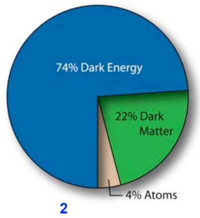
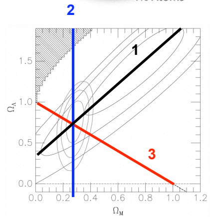
Cosmic Microwave Background

Flat Geometry

$$\Omega_0 = \Omega_M + \Omega_\Lambda$$

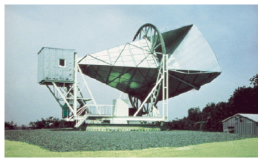
$$\approx 1.0$$





AS 4022 Cosmology

1965 -- Penzias + Wilson



Bell Labs telecommunications engineers
find excess microwave noise from the sky.

~1% of thermal (T ~ 300° K) noise ---> T ~ 3° K

Afterglow of the Big Bang

CMB = Cosmic Microwave Background

Confirms a forgotten 1948 prediction by Gamow.

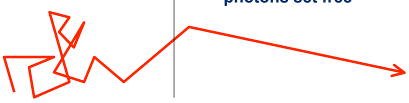
Nobel Prize -> P+W

AS 4022 Cosmology

Recombination Epoch (z~1100)

ionised plasma --> neutral gas

ionised plasma	neutral gas
• Redshift z > 1100	• z < 1100
• Temp T > 3000 K	• T < 3000 K
• H ionised	• H recombined
• electron -- photon Thompson scattering	• almost no electrons
	• neutral atoms
	• photons set free

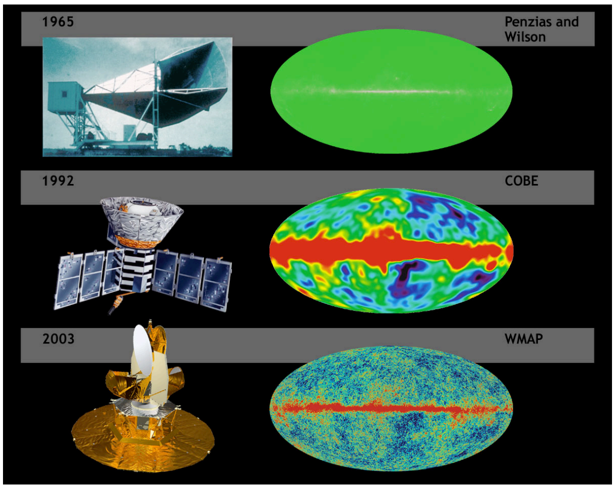


e - scattering optical depth

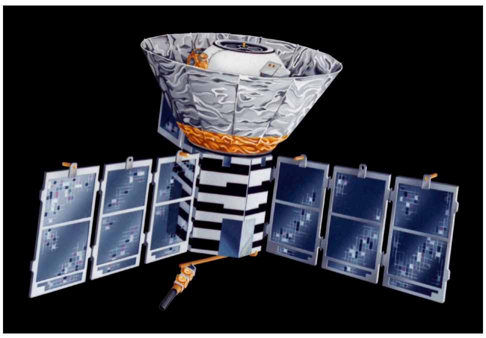
$$\tau(z) \approx \left(\frac{z}{1080}\right)^{13}$$

thin surface of last scattering

AS 4022 Cosmology

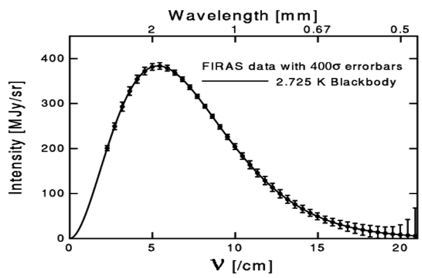


NASA 1992 - COBE
COsmic Background Explorer



AS 4022 Cosmology

COBE spectrum of CMB



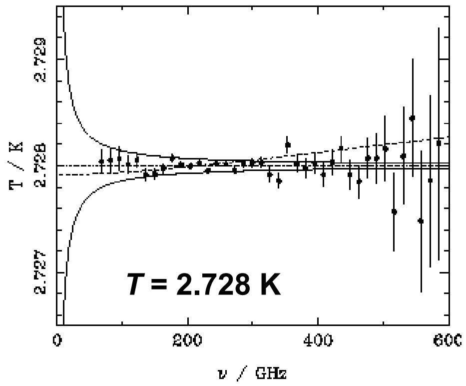
A perfect Blackbody !

No spectral lines -- strong test of Big Bang.
Expansion preserves the blackbody spectrum.

