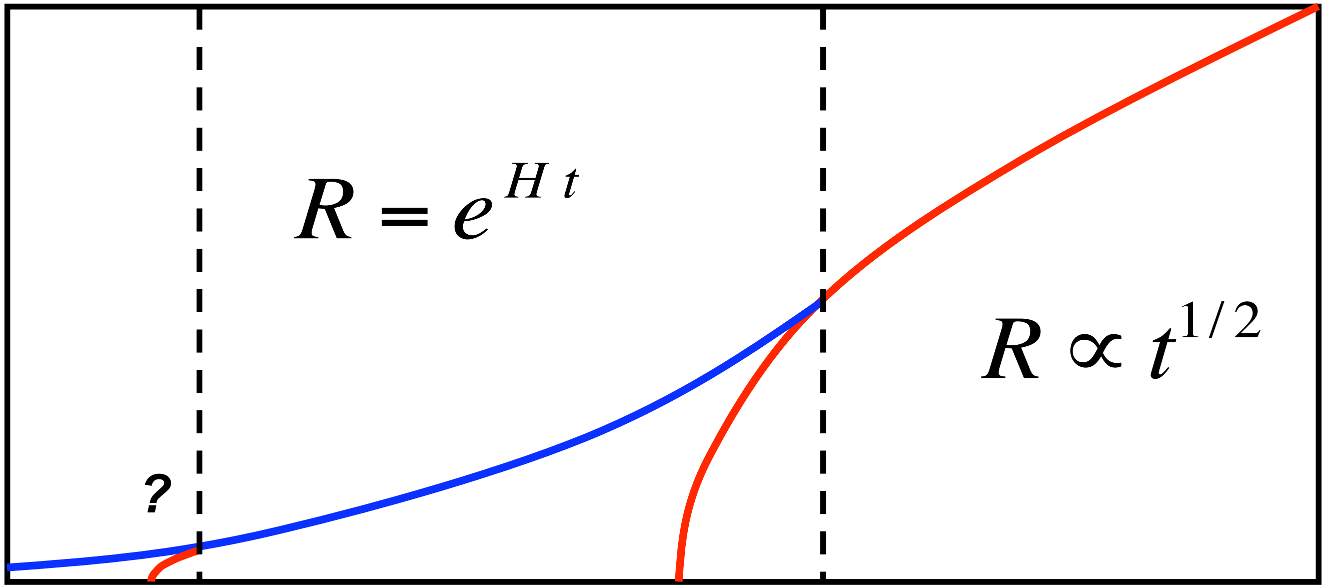
Phase Transitions among Forces



Today:

$kT \sim 10^{-3.6}\,\mathrm{eV}$

Inflation launches the Big Bang



expand by factor $f = e^{H\,t}$ in time $t = \dfrac{\ln f}{H}$ $\quad H \sim \left(\dfrac{8\pi\, G\, \rho_{vac}}{3} \right)^{1/2}$

What came before t = 0 ?
Early inflation replaces the Big Bang singularity.
Launches the Hot Big Bang.

Horizon problem: Expands a causally-connected region.
Flatness problem: Flattens the geometry.
Monopole problem: Moves primordial monopoles beyond horizon.
Seeding structures: Stretches out small quantum fluctuations.

During inflation:

Particle horizon expands

$$R \propto e^{H\,t} \qquad L_H(t_2) = \int_{t_1}^{t_2} \frac{R(t_2)\,c\;dt}{R(t)} = \frac{c}{H}\,e^{H(t_2 - t_1)}$$

Dramatic cooling

$$x \equiv 1 + z \propto \frac{1}{R} \propto e^{-H\,t} \qquad T = T_0(1+z) \propto \frac{1}{R} \propto e^{-H\,t}$$

Geometry flattens

$$\Omega(x) = \frac{8\pi G\rho}{3H^2} = \frac{\Omega_R x^4 + \Omega_M x^3 + \Omega_\Lambda}{\Omega_R x^4 + \Omega_M x^3 + \Omega_\Lambda + (1-\Omega_0)\,x^2}$$

$$\Rightarrow \frac{\Omega_\Lambda}{\Omega_\Lambda + (1-\Omega_0)\,x^2} \approx 1 - \frac{(1-\Omega_0)}{\Omega_\Lambda}\,x^2$$

How much inflation needed?

