Lecture 8

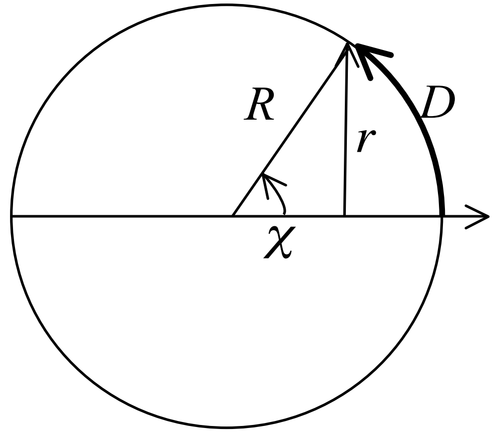
Observational Cosmology

Parameters of Our Universe

The Concordance Model

Time and Distance vs Redshift

$$\frac{d}{dt}\left(x = 1 + z = \frac{R_0}{R} \right) \quad \rightarrow \quad dt = \frac{-dx}{x\,H(x)}$$



Friedmann: $H(x) = H_0 \sqrt{\Omega_M \; x^3 + \Omega_\Lambda + (1-\Omega_0)\, x^2}$

$$\Omega_0 \equiv \Omega_M + \Omega_\Lambda \quad (\, \Omega_R = 0 \,)$$

look - back time:

$$t(z) = \int_{t_e}^{t_0} dt = \int_1^{1+z} \frac{dx}{x\,H(x)} = \frac{1}{H_0} \int_1^{1+z} \frac{dx}{x \sqrt{\Omega_M \; x^3 + \Omega_\Lambda + (1-\Omega_0)\; x^2}}$$

radial distance:

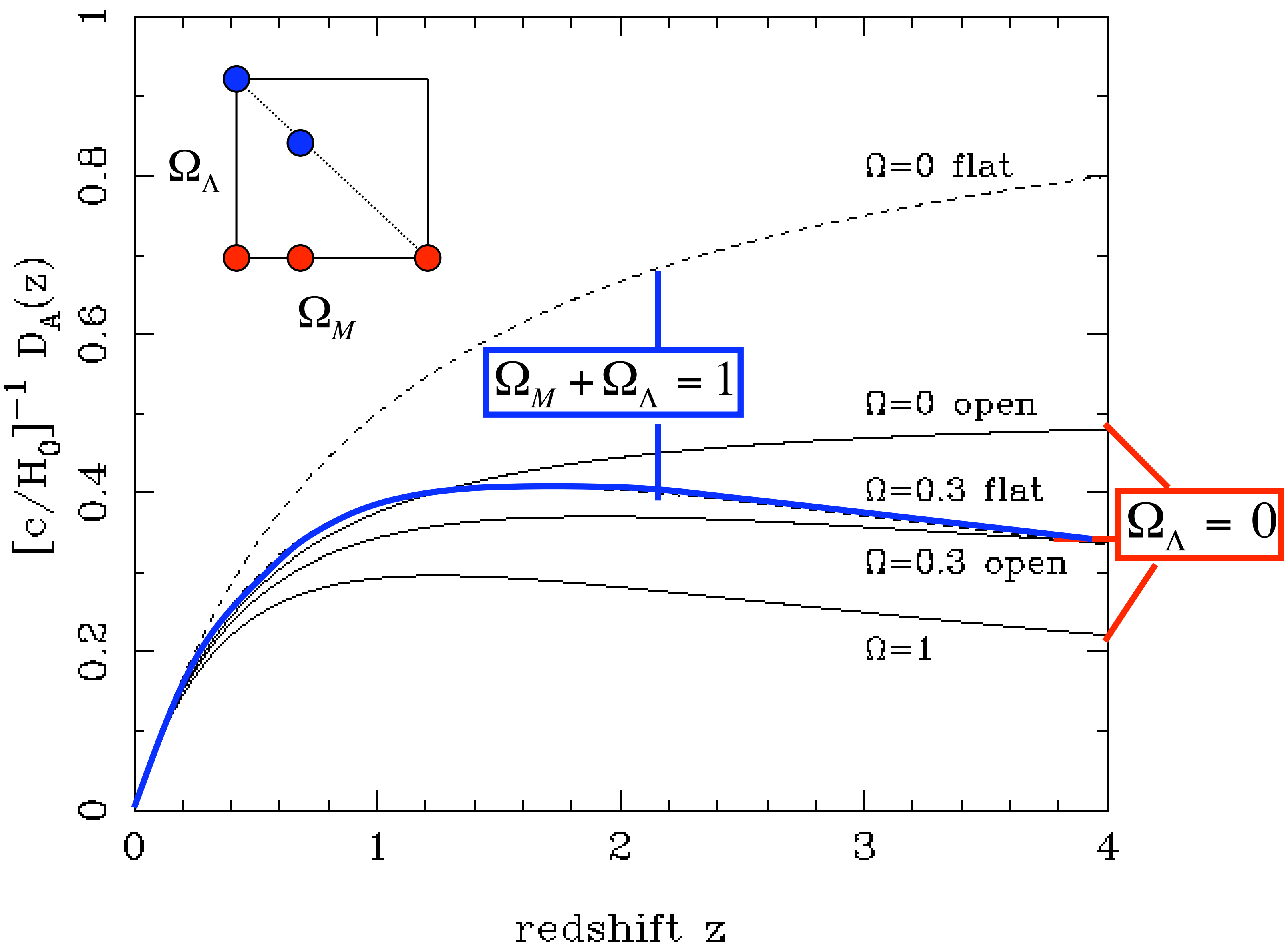
$$D_0(z) = R_0\, \chi = \int_{t_e}^{t_0} \frac{R_0}{R(t)}\, c \; dt = \int_1^{1+z} \frac{x\; c\; dx}{x\,H(x)} = \frac{c}{H_0} \int_1^{1+z} \frac{dx}{\sqrt{\Omega_M \; x^3 + \Omega_\Lambda + (1-\Omega_0)\; x^2}}$$

