Lecture 10

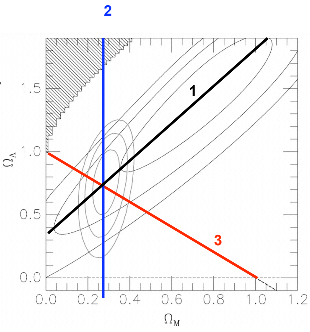
Checking the Distance Ladder:

Sunyaev-Zeldovich Effect

Gravitational Lensing

"Concordance" Model

1. Supernova Hubble Diagram
2. Galaxy Counts + M/L ratios
   $\Omega_M \sim 0.3$
3. Flat Geometry
   (inflation, CMB fluctuations)
   $\Omega_0 = \Omega_M + \Omega_\Lambda = 1$



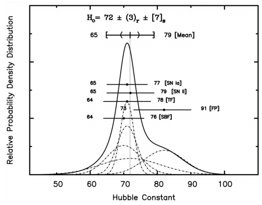
concordance model

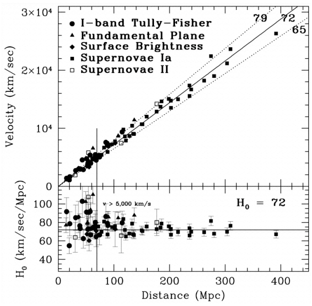
$H_0 \approx 72 \quad \Omega_M \approx 0.3 \quad \Omega_\Lambda \approx 0.7$

HST Key Project

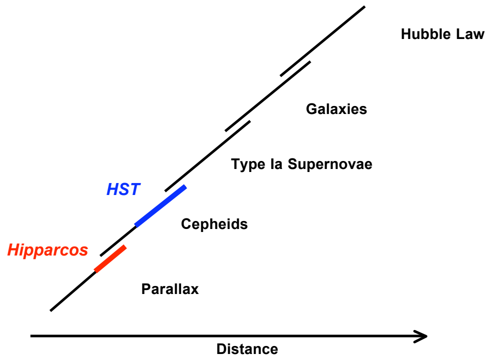
$$H_0 \approx 72 \pm 3 \pm 7 \quad \text{km s}^{-1}\ \text{Mpc}^{-1}$$

Freedman, et al. 2001 ApJ 553, 47.



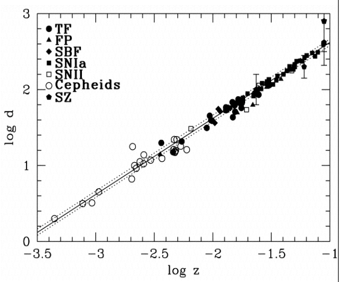


Cosmic Distance Ladder



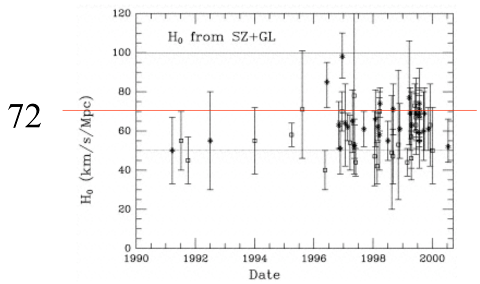
Frailty of the Distance Ladder

- **Parallax**
  - **0 - 300 pc**
  - **(GAIA 2015 5 kpc)**
- **Cepheids**
  - **~100 pc - 20 Mpc (HST)**
- **Type Ia SNe**
  - **20 - 400 Mpc (8m)**
  - **z ~ 1.5 (HST)**
- **Little overlap between Cepheids and SN Ia.**



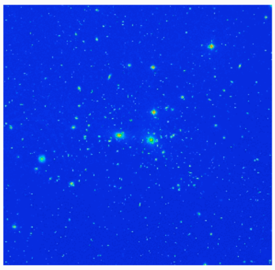
Only 3 galaxies with both Cepheids and SN Ia

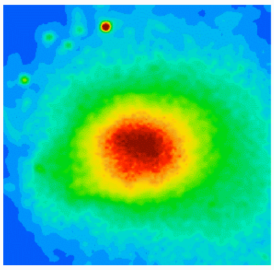
H_0 from SZ and GL



Cepheid-independent methods give lower values : (?

Galaxy Clusters are filled with hot X-ray gas





optical (galaxies) Coma cluster X-ray (hot gas)

Sunyaev - Zeldovich (SZ) effect

Silhouettes of the Hot Cluster Gas seen against the CMB.