$T(z) = T_0 (1+z)$ $T_0 \sim 3000$ K $z \sim 1100$

AS 4022 Cosmology

COBE spectrum



AS 4022 Cosmology

Radiation -> Matter -> Vacuum

$T = 2.728$ K

radiation energy density :

$$\rho_R = \frac{u(T)}{c^2} = \frac{4\,\sigma}{c^3} T^4$$

$$\Omega_R = 8.6\times10^{-5}\left(\frac{0.7}{h}\right)^2 \sim 0.01\%$$

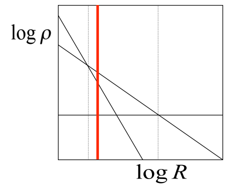
(*including neutrinos*)

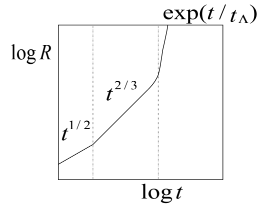
matter - radiation equality :

$$\frac{\Omega_R(z)}{\Omega_M(z)} = \frac{(1+z)^4\,\Omega_R}{(1+z)^3\,\Omega_M} = 1$$

$$(1+z_{eq}) = \frac{\Omega_M}{\Omega_R} = 3500\left(\frac{\Omega_M}{0.3}\right)\left(\frac{2.73K}{T}\right)^4$$

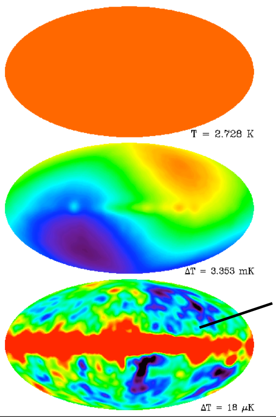
Matter-dominated at z=1100





AS 4022 Cosmology

Cosmic Microwave Background



Almost isotropic

$T = 2.728$ K

Dipole anisotropy

$$\frac{V}{c} = \frac{\Delta\lambda}{\lambda} = \frac{\Delta T}{T} \approx 10^{-3}$$

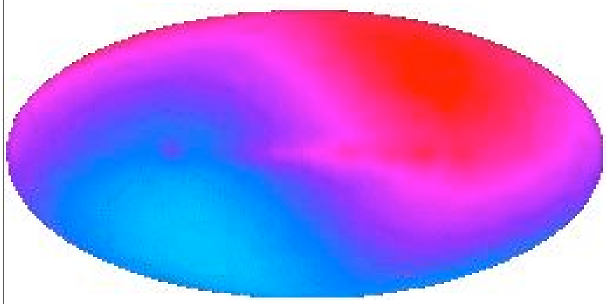
Our velocity:

$V \approx 600$ km/s

Milky Way sources

$+$ anisotropies $\frac{\Delta T}{T} \sim 10^{-5}$

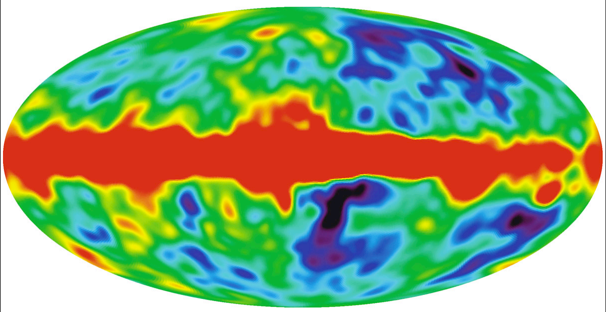
0.1% CMB dipole anisotropy velocity relative to CMB photons



$$T(\theta) = T_0\left(1 + \frac{V}{c}\cos\theta + ...\right) \rightarrow \begin{matrix} V_{SUN} = 371 \pm 1\,\mathrm{km\,s^{-1}} \\ V_{MW} \approx 600\,\mathrm{km\,s^{-1}} \end{matrix}$$

AS 4022 Cosmology

COBE 4-year map

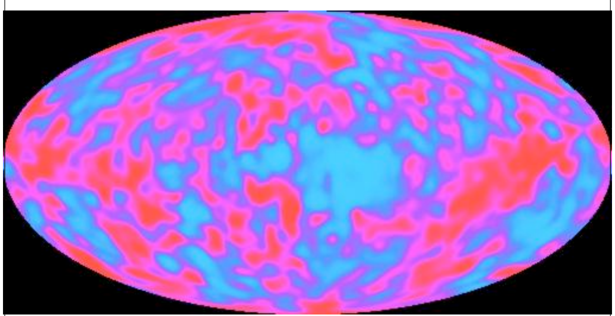


Milky Way emission

subtract by using maps at several frequencies

AS 4022 Cosmology

COBE - tiny ripples

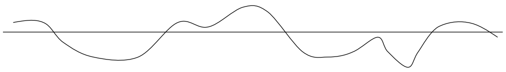


$$\frac{\delta T}{T} \sim 10^{-5}$$

Resolution ~ 7°

AS 4022 Cosmology

Tiny Ripples at Redshift 1100



$$\frac{\Delta T}{T} \approx \frac{\Delta \rho}{4\,\rho} \sim 10^{-5} \text{ at } z = 1100$$