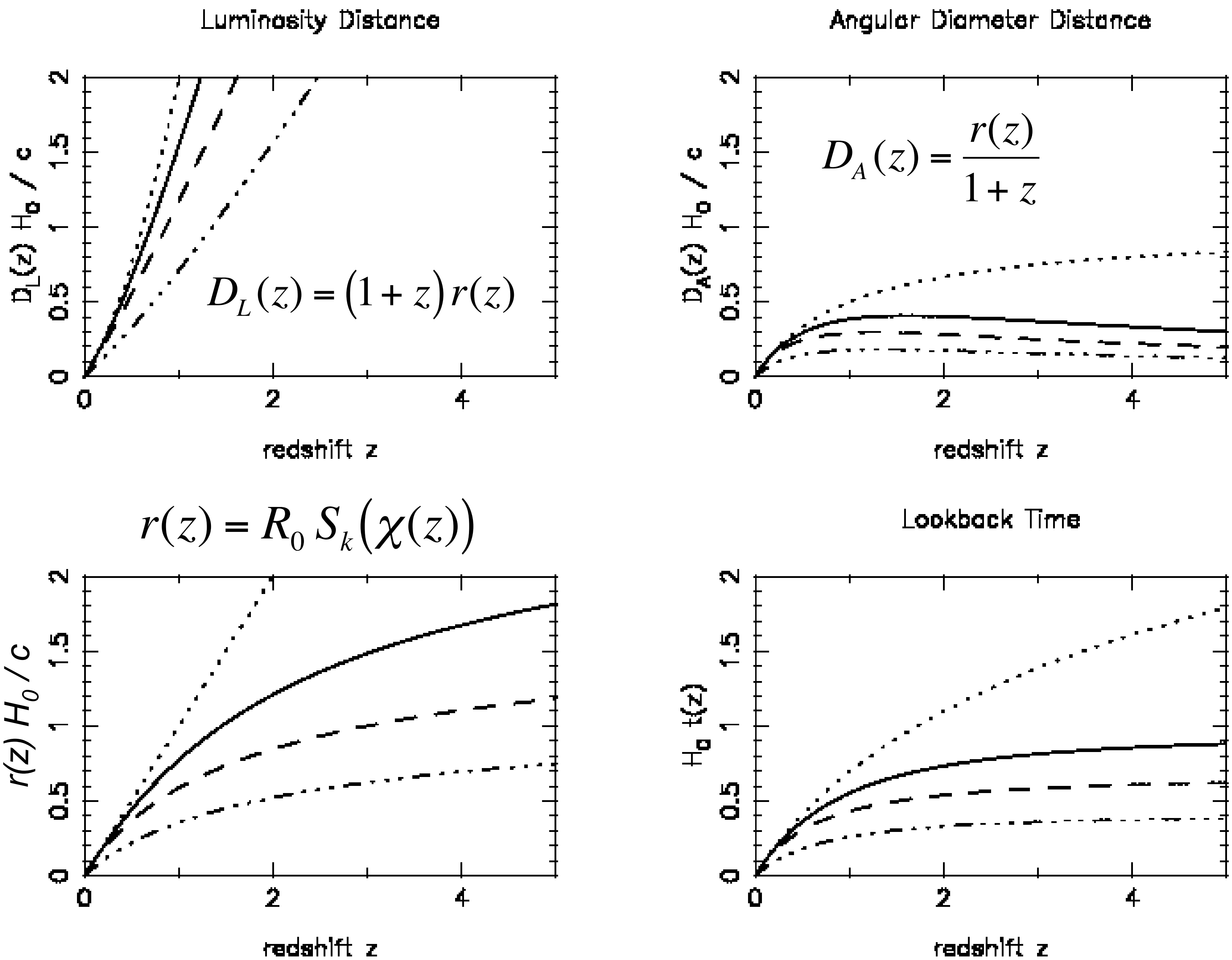
circumferencial distance: $r_0 = R_0\; S_k(\chi) \quad R_0 = \frac{c}{H_0}\left(\frac{k}{\Omega_0 - 1} \right)^{1/2}$

angular diameter distance: $D_A = r_0 \,/\, (1+z)$

luminosity distance: $D_L = (1+z)\; r_0$

Angular Diameter Distance




Luminosity Distance
$D_L(z) = (1+z)\, r(z)$
$D_L(z)\, H_0 / c$
redshift z
Angular Diameter Distance
$D_A(z) = \frac{r(z)}{1+z}$
$D_A(z)\, H_0 / c$
redshift z
$r(z) = R_0\, S_k(\chi(z))$
$r(z)\, H_0 / c$
redshift z
Lookback Time
$H_0\, t(z)$
redshift z

"Concordance Model" Parameters

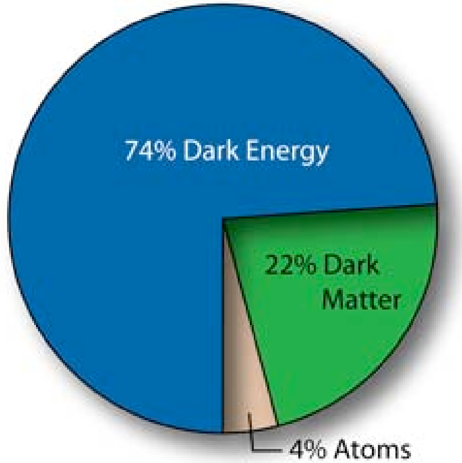
$$H_0 \equiv 100\, h\, \frac{\text{km/s}}{\text{Mpc}} \approx 70 \frac{\text{km/s}}{\text{Mpc}} \qquad h \approx 0.7$$

$$\Omega_R \approx 4.2 \times 10^{-5} h^{-2} \approx 8.4 \times 10^{-5}\ (CMB\ photons + neutrinos)$$

$$\Omega_B \sim 0.02\, h^{-2} \sim 0.04 \quad (baryons)$$

$$\Omega_M \sim 0.3 \quad (Dark\ Matter)$$

$$\Omega_\Lambda \sim 0.7 \quad (Dark\ Energy)$$



$$\Omega_0 \equiv \Omega_R + \Omega_M + \Omega_\Lambda = 1.0 \quad \rightarrow \quad Flat\ Geometry$$