Present horizon: *$R_H \sim c / H_0 \sim$ 4300 Mpc $\sim 10^{26}$ m*

$$\text{Planck time}: \quad t_P \sim 10^{-43}\,\text{s} \qquad \text{horizon} \sim c\, t_P \sim L_P \sim 10^{-35}\,\text{m}$$

$$\text{post-inflation redshift}: \quad z_P \sim \frac{E_P}{k\,T_{CMB}} \sim \frac{10^{19}\,\text{GeV}}{10^{-3.6}\,\text{eV}} \sim 10^{32}$$

$$L_P \Rightarrow L_P\, f\, z_P \sim f \times 10^{-3}\,\text{m}$$

Need inflation factor *$f \sim 10^{29} \sim e^{67}$*

$$\text{GUT phase transition}: \quad E_{GUT} \sim 10^{15}\,\text{GeV} \qquad E \sim kT \sim R^{-1} \sim t^{-1/2}$$

$$t_{GUT} \sim \left(\frac{E_P}{E_{GUT}}\right)^2 t_P \sim 10^8\, t_P \sim 10^{-35}\,\text{s} \qquad \text{horizon} \sim c\, t_{GUT} \sim 10^8\, L_P \sim 10^{-27}\,\text{m}$$

$$\text{redshift} \sim z_{GUT} \sim \frac{E_{GUT}}{k\,T_{CMB}} \sim \frac{10^{15}\,\text{GeV}}{10^{-3.6}\,\text{eV}} \sim 10^{28}$$

$$L_{GUT} \Rightarrow L_{GUT}\, f\, z_{GUT} \sim f \times 10^{9}\,\text{m}$$

Inflation factor *$f \sim 10^{17} \sim e^{40}$*

Time $t \sim \ln(f) / H$

$\sim 40\, t_{GUT} \sim 10^{-33}$ s

Requirements for Inflation:

Friedmann momentum equation:

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3c^2}(\varepsilon + 3p)$$

$$= -H_0^{\,2}\sum_w \Omega_w \, x^{3(1+w)} \, \frac{1+3w}{2}$$

Acceleration requires a sufficiently negative pressure:

p < - ε / 3

w < -1 / 3

Inflation requires the ***dominant*** component to have *w < -1 / 3.*

"Dark Energy" is driving inflation now:

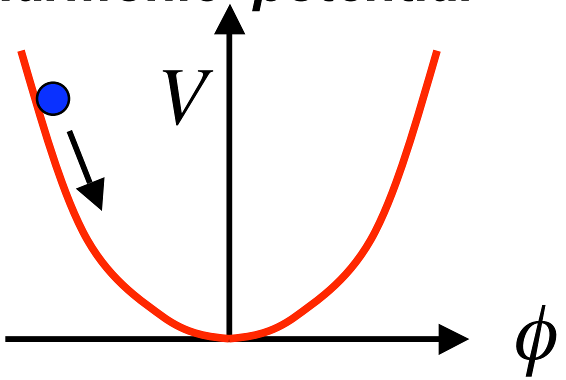
$$w_\Lambda = -1 \qquad p_\Lambda = -\varepsilon_\Lambda$$