Ripples are :

- **relics of the Big Bang**
- **initial quantum fluctuations expanded by early inflation**
- **the seeds of later galaxy/cluster formation.**
- **standard yardsticks for measuring curvature**
  - **(and other cosmology parameters)**

1999 - Boomerang in Antarctica

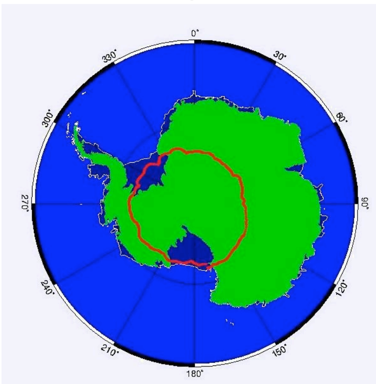
Baloon Observations Of Millimetric Extragalactic Radiation ANisotropy and Geophysics

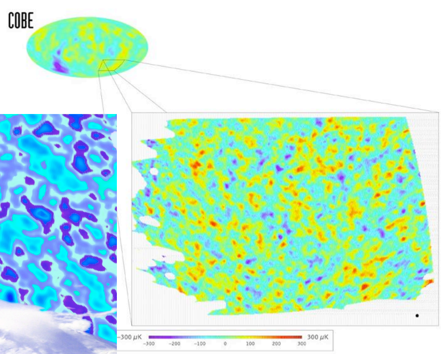
Boomerang's Baloon

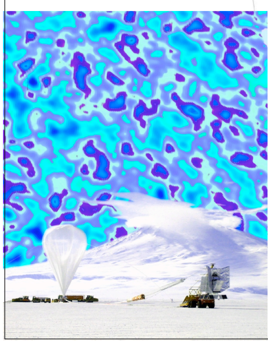
Boomerang's Stratospheric Flight Track

Altitude 37 km

10 days

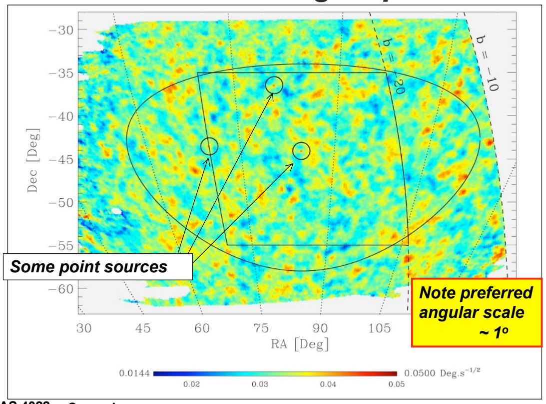






Resolution ~ 0.3°

Boomerang Map



Spherical Harmonics

Fit temperature map with a series of spherical harmonics

$$\frac{\Delta T}{T}(\theta,\phi) = \sum_{l,m} a_{l\,m} Y_{l\,m}(\theta,\phi)$$

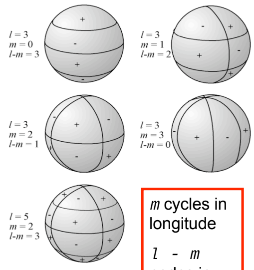
angular power spectrum

$$C_l = \left\langle \left| a_{l\,m} \right|^2 \right\rangle \quad \text{average} \ -l \le m \le l$$

dimensionless power spectrum

$$l(l+1)\,C_l \propto \frac{d\left\langle (\Delta T/T)^2 \right\rangle}{d\ln l}$$

angular scale: $\Delta\theta \approx \frac{\pi}{l} = \frac{180^\circ}{l}$



m cycles in longitude

l - *m* nodes in latitude

Spherical Harmonics

$$Y_0^0(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{1}{\pi}}$$

$$Y_1^{-1}(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{3}{2\pi}}\sin\theta\, e^{-i\varphi}$$

$$Y_1^{0}(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{3}{\pi}}\cos\theta$$

$$Y_1^{1}(\theta,\varphi) = \frac{-1}{2}\sqrt{\frac{3}{2\pi}}\sin\theta\, e^{i\varphi}$$

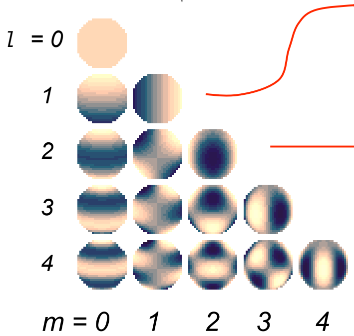
$$Y_2^{-2}(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{-2i\varphi}$$

$$Y_2^{-1}(\theta,\varphi) = \frac{1}{2}\sqrt{\frac{15}{2\pi}}\sin\theta\cos\theta\, e^{-i\varphi}$$