Our (Crazy?) Universe

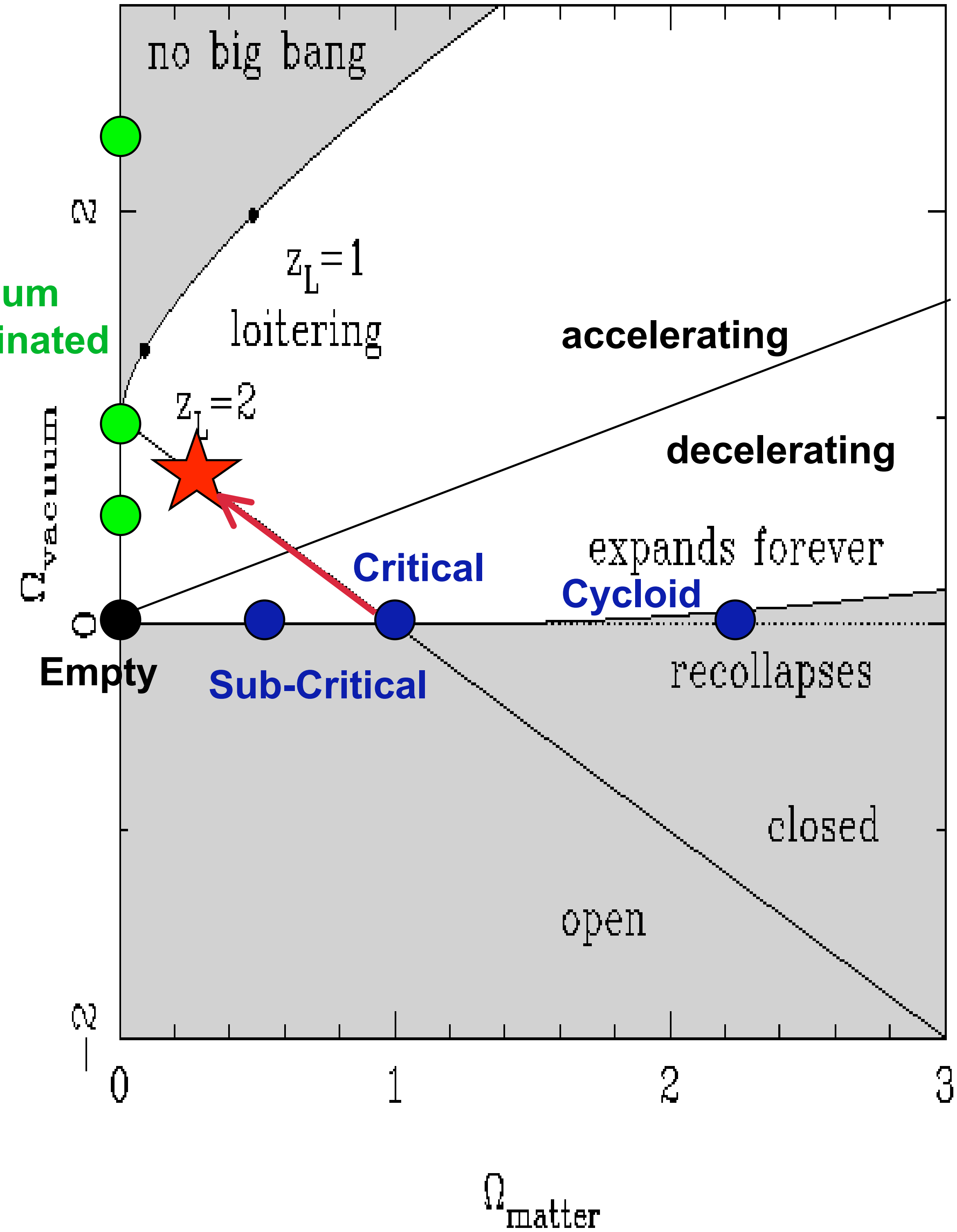
$$H_0 \approx 70 \frac{\text{km/s}}{\text{Mpc}}$$

$$\Omega_M \sim 0.3$$

$$\Omega_\Lambda \sim 0.7$$

$$\Omega_R \sim 8 \times 10^{-5}$$

$$\Omega_0 = 1.0$$



Evolution of Ω

Density at a past/future epoch in units of $\rho_c = 3\,H_0^{\,2}/8\pi\,G$

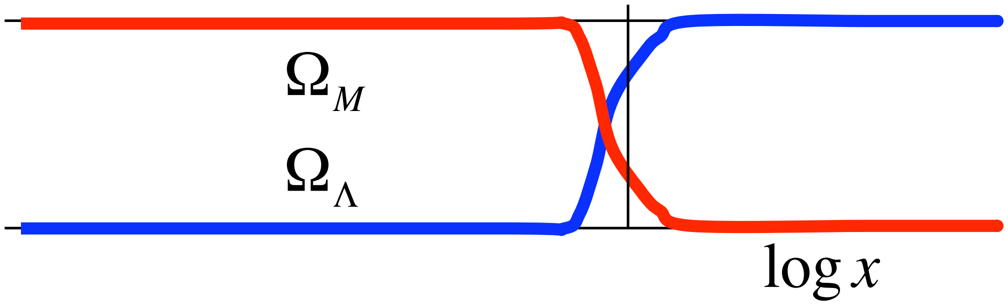
$$\Omega \equiv \frac{\rho}{\rho_c} = \sum_w \Omega_w x^{3(1+w)} = \Omega_R x^4 + \Omega_M x^3 + \Omega_\Lambda \qquad x \equiv 1+z = R_0/R$$

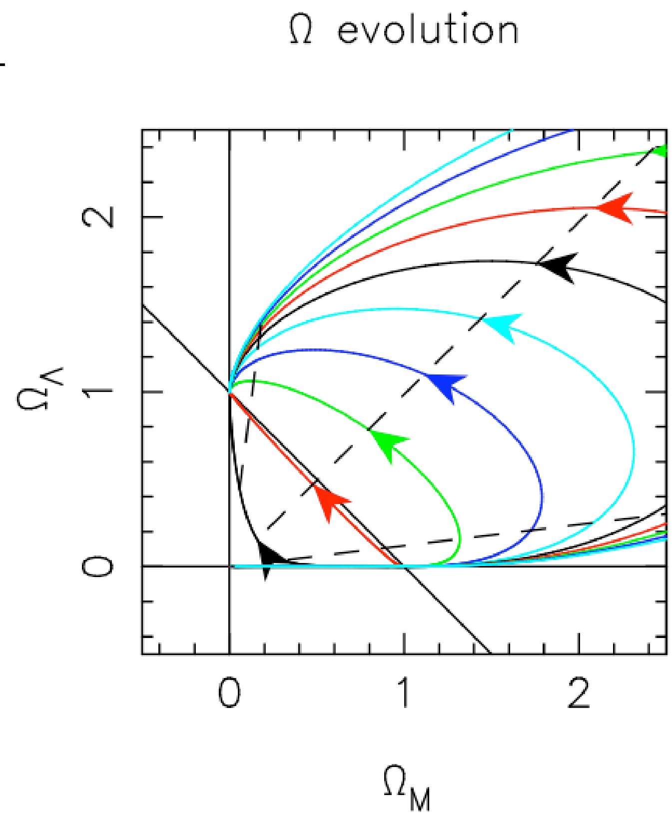
in units of critical density $3\,H^2/8\pi\,G$ at the past/future epoch:

$$\Omega_M(x) = \frac{H_0^{\,2}}{H^2}\Omega_M x^3 = \frac{\Omega_M x^3}{\Omega_M x^3 + \Omega_\Lambda + (1-\Omega_0)x^2}$$

$$\Omega_\Lambda(x) = \frac{H_0^{\,2}}{H^2}\Omega_\Lambda = \frac{\Omega_\Lambda}{\Omega_M x^3 + \Omega_\Lambda + (1-\Omega_0)x^2}$$

Do we live at a special time ?

