Lecture 10: Chemical Evolution of Galaxies

Metalicity evolution $Z(t)$ (vs galaxy type)

Processes that alter the metalicity:

1. Type-II SNe enrich the ISM.
2. Low-mass stars form from enriched ISM and "lock-up" metals.
3. Primordial gas falls in from IGM.
4. ISM ejected into IGM. (e.g. SN explosions, galaxy collisions)

Closed Box model: 1 and 2 only.

Accreting Box: 1,2,3. **Leaky Box**: 1,2,4.

Metalicity Evolution: $Z(t)$

M_0 = total mass

$M_G(t)$ = mass of gas in ISM

$M_Z(t)$ = mass of metals in ISM

$M_*(t)$ = mass locked up in stars and remnants

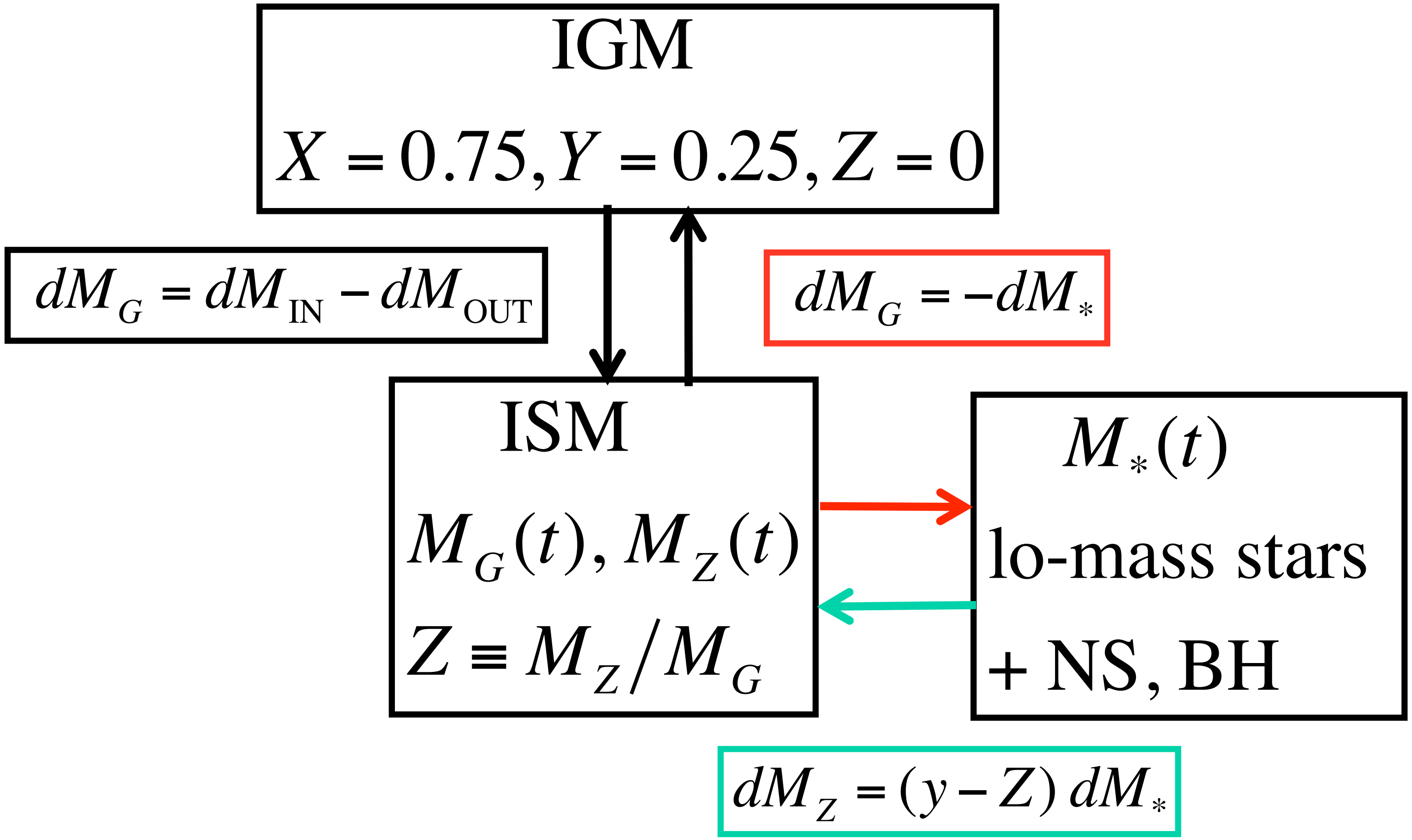
Mass conservation: $M_*(t) = M_0 - M_G(t)$