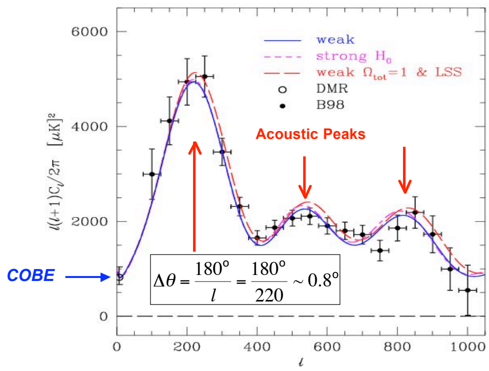
$$Y_2^{0}(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{5}{\pi}}(3\cos^2\theta - 1)$$

$$Y_2^{1}(\theta,\varphi) = \frac{-1}{2}\sqrt{\frac{15}{2\pi}}\sin\theta\cos\theta\, e^{i\varphi}$$

$$Y_2^{2}(\theta,\varphi) = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\varphi}$$



Boomerang Power Spectrum

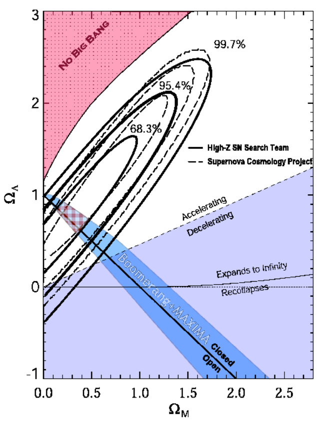


Supernovae + CMB ripples

Pre-WMAP constraints

From BOOMERANG and MAXIMA

circa 2002



WMAP

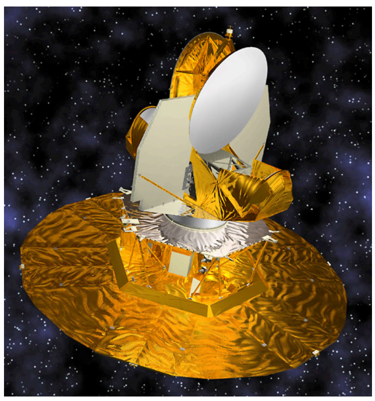
NASA 2001...

Wilkinson

Microwave

Anisotropy

Probe



COBE

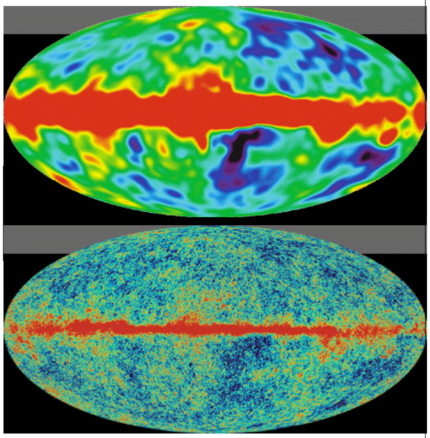
1992

7 degree resolution

WMAP

2003

20 arcmin resolution



COBE

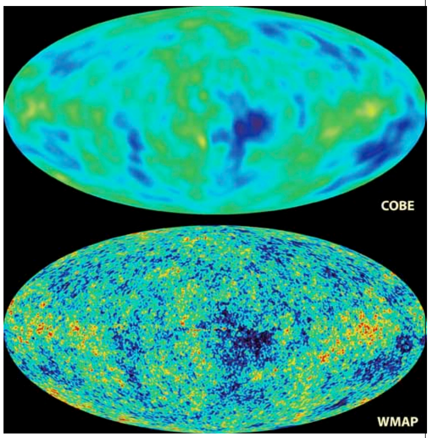
1992

7 degree resolution

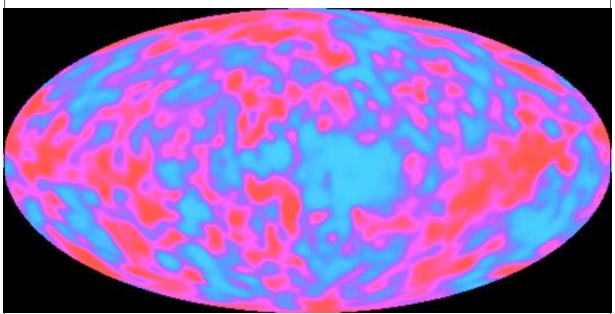
WMAP

2003

20 arcmin resolution



COBE - temperature ripples

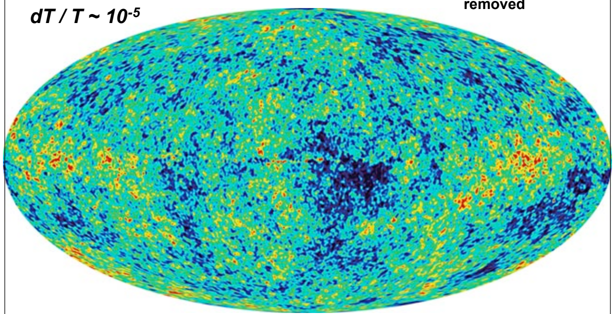


2003 WMAP all-sky

***T* = 2.73 K**

***dT / T* ~ 10^{-5}**

Dipole and foreground galaxy removed

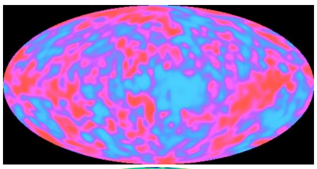


Snapshot at z=1100 of quantum fluctuations stretched by inflation.