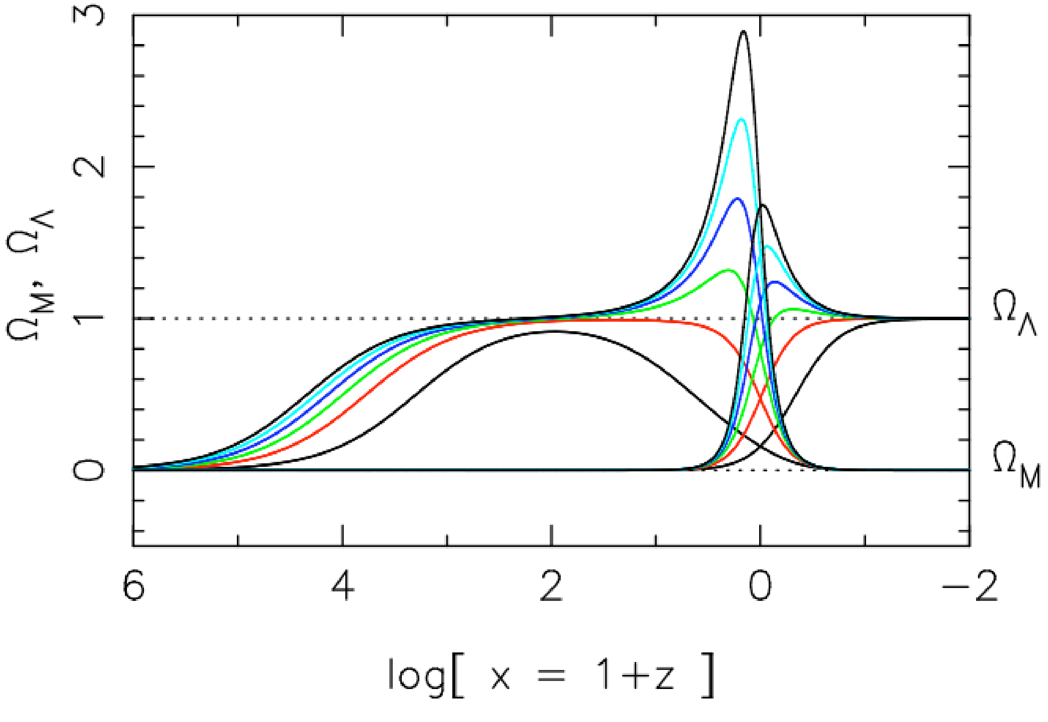


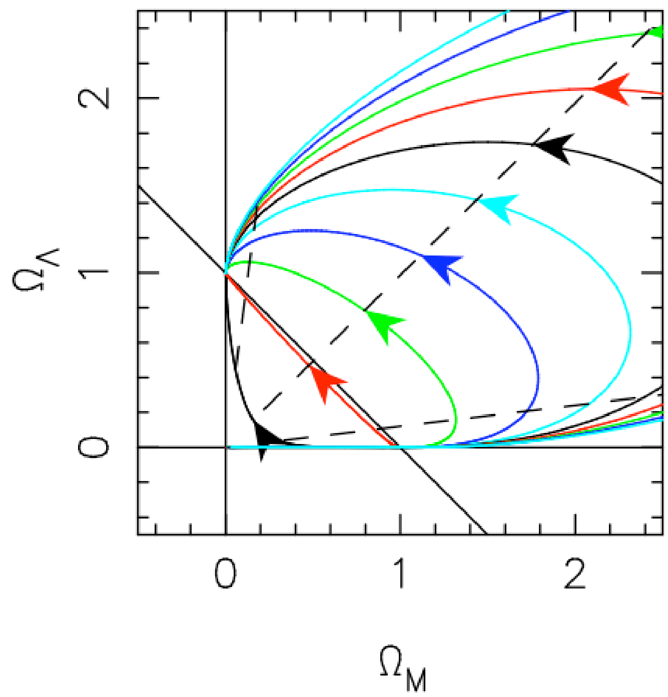


Evolution of Ω

$$\Omega_M(x) = \frac{\Omega_M x^3}{\Omega_R x^4 + \Omega_M x^3 + \Omega_\Lambda + (1-\Omega_0)x^2}$$

$$\Omega_\Lambda(x) = \frac{\Omega_\Lambda}{\Omega_R x^4 + \Omega_M x^3 + \Omega_\Lambda + (1-\Omega_0)x^2}$$





"Concordance" Model

Three main constraints:

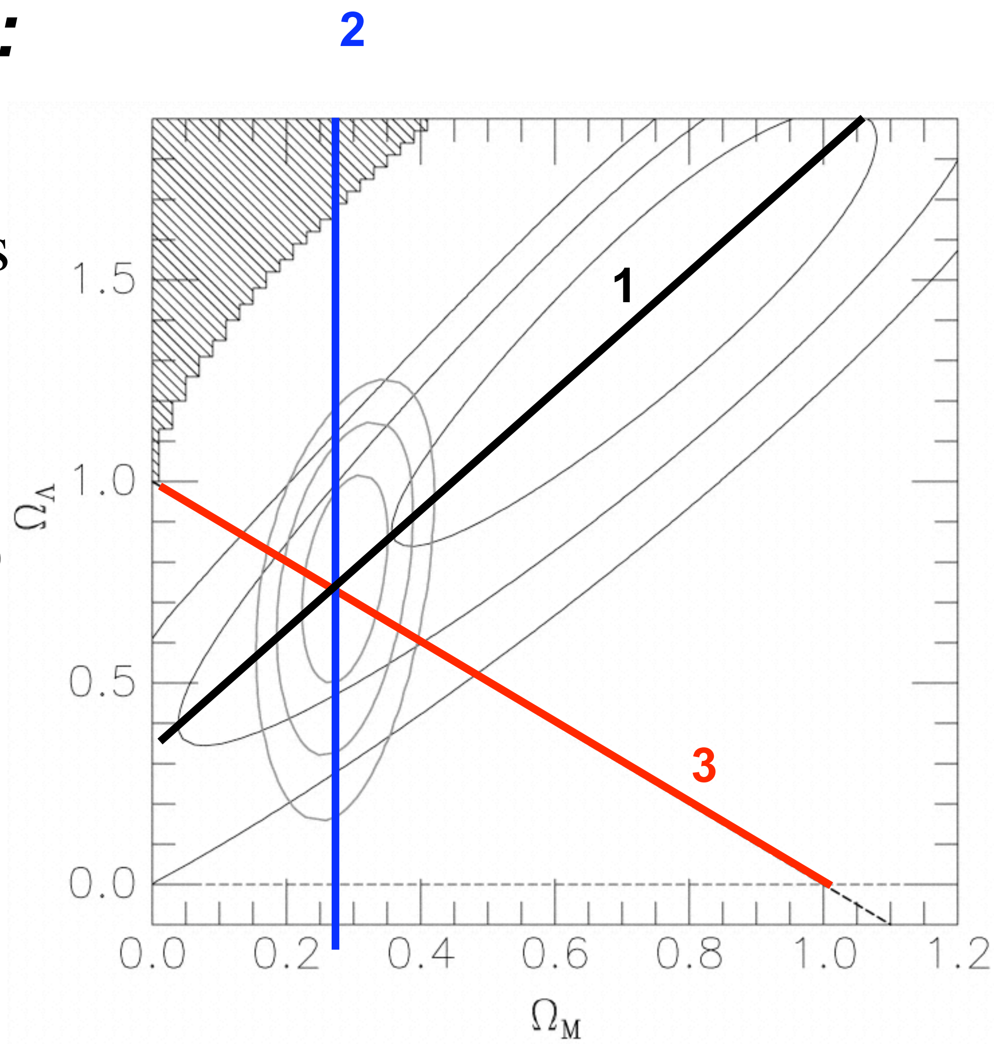
1. Supernova Hubble Diagram
2. Galaxy Counts + M/L ratios