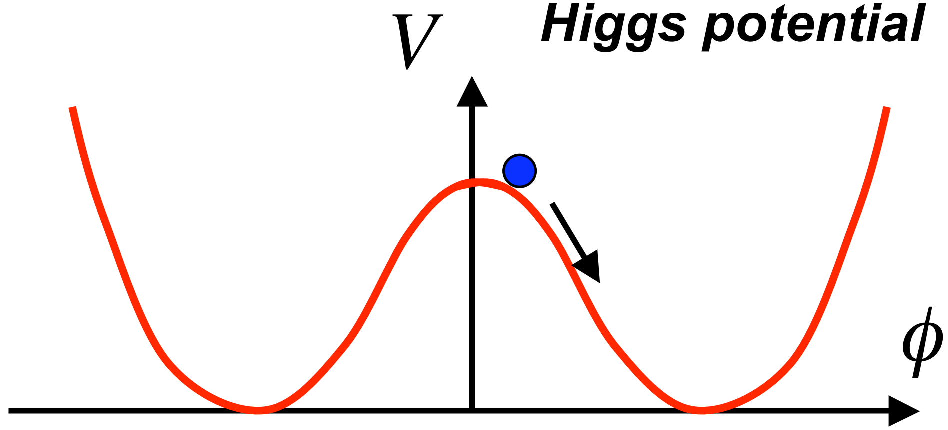
$$\frac{\ddot{R}}{R} \Rightarrow +\frac{8\pi G}{3c^2}\varepsilon_\Lambda = H_0^{\,2}\Omega_\Lambda$$

$$\Rightarrow R \propto \exp(H_\Lambda \, t) \qquad H_\Lambda \equiv H_0\sqrt{\Omega_\Lambda} = \frac{\Lambda}{3}$$

But today's Dark Energy is negligible at early times.

What causes Early Inflation? An evolving Scalar Field?





Klein-Gordon equation
= wave equation
for a massive scalar field.

$$E^2 = p^2 c^2 + m^2 c^4$$

$$E \rightarrow i\hbar \frac{\partial}{\partial t} \qquad p \rightarrow -i\hbar \nabla$$

$$\ddot{\phi} - c^2 \nabla^2 \phi = \frac{m^2 c^4}{\hbar^2} \phi = \frac{m^2 \phi}{(M_P t_P)^2}$$

For a spatially-uniform field:

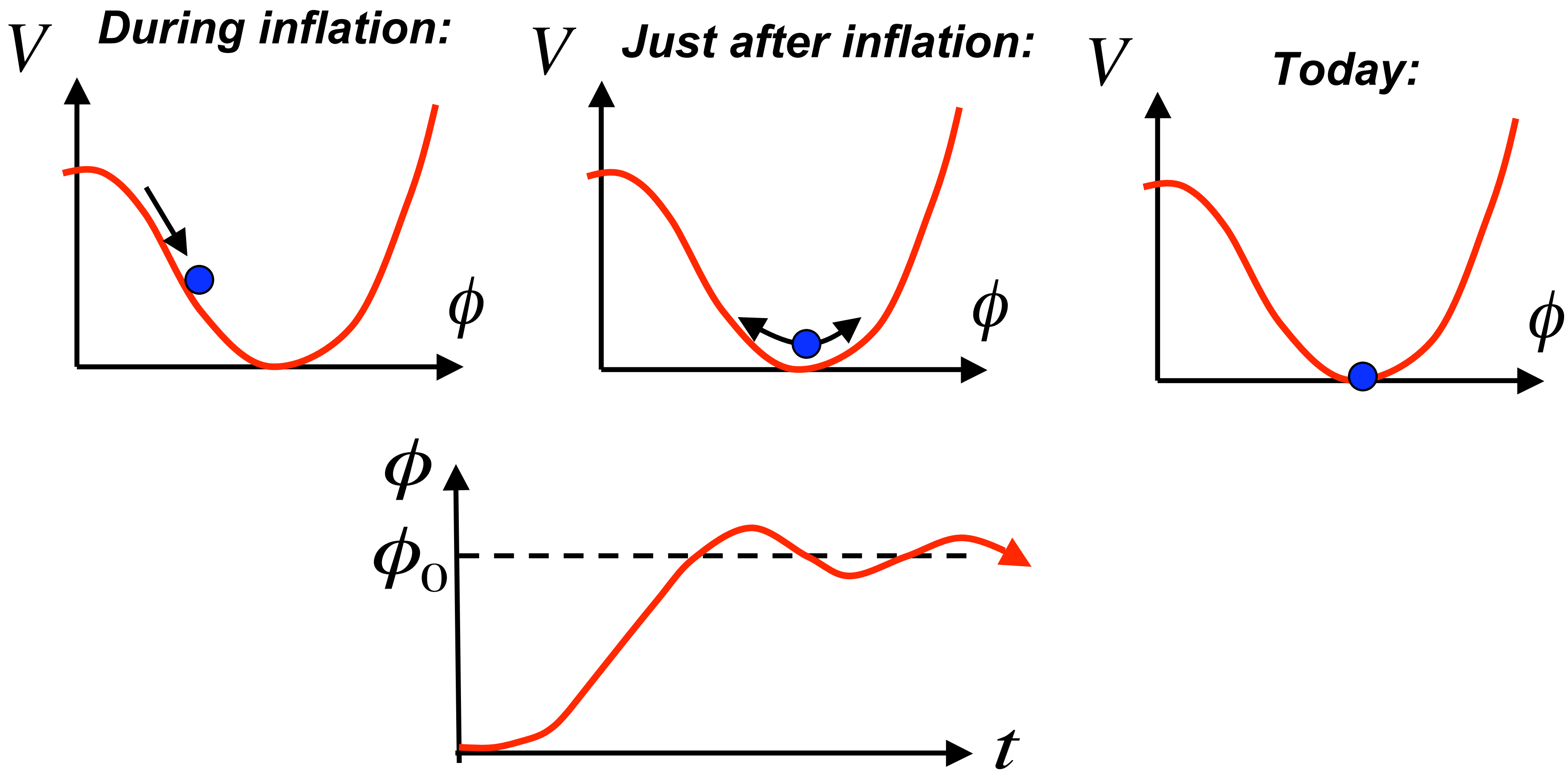
$$\nabla \phi \approx 0 \qquad \ddot{\phi} = -\frac{dV}{d\phi}$$