Dark matter potential wells that seed later galaxy formation.

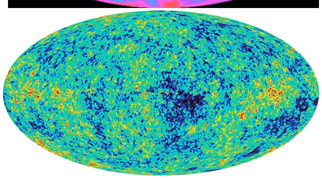
CMB Anisotropies

COBE 1994



$$\frac{\Delta T}{T} \sim 10^{-5}$$

WMAP 2004

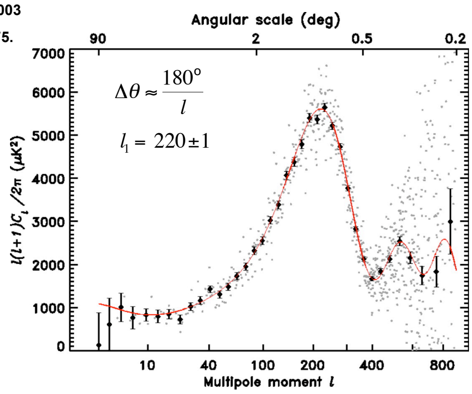


$$\Delta\theta \sim 1^\circ$$

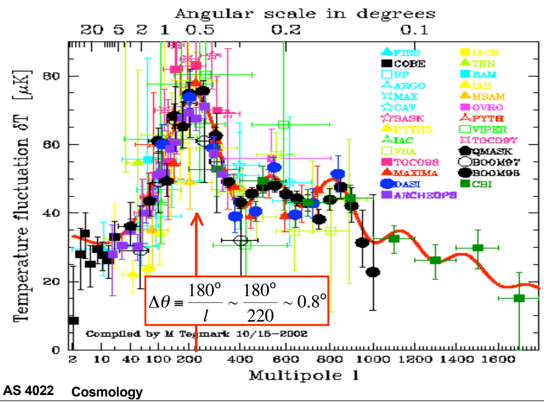
Snapshot of Universe at z = 1100
Seeds that later form galaxies.

2003 -- WMAP Power Spectrum

Spergel et al. 2003
ApJSup 148,175.



2002 - CMB Power Spectrum



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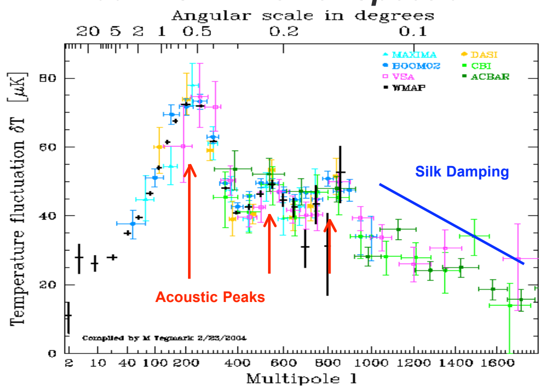
2004 - CMB Power Spectrum



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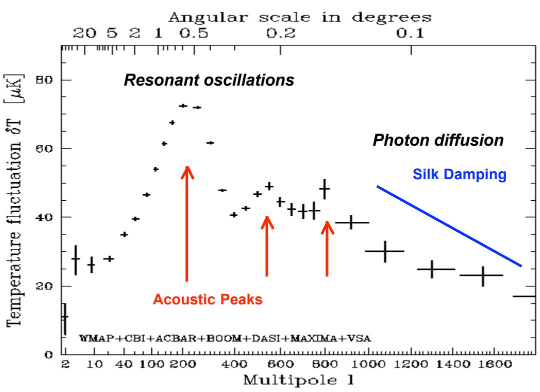
2004 - CMB Binned



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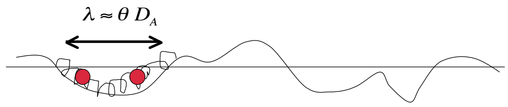
Acoustic Oscillations



Dark Matter potential wells - many sizes.

photon-electron-baryon fluid

fluid falls into DM wells

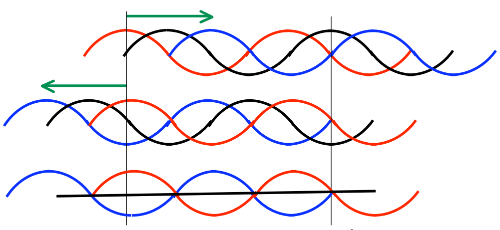
photon pressure pushes it out again

oscillations starting at t = 0 (post-inflation)

stopping at z = 1100 (recombination)