CMB photons scattered by hot electrons

$T \rightarrow T\, e^{-\tau} \qquad \Delta T \approx T\, \tau$

scattering optical depth :

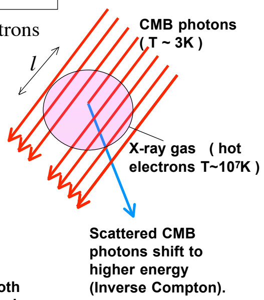
$\tau = n_e\, \sigma\, l$

X - ray emission by hot gas :

$F_X = \frac{L_X}{4\pi\, D_L^{\,2}} \qquad L_x \approx a(T_x)\, n_e^{\,2}\, l^3$

angular diameter :

$\theta = \frac{l}{D_A}$



Note: assume smooth density and spherical symmetry of hot gas

SZ Distances

Eliminate unknown n_e

$$\frac{L_X}{\tau^2} = \frac{a\, n_e^{\,2}\, l^3}{\left(n_e\, \sigma\, l\right)^2} = \frac{a}{\sigma^2}\, l = \frac{a}{\sigma^2}\, \theta\, D_A$$

$$= \frac{4\pi\, D_L^{\,2}\, F_X}{\left(\Delta T / T\right)^2}$$

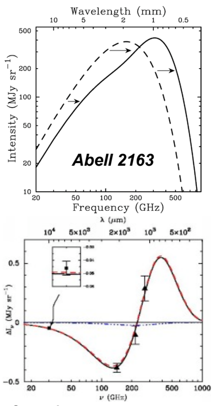
Solve for distance:

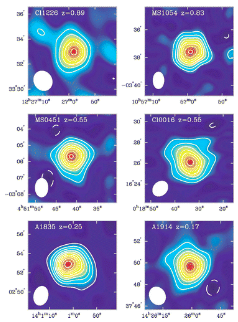
$$\frac{D_L^{\,2}}{D_A} = \frac{\left[r_0\,(1+z)\right]^2}{r_0/(1+z)} = r_0\,(1+z)^3 = \frac{a(T_X)}{4\pi\, \sigma^2} \left(\frac{\Delta T}{T}\right)^2 \frac{\theta}{F_X}$$

Mixture of D_L and D_A

Another observable distance !

SZ effect

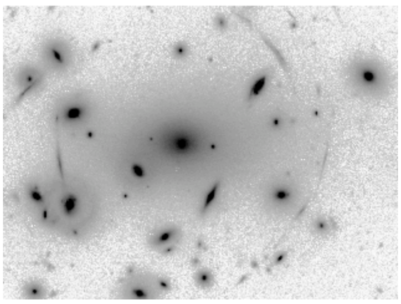




Carlstrom et al., 2002.

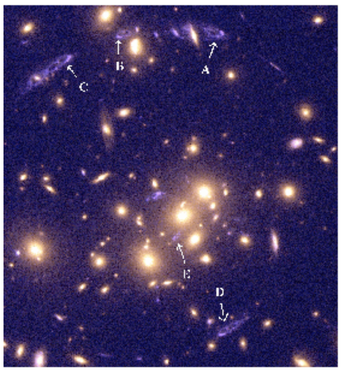
Gravitational Lensing

- Luminous arcs in clusters of galaxies

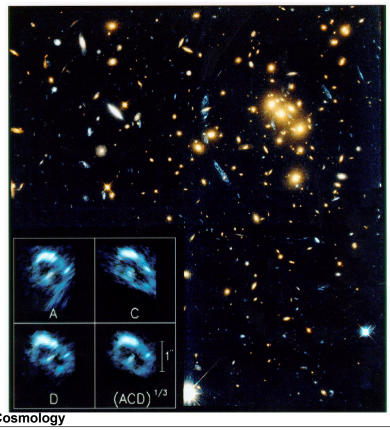


Gravitational Lensing

multiple images of background galaxy lensed by the cluster

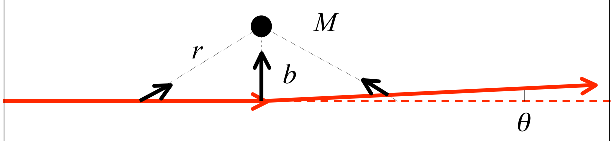


The Lensed Galaxy

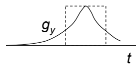


AS 4022 Cosmology

Newtonian Bend Angle



vertical acceleration $g_y = \left(\frac{G\,M}{r^2}\right)\left(\frac{b}{r}\right) \le \frac{G\,M}{b^2} = g_{\max}$