$$\text{massive field}: \quad V = \frac{m^2 \phi^2}{2(M_P t_P)^2}$$

$$\text{Higgs field}: \quad V \propto -a\phi^2 + b\phi^4$$

Vacuum energy starts large, declines to 0 at late times.

Scalar Field Dynamics

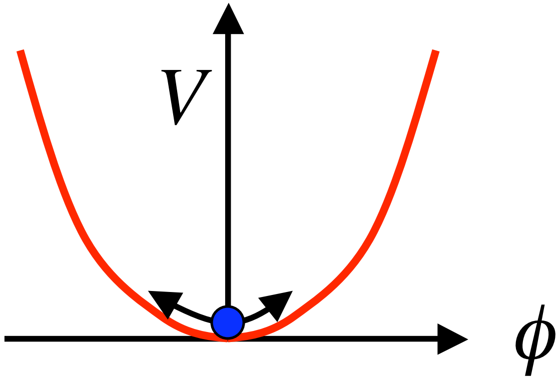


Kinetic energy of the oscillations is damped.
Re-heats the Universe, creating all types of particle-antiparticle pairs, launching the Hot Big Bang.

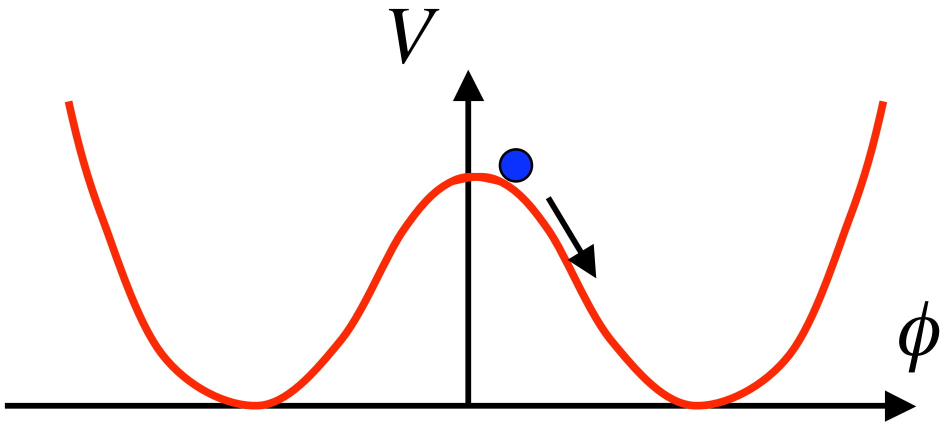