   $\Omega_M \sim 0.3$

3. Flat Geometry

   (inflation, CMB fluctuations)

   $\Omega_0 = \Omega_M + \Omega_\Lambda = 1$



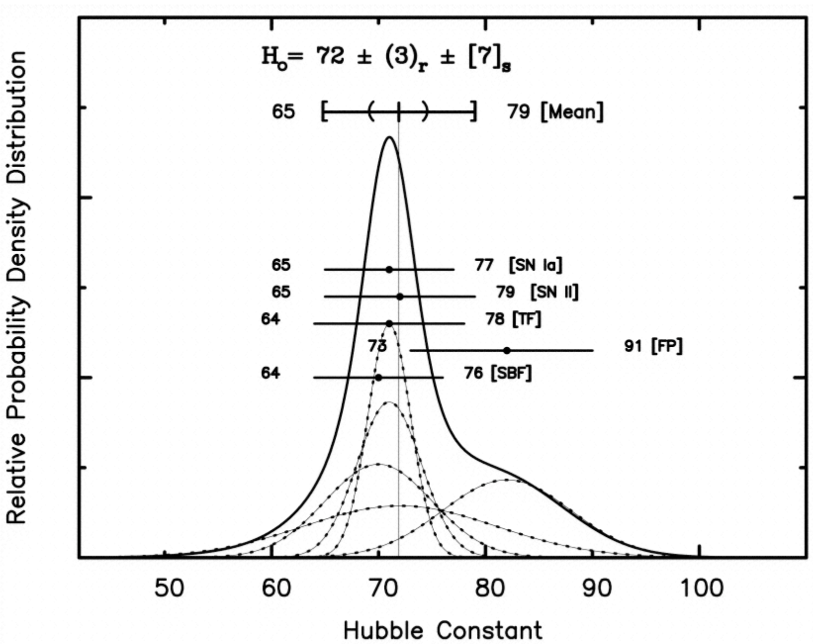
concordance model

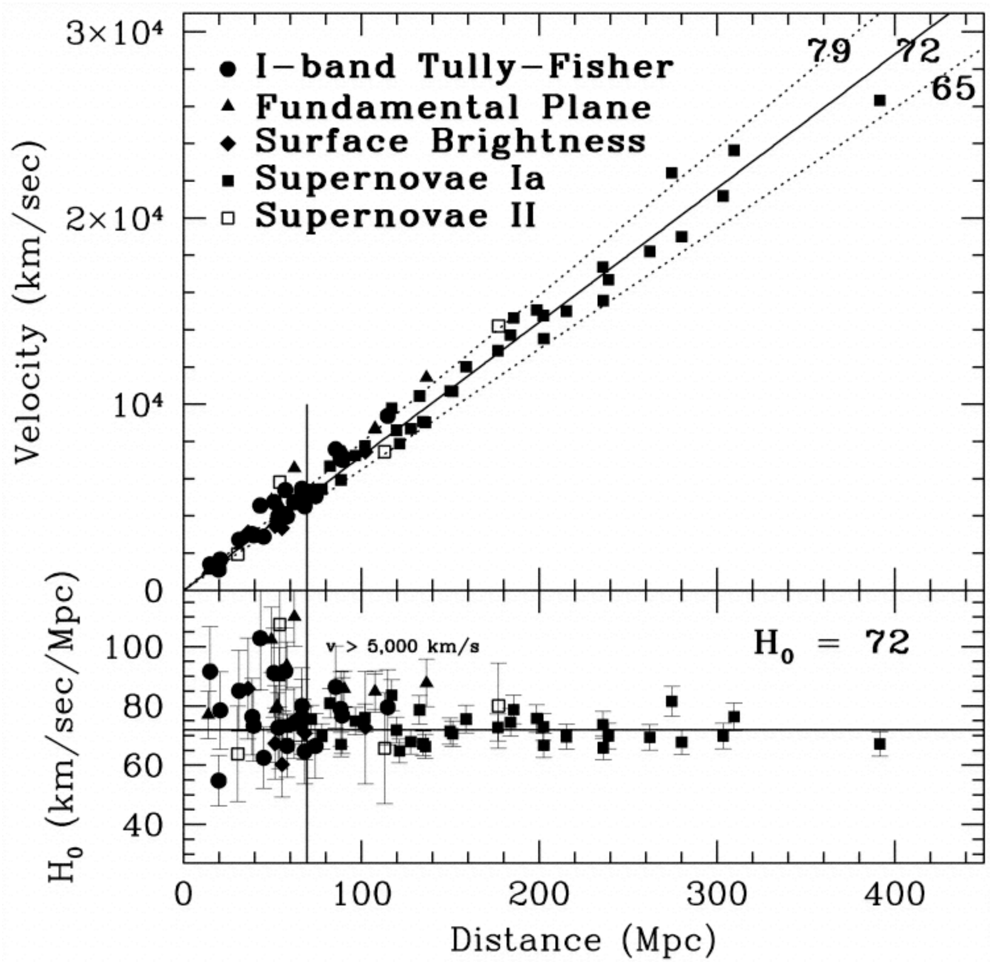
$H_0 \approx 72 \quad \Omega_M \approx 0.3 \quad \Omega_\Lambda \approx 0.7$

HST Key Project

$$H_0 \approx 72 \pm 3 \pm 7 \quad \mathrm{km\ s^{-1}\ Mpc^{-1}}$$

Freedman, et al.
2001 ApJ 553, 47.





Hubble time and radius

Hubble constant :

$$H \equiv \frac{\dot{R}}{R} \qquad H_0 \equiv \left(\frac{\dot{R}}{R}\right)_0 = 100\, h \,\frac{\text{km/s}}{\text{Mpc}} \approx 70\, \frac{\text{km/s}}{\text{Mpc}}$$

Hubble time :

$$t_H \equiv \frac{1}{H_0} = 10^{10}\, h^{-1}\ \text{yr} \approx 14 \times 10^9\ \text{yr}$$

$\sim$ age of Universe

Hubble radius :

$$R_H \equiv \frac{c}{H_0} = 3000\ h^{-1}\ \text{Mpc} \approx 4 \times 10^9 \text{pc}$$

= distance light travels in a Hubble time.