We also know: $\mu(t) \equiv \frac{M_G(t)}{M_0}$ $\quad \mu(0) = 1$

To derive: $Z(t) \equiv \frac{M_Z(t)}{M_G(t)}$ $\quad Z(0) = 0$

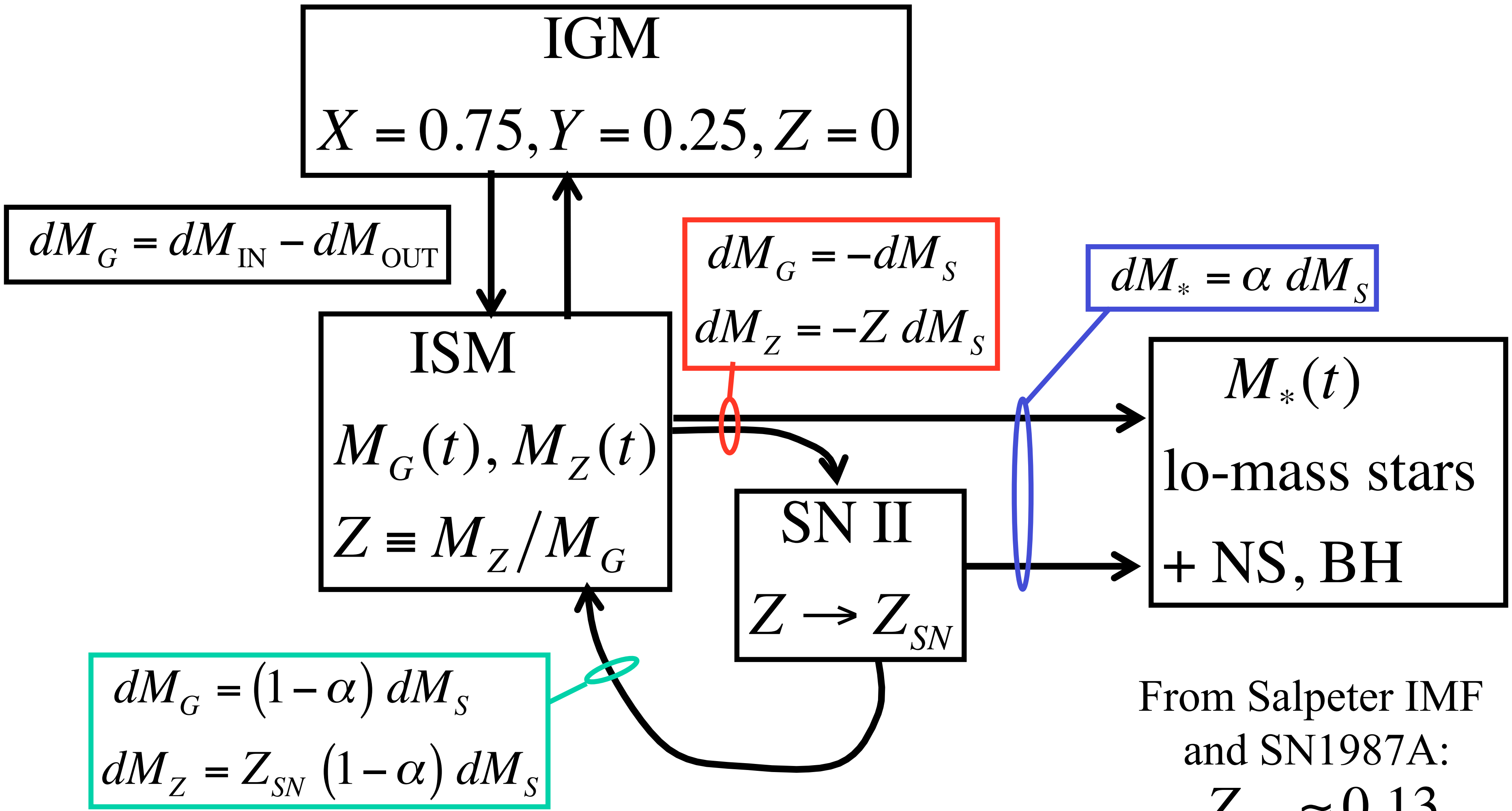
We will find: $Z(\mu(t))$

ISM Recycling Model



Yield: y = mass of metals returned to ISM per mass turned into low-mass stars and remnants

ISM Recycling Model



From Salpeter IMF and SN1987A:

$$Z_{SN} \approx 0.13$$

$$\alpha \approx 0.93$$

The "Yield"