Spontaneous Symmetry Breaking

Why start in the false vacuum, not near V~0 ?

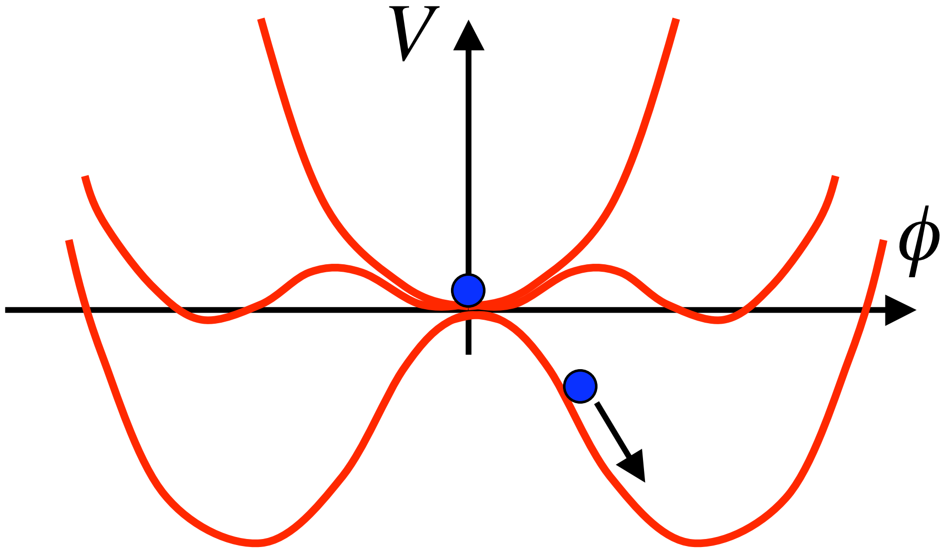
Before GUT phase transition:



After GUT phase transition:



false vacuum



true vacuum

Tunnel to true vacuum ? No.

Latent heat released, converts to particle-antiparticle pairs, re-heating the universe.

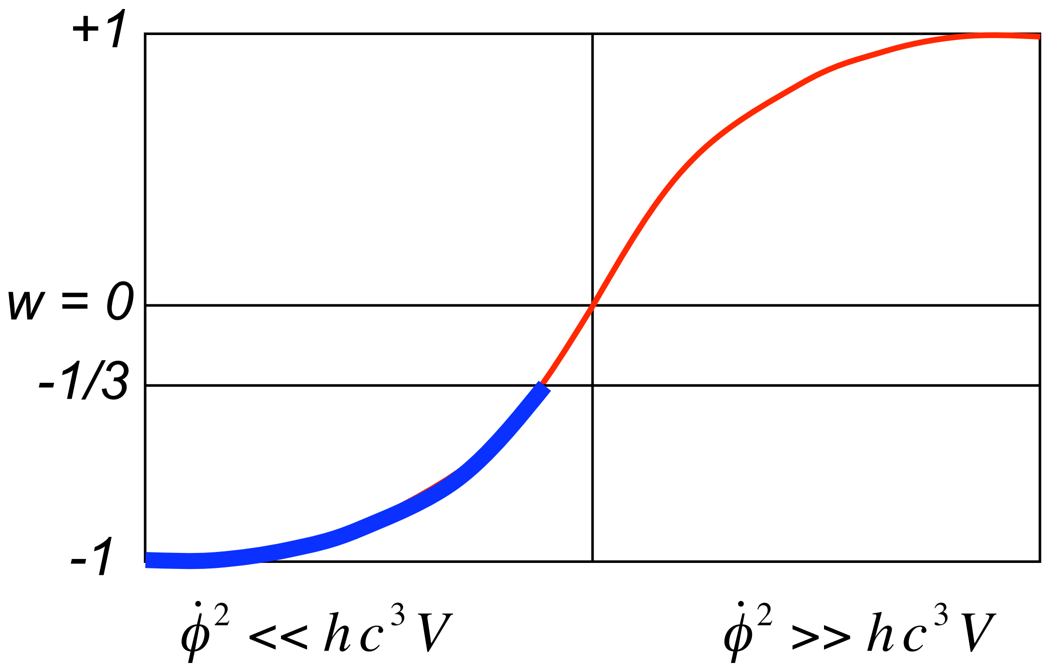
Small excess (10^{-9}) of particles.