$\sim$ distance to the Horizon.

Age vs Hubble time

$$H = \frac{\dot{R}}{R} \qquad H = \left.\frac{\dot{R}}{R}\right|_{t=t_0}$$

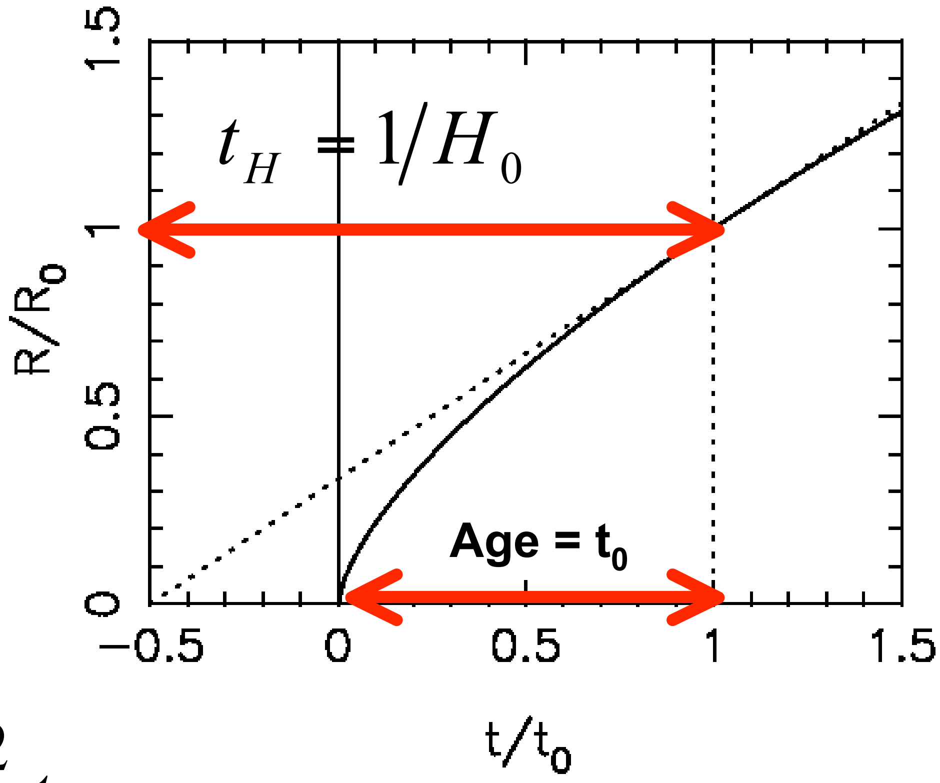
deceleration decreases age

acceleration increases age

e.g. matter dominated

$$R \propto t^{2/3} \quad \Rightarrow \quad \dot{R} = \frac{2}{3}\frac{R}{t}$$

$$H = \frac{\dot{R}}{R} = \frac{2}{3}\frac{1}{t} \quad \Rightarrow \quad t_0 = \frac{2}{3}\frac{1}{H_0} = \frac{2}{3}t_H$$



Beyond H_0

- Globular cluster ages: $t < t_0$ --> acceleration
- Radio jet lengths: $D_A(z)$ --> deceleration
- Hi-Redshift Supernovae: $D_L(z)$ --> acceleration

- Dark Matter estimates ---> $\Omega_M \sim 0.3$

- Inflation ---> Flat Geometry $\Omega_0 \approx 1.0$
- CMB power spectra

- "Concordance Model"

$$\Omega_M \sim 0.3 \qquad \Omega_\Lambda \sim 0.7$$

$$\Omega_0 = \Omega_M + \Omega_\Lambda \approx 1.0$$

Age Constraints

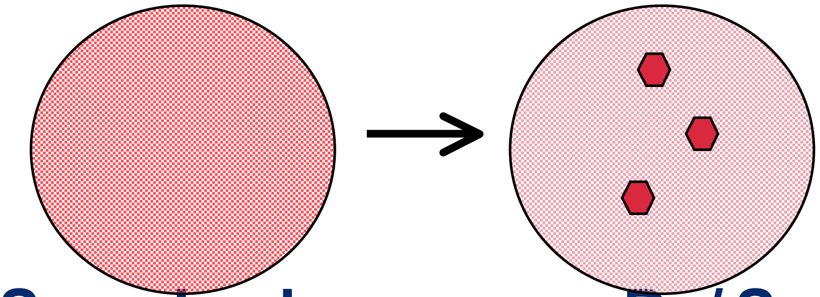
- **Nuclear decay (U, Th -> Pb)**
  - Decay times for (^{232}Th,^{235}U,^{238}U) = (20.3, 1.02, 6.45) Gyr
  - 3.7 Gyr = oldest Earth rocks
  - 4.57 Gyr = meteorites
  - ~10 Gyr = time since supernova produced U, Th
    - (^{235}U / ^{238}U = 1.3 --> 0.33, ^{232}Th / ^{238}U = 1.7 --> 2.3)

- **Stellar evolution**
  - 13-17 Gyr = oldest globular clusters

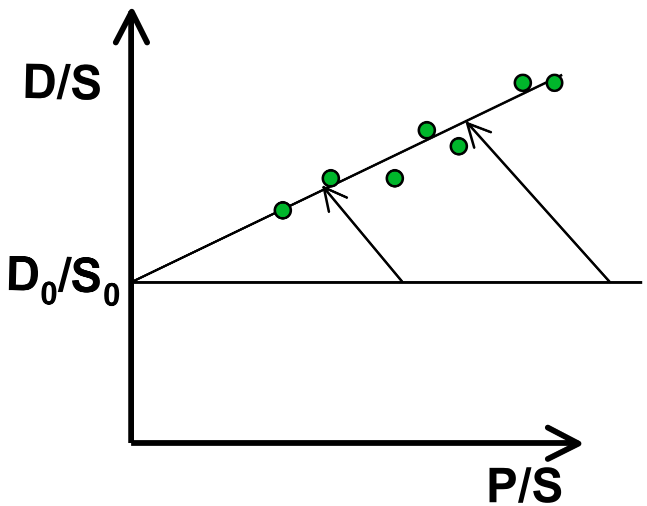
- **White dwarf cooling**
  - ~13 Gyr = coolest white dwarfs in M4