Mass is conserved (gas $\Rightarrow$ stars)

$$dM_G = -\,dM_* = -\alpha\, dM_S$$

Metals are lost to stars, but enriched gas is returned by SNe:

$$dM_Z = \quad -Z\, dM_S \quad + \quad Z_{SN}\left(1-\alpha\right) dM_S$$

$$= \left[\left(-\alpha Z + \alpha Z\right) - Z + Z_{SN}\left(1-\alpha\right)\right]\left(\frac{dM_*}{\alpha}\right)$$

$$= \left[\frac{\left(Z_{SN}-Z\right)\left(1-\alpha\right)}{\alpha} - Z\right] dM_* \qquad \boxed{\equiv \left(y - Z\right) dM_*}$$

$$\boxed{\text{Yield: } y = \left(Z_{SN} - Z\right)\left(\frac{1-\alpha}{\alpha}\right)}$$

From Salpeter IMF and SN1987A:

$$Z_{SN} \approx 0.13$$

$$\alpha \approx 0.93$$

Initial yield: $y_0 = Z_{SN}\left(\frac{1-\alpha}{\alpha}\right) = (0.13)\frac{0.07}{0.93} = 0.01$

Metalicity Evolution Z(t)

Differentiate: $Z(t) \equiv \frac{M_Z(t)}{M_G(t)}$

$$\delta Z = \delta\left(\frac{M_Z}{M_G}\right) = \frac{\delta M_Z}{M_G} + M_Z\,\delta\left(\frac{1}{M_G}\right)$$

$$= \frac{\delta M_Z}{M_G} + M_Z\left(-\frac{\delta M_G}{M_G^2}\right)$$

$$y = 1/x$$

$$\frac{\delta y}{\delta x} = -1/x^2$$

$$\delta y = -\delta x / x^2$$

$$= \frac{1}{M_G}\left(\delta M_Z - \frac{M_Z}{M_G}\,\delta M_G\right)$$

Definition of yield:

$$\delta M_Z = \left(y - Z\right)\delta M_*$$

$$= \left(Z - y\right)\delta M_G$$

$$= \frac{1}{M_G}\left(\left(Z - y\right)\delta M_G - Z\,\delta M_G\right)$$

$$= -y\,\frac{\delta M_G}{M_G} = -y\,\delta\left(\ln(M_G)\right)$$

Closed Box with constant Yield

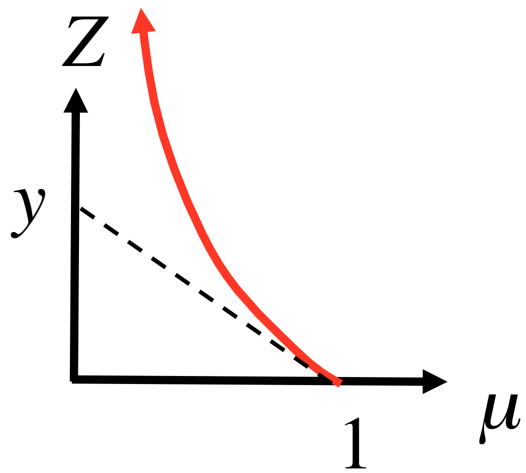
Integrate $\delta Z = -y \dfrac{\delta M_G}{M_G}$ (with y = constant) :

$$Z = -y \ln(M_G) + C$$

At $Z = 0$, $M_G = M_0$:

$$0 = -y \ln(M_0) + C \quad \Rightarrow \quad C = y \ln(M_0)$$

$$\therefore \quad Z = -y \ln\left(\frac{M_G}{M_0}\right) = -y \ln(\mu)$$



Note that as μ => 0, Z => ∞

Impossible ! :-(

What went wrong? Yield is not quite constant.

Closed Box with varying Yield

$$y = (Z_{SN} - Z)\left(\frac{1-\alpha}{\alpha}\right) \qquad y_0 = Z_{SN}\left(\frac{1-\alpha}{\alpha}\right)$$

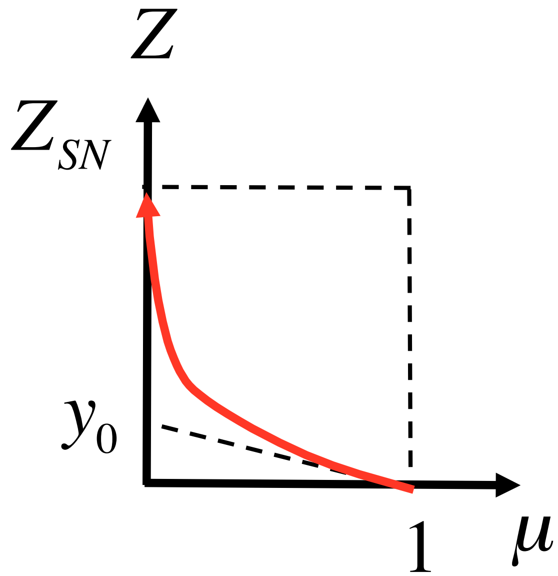
$$\delta Z = -y\,\delta(\ln\mu) = (Z - Z_{SN})\left(\frac{1-\alpha}{\alpha}\right)\delta(\ln\mu)$$