Scalar Field Equation of State

Equation of State: $w = p / \varepsilon$

$$p_\phi = \frac{1}{2}\frac{1}{\hbar c^3}\left(\dot{\phi}^2 + c^2 \nabla \varphi^2\right) - V(\phi)$$

$$\varepsilon_\phi = \frac{1}{2}\frac{1}{\hbar c^3}\left(\dot{\phi}^2 + c^2 \nabla \varphi^2\right) + V(\phi)$$



Uniform field: $c^2 (\nabla \phi)^2 \quad << \quad \dot{\phi}^2$

(Inflation makes spatial gradients small.)

Required for inflation:

$$w = \frac{p}{\varepsilon} = \frac{\dot{\phi}^2 - 2\hbar c^3 V}{\dot{\phi}^2 + 2\hbar c^3 V} < -\frac{1}{3}$$

$$\varepsilon + 3p = 2\left(\frac{\dot{\phi}^2}{\hbar c^3} - V\right) < 0$$

$$\frac{\dot{\phi}^2}{\hbar c^3 V} < 1$$

The potential energy must dominate the kinetic energy.

Scalar Field Dynamics

Equation of State: $w = p / \varepsilon$

$$p_\phi = \frac{\dot{\phi}^2}{2\hbar c^3} - V(\phi)$$

$$\varepsilon_\phi = \frac{\dot{\phi}^2}{2\hbar c^3} + V(\phi)$$

Evolution of energy density:

$$\varepsilon \propto x^{3(1+w)} \qquad dt = -dx/(x H)$$

$$\dot{\varepsilon} = \frac{d\varepsilon}{dx}\frac{dx}{dt} = \left(3(1+w)\frac{\varepsilon}{x}\right)\left(-x H\right) = -3H\left(\varepsilon + p\right)$$

Evolution of uniform scalar field:

$$\dot{\varepsilon} = \frac{\dot{\phi}\,\ddot{\phi}}{\hbar c^3} + \frac{\partial V}{\partial \phi}\dot{\phi} = -3H(\varepsilon + p) = -3H\frac{\dot{\phi}^2}{\hbar c^3}$$

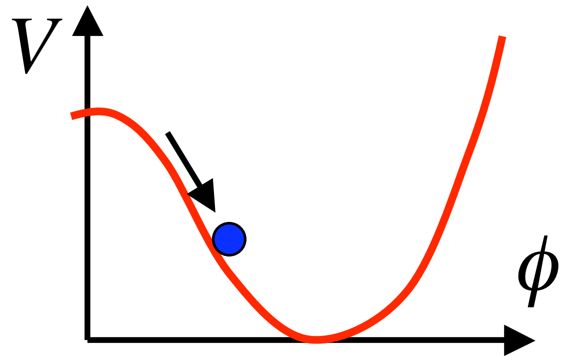
$$\ddot{\phi} + 3H\dot{\phi} = -\hbar c^3\frac{\partial V}{\partial \phi}$$

Scalar Field Dynamics

Hubble Drag:
Acceleration damped by expansion:

$$\ddot{\phi} + 3H\dot{\phi} - c^2\nabla^2\phi = -\hbar c^3 \frac{\partial V}{\partial \phi}$$

(Inflation makes spatial gradients small.)