Nuclear Decay Chronology

- **P=parent D=daughter S=stable isotope of D**
- **Chemical fractionation changes P/S but not D/S:**



e.g. concentrate D+S in crystals



- **Samples have same D_0 / S_0**
  various P_0 / S_0
- **P decays to D:**

$$P(t) = P_0\, e^{-t/\tau}$$

$$D(t) = D_0 + P_0\left(1 - e^{-t/\tau}\right) \qquad S(t) = S_0$$

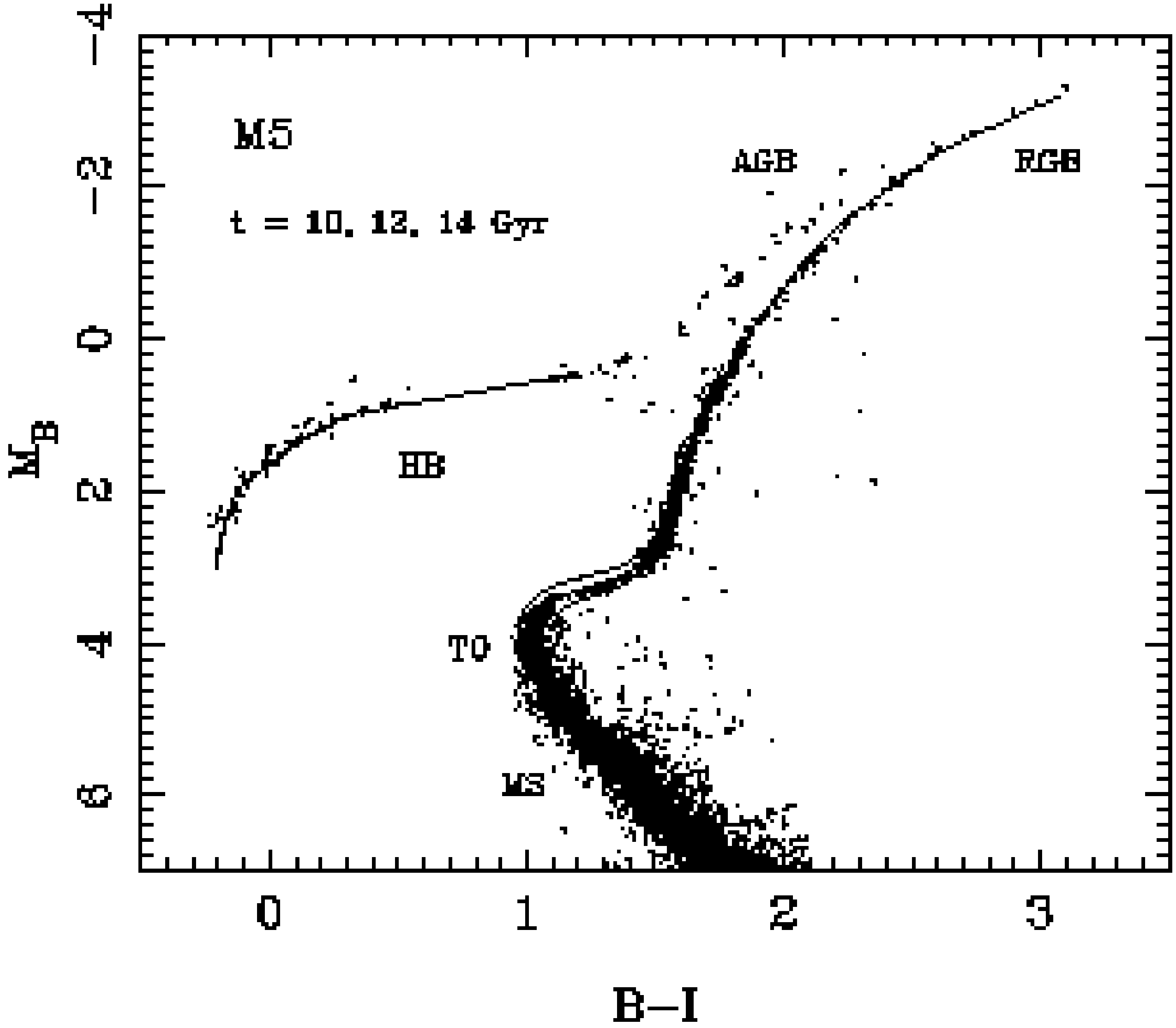
$$= D_0 + P(t)\left(e^{t/\tau} - 1\right)$$

$$\frac{D(t)}{S(t)} = \frac{D_0}{S_0} + \frac{P(t)}{S_0}\left(e^{t/\tau} - 1\right)$$