AS 4022 Cosmology

Standing Sound Waves



temperature oscillations: $c_s = \frac{\lambda}{P} = \frac{\omega}{k} = \frac{c}{\sqrt{3}}$

$\Delta T(x,t) = a\cos(\omega t)\cos(k x) \qquad \omega = \frac{2\pi}{P} \qquad k = |\mathbf{k}| = \frac{2\pi}{\lambda}$

$\Delta T(\mathbf{x},t) = \sum_{\mathbf{k}} a(\mathbf{k})\cos(\omega t)\cos(\mathbf{k}\bullet\mathbf{x}) \qquad \langle a(k)\rangle = a_0\left(\frac{k}{k_0}\right)^{n_S}$

AS 4022 Cosmology

Resonant Oscillations

size of potential well λ

oscillation period $P \approx \frac{\lambda}{c_s}$

sound speed $c_s = \frac{c}{\sqrt{3}}$

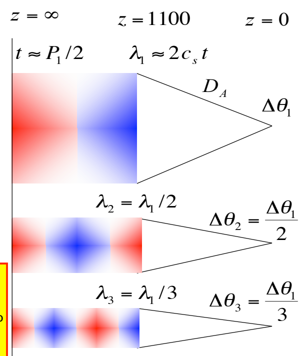
temperature oscillations

$\Delta T(t) = \Delta T(0)\cos(2\pi\, t/P)$

$\max|\Delta T|$ at $t = \frac{nP}{2} \sim \frac{n\lambda}{2c_s}$

angular size

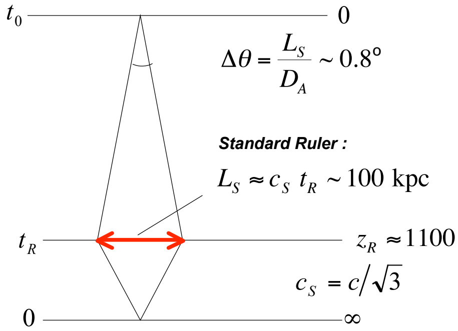
$\Delta\theta_n = \frac{\lambda_n}{D_A} = \frac{\Delta\theta_1}{n} \qquad \Delta\theta_1 \approx \frac{2c_s t}{D_A} \sim 0.8^\circ$



Smaller wells oscillate faster.

AS 4022 Cosmology

Sound Horizon at z = 1100



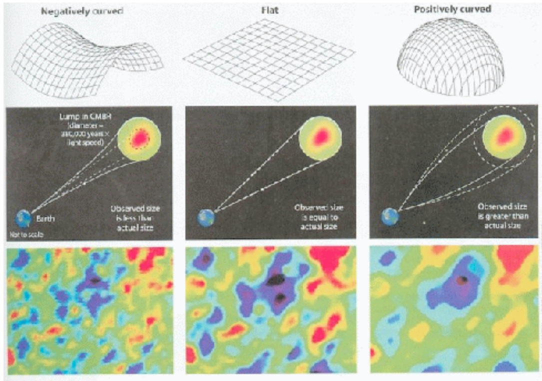
$$\Delta\theta = \frac{L_S}{D_A} \sim 0.8^\circ$$

Standard Ruler :

$$L_S \approx c_S \, t_R \sim 100 \text{ kpc}$$

$$c_S = c/\sqrt{3}$$

Angular scale --> Geometry



Sound Horizon at z = 1100

distance travelled by a sound wave

$c_S \, dt$

expand each step by factor $R(t_R)/R(t)$:

recombination at ***z = 1100***

$$x \equiv 1+z = \frac{R_0}{R(t)}$$

$$dt = \frac{-dx}{x\,H(x)}$$

sound speed

$$c_S \approx \frac{c}{\sqrt{3}}$$

$$L_S(t_R) = R(t_R)\int_0^{t_R} \frac{c_S\,dt}{R(t)}$$

dt = - dx / x H(x)
R(t) = R₀ / x

$$= \frac{R_0}{1+z}\int_{1+z}^{\infty} \frac{x}{R_0}\,\frac{c_S\,dx}{x\,H(x)}$$

$$= \frac{c_S}{(1+z)}\int_{1+z}^{\infty} \frac{dx}{H(x)}$$

***H(x)* from Friedmann Eqn.**

$$= \frac{c_S}{(1+z)\,H_0}\int_{1+z}^{\infty} \frac{dx}{\sqrt{x^4\,\Omega_R + x^3\,\Omega_M + \Omega_\Lambda + (1-\Omega_0)\,x^2}}$$

$$\approx \frac{c_S}{(1+z)\,H_0}\int_{1+z}^{\infty} \frac{dx}{\sqrt{x^4\,\Omega_R + x^3\,\Omega_M}}$$

keep 2 largest terms.