"Slow-Roll" Approximation:

$$3H\dot{\phi} = -\hbar c^3 \frac{\partial V}{\partial \phi}$$

Terminal velocity:

$$\dot{\phi} \Rightarrow -\frac{\hbar c^3}{3H}\frac{\partial V}{\partial \phi}$$

"Like a snowflake falling"

Friedmann equation:

$$H^2 \approx \frac{8\pi G}{3}\frac{\varepsilon}{c^2} \approx \frac{8\pi G}{3}\frac{V}{c^2}$$

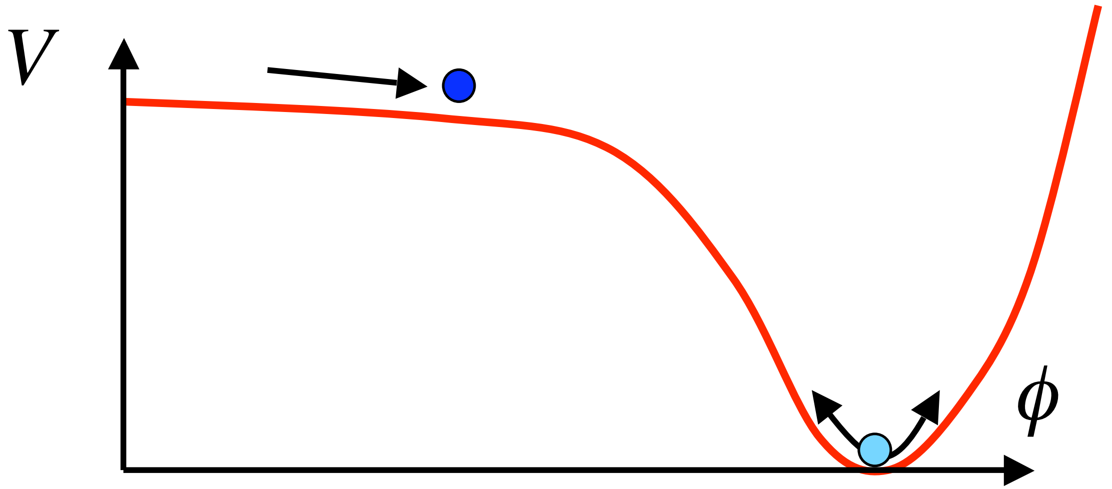
Required for Inflation:

$$\frac{\dot{\phi}^2}{\hbar c^3 V} \Rightarrow \frac{\hbar c^3}{9H^2 V}\left(\frac{\partial V}{\partial \phi}\right)^2$$

$$= \frac{\hbar c^5}{24\pi G}\left(\frac{\partial V/\partial \phi}{V}\right)^2 << 1$$

Inflation requires a "very flat" potential.

Long Slow Roll Inflation Ending in a Hot Big Bang.



Long slow roll across the "plateau" (potential dominated) gives > 60 e-foldings of inflation (cooling,flattening).

Latent heat released during (kinetic energy dominated) rapid roll and damped oscillations at the end fills the universe with photons and particle-antiparticle pairs, launching the Hot Big Bang.

Initial quantum vacuum fluctuations mean different regions finish at slightly different times, giving the small (10^{-5}) temperature/density ripples we see on the CMB.

A Multi-verse? Chaotic Inflation?

Vacuum with quantum fluctuations.

Many bubbles of false vacuum inflate,
with:
Different physical constants.
Different spatial dimensions.

ANTHROPIC PRINCIPLE:

Most bubbles quickly re-collapse
or expand too fast to form stars
or are otherwise unsuitable
as habitats for Life.

Is our visible universe
part of a big bubble
lasting long enough,
and with suitable physical laws,
to allow beings like us to evolve?