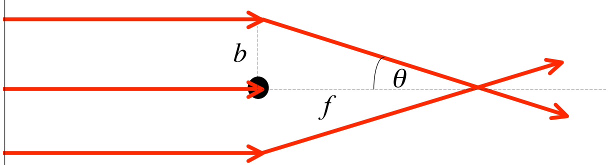
time to pass $\Delta t \approx 2\,b / V_x$

vertical velocity $V_y = \int g_y\, dt \approx g_{\max} \Delta t \approx \left(\frac{G\,M}{b^2}\right)\left(\frac{2\,b}{V_x}\right) = \frac{2\,G\,M}{b V_x}$

bend angle $\theta \approx \frac{V_y}{V_x} \approx \frac{2\,G\,M}{b V_x^{\,2}} \Rightarrow \frac{2\,G\,M}{b c^2}$

AS 4022 Cosmology

Focal Length of Gravitational Lens

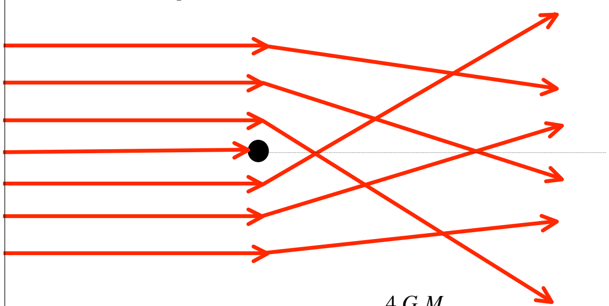


Einstein's bend angle $\theta = \frac{4\,G\,M}{b\,c^2}$

Focal length: $f = \frac{b}{\theta} = \frac{b^2\,c^2}{4\,G\,M}$

AS 4022 Cosmology

Spherical Aberration



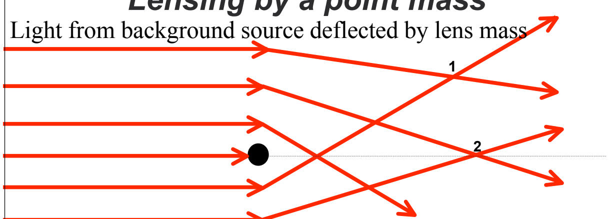
Einstein's bend angle $\theta = \frac{4\,G\,M}{b\,c^2}$

Focal length: $f = \frac{b}{\theta} = \frac{b^2\,c^2}{4\,G\,M}$

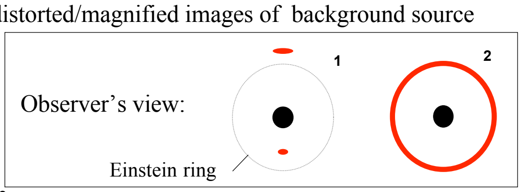
AS 4022 Cosmology

Lensing by a point mass

Light from background source deflected by lens mass



Two distorted/magnified images of background source



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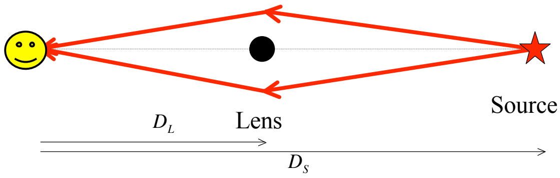
Einstein Ring Radius



Geometric optics:

$$\frac{1}{D_S - D_L} + \frac{1}{D_L} = \frac{1}{f} = \frac{4\,G\,M}{c^2\,b^2}$$

Einstein Ring Radius:

$$b = R_E = \sqrt{\frac{4\,G\,M}{c^2}\,\frac{D_L\,(D_S - D_L)}{D_S}}$$

$$\theta_E = \frac{R_E}{D_L} = \left(\frac{M}{10^{11.1}\,M_{sun}}\right)^{1/2}\left(\frac{D_L D_S / D_{LS}}{\text{Gpc}}\right)^{-1/2} \text{arcsec}$$

AS 4022 Cosmology

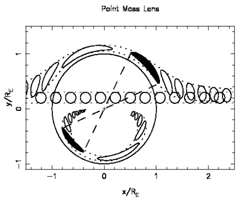
Lensing by a Point Mass

2 images

opposite sides of lens

major image outside ring

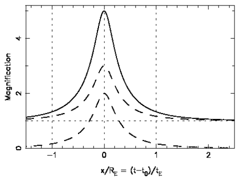
minor image inside ring



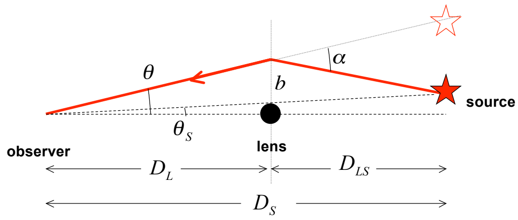
net magnification

(sum of 2 images)

vs time



Off-Axis Lensing Geometry



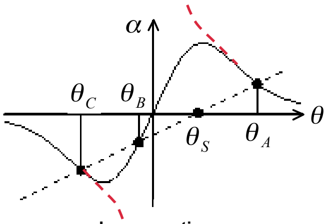
angular diameter distances from redshifts: z_L, z_S