Sound Horizon at z = 1100

$$L_S(t_R) = \frac{c_S}{(1+z)}\int_{1+z}^{\infty}\frac{dx}{H(x)} \approx \frac{c_S}{(1+z)\,H_0}\int_{1+z}^{\infty}\frac{dx}{\sqrt{x^4\,\Omega_R + x^3\,\Omega_M}}$$

$$= \frac{c_S}{(1+z)\,H_0\sqrt{\Omega_R}}\int_{1+z}^{\infty}\frac{dx}{\sqrt{x^3(x+x_0)}} \qquad x_0 \equiv \frac{\Omega_M}{\Omega_R} \approx 3500\left(\frac{\Omega_M}{0.3}\right)$$

$$= \frac{c_S}{(1+z)\,H_0\sqrt{\Omega_R}}\left(-\frac{2}{x_0}\sqrt{1+\frac{x_0}{x}}\right)_{1+z}^{\infty}$$

$$= \frac{2\,c_S}{(1+z)\,H_0\sqrt{\Omega_M\,x_0}}\left(\sqrt{1+\frac{x_0}{1+z}}-1\right) \qquad c_S = \frac{c}{\sqrt{3}}$$

$$= \frac{c}{H_0}\,\frac{2\left(\sqrt{4.6}-1\right)}{1100\,\sqrt{3\times 0.3\times 3500}}$$

$$= 3.4\times 10^{-5}\,\frac{c}{H_0} \approx 110\left(\frac{0.7}{h}\right)\left(\frac{0.3}{\Omega_M}\right)^{1/2} \text{ kpc}$$

Expands by factor 1 + z = 1100 to ~120 Mpc today.

Angular Scale measures Ω_0

sound horizon :

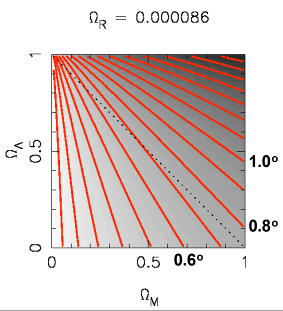
$$L_S(z) = \frac{1}{1+z}\int_{1+z}^{\infty}\frac{c_S\,dx}{H(x)}$$

angular diameter distance :

$$D_A(z) = \frac{R_0\,S_K(\chi)}{1+z} \qquad \chi = \int_t^{t_0}\frac{c\,dt}{R(t)} = \frac{c}{R_0}\int_1^{1+z}\frac{dx}{H(x)}$$

angular scale

$$\theta = \frac{L_S(z)}{D_A(z)} = \frac{\int_{1+z}^{\infty}\frac{c_S\,dx}{H(x)}}{R_0\,S_k\!\left(\frac{c}{R_0}\int_1^{1+z}\frac{dx}{H(x)}\right)}$$



Angular scale depends mainly on the curvature.

Gives θ ~ 0.8° for flat geometry, ***$\Omega_0 = \Omega_M + \Omega_\Lambda = 1$***

Finer Details: measure Ω_b and Ω_M

Sound speed not constant :

$$c_S(z) = \frac{c}{\sqrt{3\,(\,1+R(z)\,)}}$$

$$R(z) \equiv \frac{3\,\rho_b(z)}{4\,\rho_R(z)} = \frac{3\,\Omega_b\,(1+z)}{4\,\Omega_R}$$

Acoustic peaks not quite equally spaced :

$$l_n = l_A\,(\,n+\delta_n\,)$$

phase shifts

$$\delta_n \approx a_n\left(\frac{r}{0.3}\right)^{0.1} \qquad a_{1,2,3} \approx 0.267\,,0.24,0.35$$

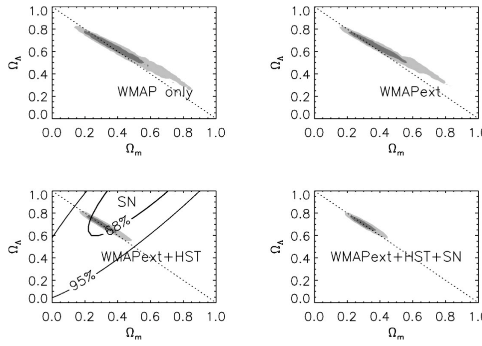
$$r \equiv \frac{\rho_M(z)}{\rho_R(z)} = \frac{\Omega_M\,(1+z)}{\Omega_R}$$

Max Tegmark's CMB Movies

Shows how the **CMB power spectrum**
(and the **baryon power spectrum**)
depend on the cosmological parameters.