Observed slope $= \left(e^{t/\tau} - 1\right)$

gives age t/τ , typically to $\sim 1\%$

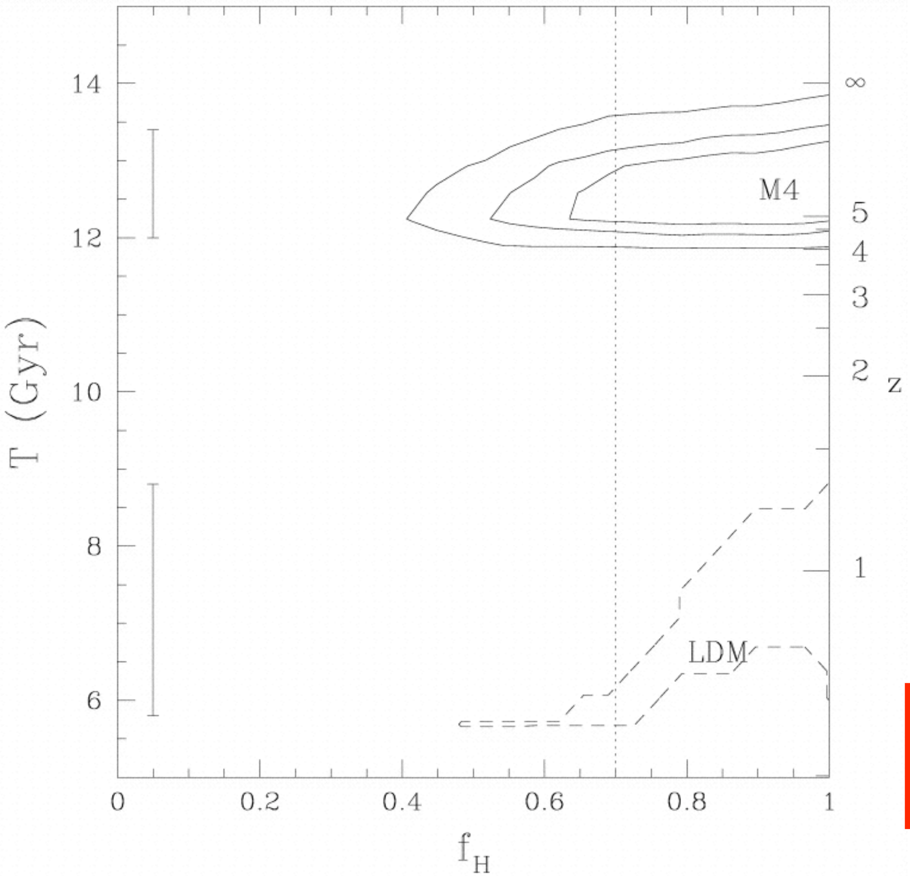
Globular Cluster Ages

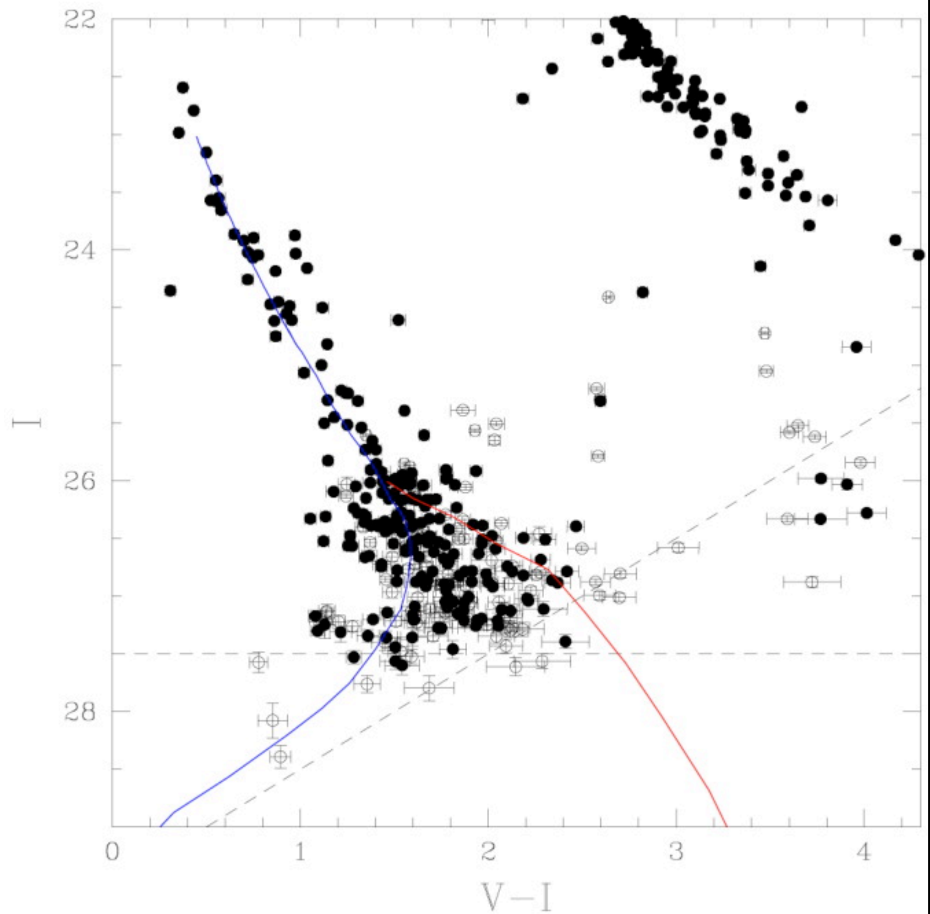


Coolest White Dwarfs

12.7 ± 0.7 Gyr

Hansen et al. 2002 ApJ 574,155





White dwarf cooling ages --> star formation at z > 5.

Cooling times have been <u>measured</u> using "ZZ Ceti" oscillation period changes.

Age Crisis (~1995)

$$H_0\ t_0 = \int_1^\infty \frac{dx}{x\sqrt{\Omega_M\ x^3 + \Omega_\Lambda + (1-\Omega_0)\ x^2}}$$

observations :

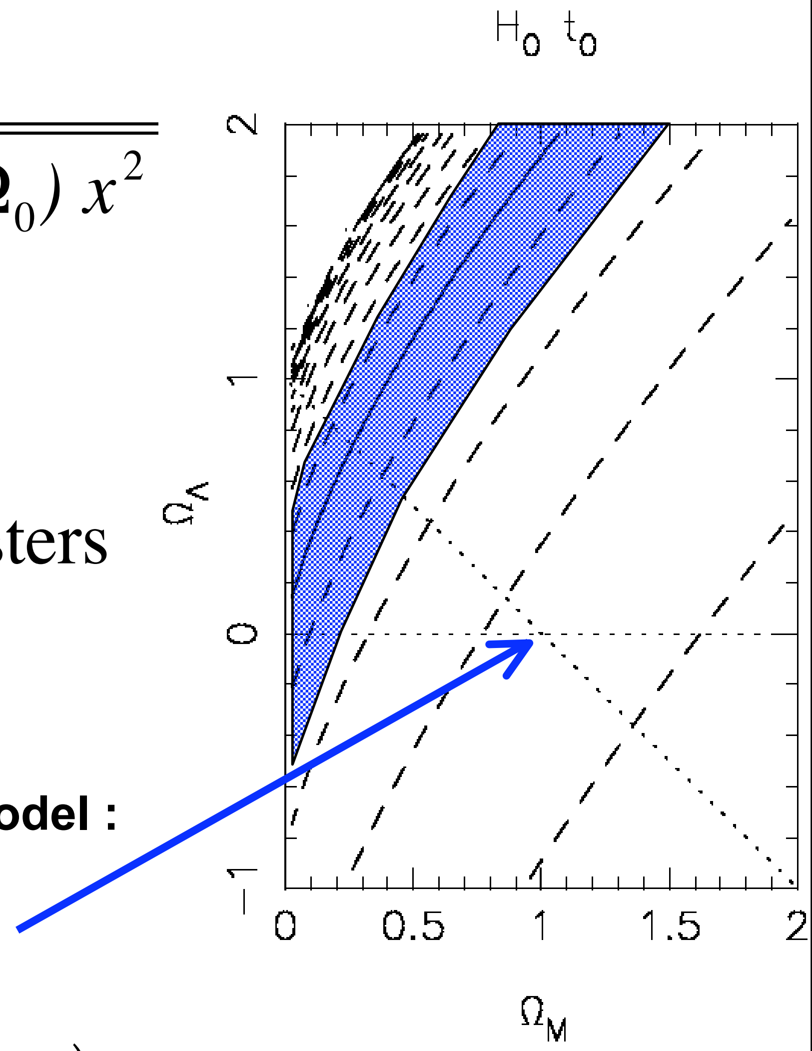
$$H_0 = 72 \pm 8 \text{ km s}^{-1}\text{ Mpc}^{-1}$$

$$t_0 \geq 14 \pm 2 \text{ Gyr} \quad \text{old globular clusters}$$

$$H_0\ t_0 \geq 1.0 \pm 0.15$$

Globular clusters older than the Universe ?
Inconsistent with critical-density matter-only model :

$$H_0\ t_0 = \frac{2}{3} \quad \text{for } \left(\Omega_M, \Omega_\Lambda\right) = (1,0)$$



Strong theoretical prejudice for inflation. $\left(\Omega_0 = 1\right)$
Doubts about stellar evolution therory (e.g. convection).