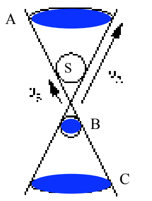
impact parameter: $b = D_L\,\theta$

source offset: $D_S\,\theta_S = D_S\,\theta - D_{LS}\,\alpha$

bend angle: $\alpha = (\theta - \theta_S)\dfrac{D_S}{D_{LS}} = \dfrac{4\,G\,M(<b)}{c^2\,b}$

Lensing by an extended mass distribution





Lens equation:

$$\alpha(\theta) = \frac{4\,G\,M(<\theta)}{c^2\,D_L\,\theta} = \frac{D_S}{D_{LS}}\left(\theta - \theta_S\right)$$

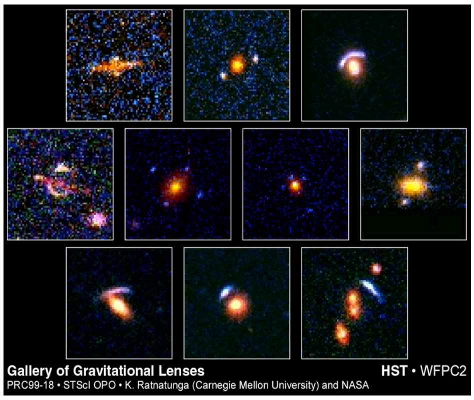
3 images on sky

Usually gives 3 images,

can be 5, 7, ...

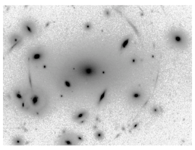
If M known, measure image angles and solve for $D_L D_S/D_{LS}$

Quasars Lensed by Galaxies



Masses from Einstein Rings

Perfect alignment gives an Einstein Ring



$$\theta_E = \frac{R_E}{D_L} = \left(\frac{4\,G\,M}{c^2}\,\frac{D_{LS}}{D_L D_S}\right)^{1/2}$$

$$\frac{\theta_E}{\text{arcsec}} = \left(\frac{M}{10^{11}\,M_{sun}}\right)^{1/2}\left(\frac{D_L D_S/D_{LS}}{\text{Gpc}}\right)^{-1/2}$$

$$\frac{M}{10^{11}\,M_{sun}} = \frac{D_L D_S/D_{LS}}{\text{Gpc}}\left(\frac{\theta_E}{\text{arcsec}}\right)^2$$

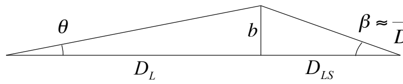
Use redshifts, z_L,z_S, for the angular diameter distances.

Or, if mass known, e.g $M \approx \dfrac{V^2 R}{G}$, then θ gives D

Mass usually less certain than distance,

so use theta and D to calculate M.

H_0 from Time Delays



$$\beta \approx \frac{b}{D_{LS}} \approx \frac{D_L}{D_{LS}}\,\theta$$

light travel time delay:

$$c\,\Delta t = \left(D_L^{\,2} + b^2\right)^{1/2} + \left(D_{LS}^{\,2} + b^2\right)^{1/2} - \left(D_L + D_{LS}\right)$$

$$= D_L\left[\left(1+\theta^2\right)^{1/2} - 1\right] + D_{LS}\left[\left(1 + \left(\frac{D_L}{D_{LS}}\,\theta\right)^2\right)^{1/2} - 1\right]$$

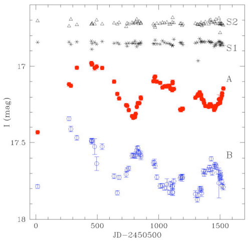
$$= D_L\,\frac{\theta^2}{2}\left(1 + \frac{D_L}{D_{LS}}\right) \approx \frac{c\,z_L}{H_0}\,\frac{\theta^2}{2}\left(1 + \frac{z_L}{z_S - z_L}\right) = \frac{c}{H_0}\,\frac{\theta^2}{2}\left(\frac{z_L\,z_S}{z_S - z_L}\right)$$

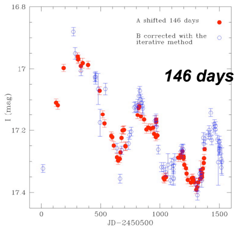
measure θ (images), z_L, z_S (spectra)

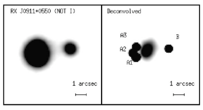
and Δt (delay from lightcurves of images).

Time Delay Measurement

Light curves of the images show a shift in time.







Hjorth et al. 2003.

But, no simple lenses.

Almost always several galaxies involved.

Prevents very accurate distance measurements.