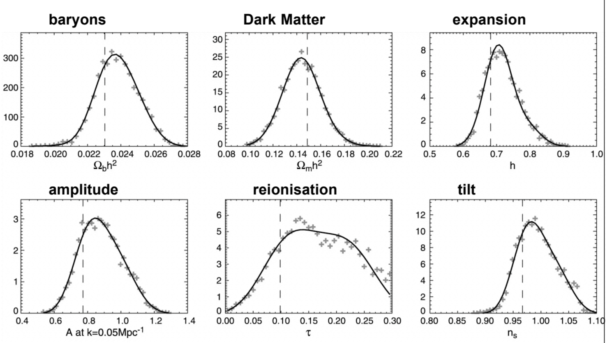
A link to these is available on the course web page.

WMAP parameter constraints

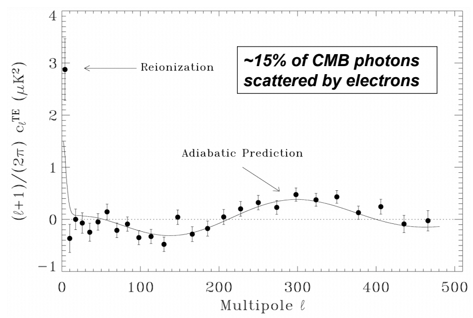


WMAP parameter constraints

Spergel et al. 2003 ApJSup 148,175.



WMAP Polarisation Power Spectrum



Spergel et al. 2003 ApJSup 148,175.

Epoch of Re-Ionisation

UV from first stars re-ionises gas.

Scatters ~15% of CMB photons yielding ~15% polarisation.

WMAP measured this !

$$dt = \frac{-dx}{x\,H(x)}$$

$$x \equiv 1+z$$

Electron scattering optical depth:

$$d\tau = n\,\sigma_T\,dr$$

$$= n_0\,(1+z)^3\,\sigma_T\,c\,dt$$

$$\tau = n_0\,\sigma_T\,c\int_1^{1+z}\frac{x^3\,dx}{x\,H(x)}$$

$$= \frac{n_0\,\sigma_T\,c}{H_0}\int_1^{1+z}\frac{x^2\,dx}{\sqrt{\Omega_M\,x^3+\Omega_\Lambda+(1-\Omega_0)\,x^2}}$$

Thompson cross-section σ_T

electron density today

$$n_0 = \frac{\Omega_b}{m_{\rm H}}\,\frac{3\,H_0^2}{8\,\pi\,G}\left(X+\frac{Y}{2}\right)$$

Gives ~15% optical depth at z ~ 20

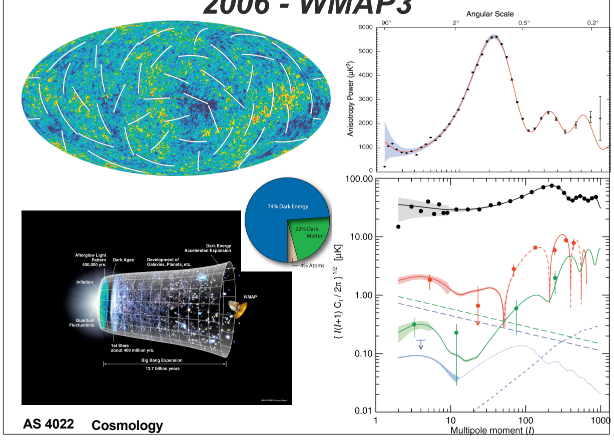
Precision Cosmology

$h = 71 \pm 3$	expanding
$\Omega = 1.02 \pm 0.02$	flat
$\Omega_b = 0.044 \pm 0.004$	baryons
$\Omega_M = 0.27 \pm 0.04$	Dark Matter
$\Omega_\Lambda = 0.73 \pm 0.04$	Dark Energy

$t_0 = 13.7 \pm 0.2 \times 10^9$ yr		now
$t_* = 180^{+220}_{-80} \times 10^6$ yr	$z_* = 20^{+10}_{-5}$	reionisation
$t_R = 379 \pm 1 \times 10^3$ yr	$z_R = 1090 \pm 1$	recombination

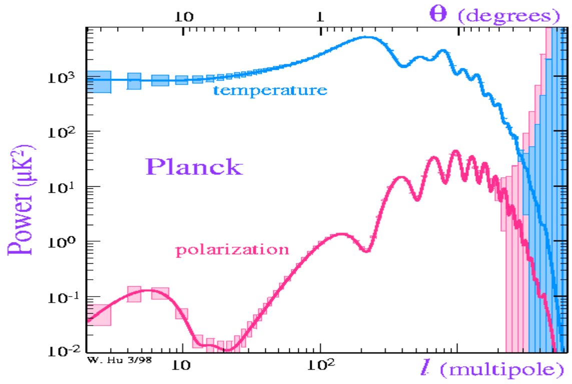
(From the WMAP 1-year data analysis)

2006 - WMAP3



AS 4022 Cosmology

Planck -- 2009 launch



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