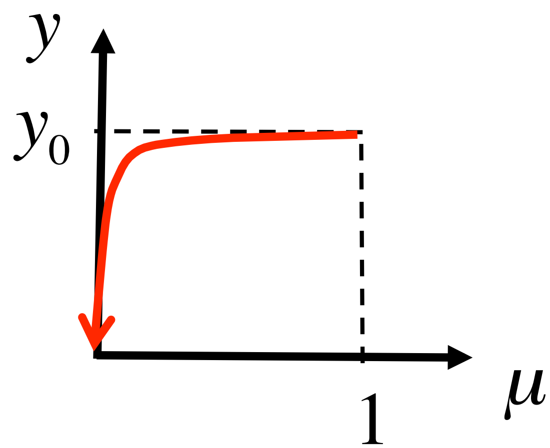
$$\frac{\delta Z}{Z - Z_{SN}} = \left(\frac{1-\alpha}{\alpha}\right)\delta(\ln\mu)$$

$$\ln(Z - Z_{SN}) = \left(\frac{1-\alpha}{\alpha}\right)\ln(\mu) + C$$

$$Z - Z_{SN} = A\,\mu^{\left(\frac{1-\alpha}{\alpha}\right)} \qquad A = e^C = -Z_{SN}$$

$$Z = Z_{SN}\left(1 - \mu^{\left(\frac{1-\alpha}{\alpha}\right)}\right) \qquad y = y_0\,\mu^{\left(\frac{1-\alpha}{\alpha}\right)}$$



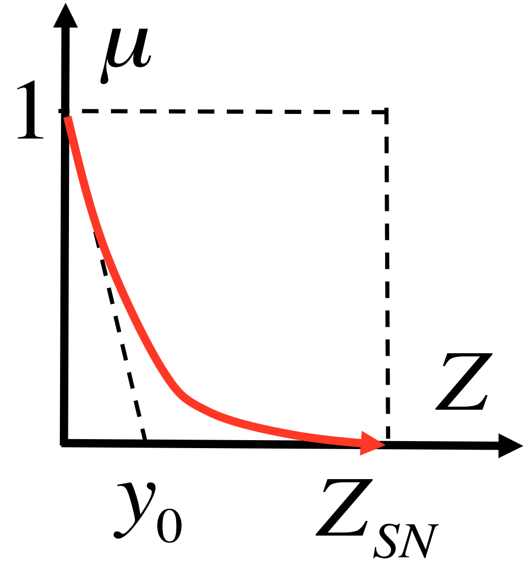


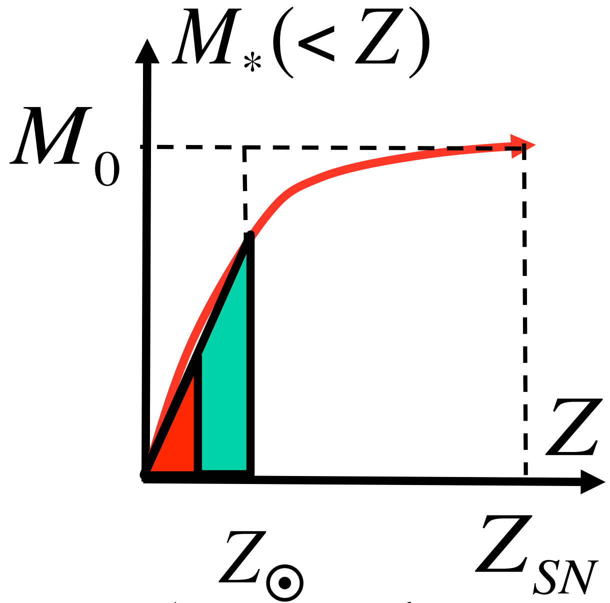
Yield is approx constant: $(1-\alpha) / \alpha \sim 0.075$, $(0.1)^{0.075}=0.98$

But $y \Rightarrow 0$ and $Z \Rightarrow Z_{SN}$ as $\mu \Rightarrow 0$ (from the last SN).

Metalicity distribution of the Stars

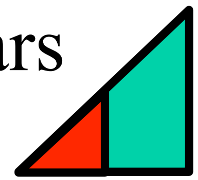
Metalicity of stars = Metalicity of gas from which they formed.





"G dwarf problem": very few sun-like stars (spectral type G) have metalicity below ½ solar.

Closed Box Model FAILS: predicts that > ¼ of stars with $Z < Z_\odot$ have $Z < \frac{1}{2} Z_\odot$



Why are there so few low-metalicity stars?

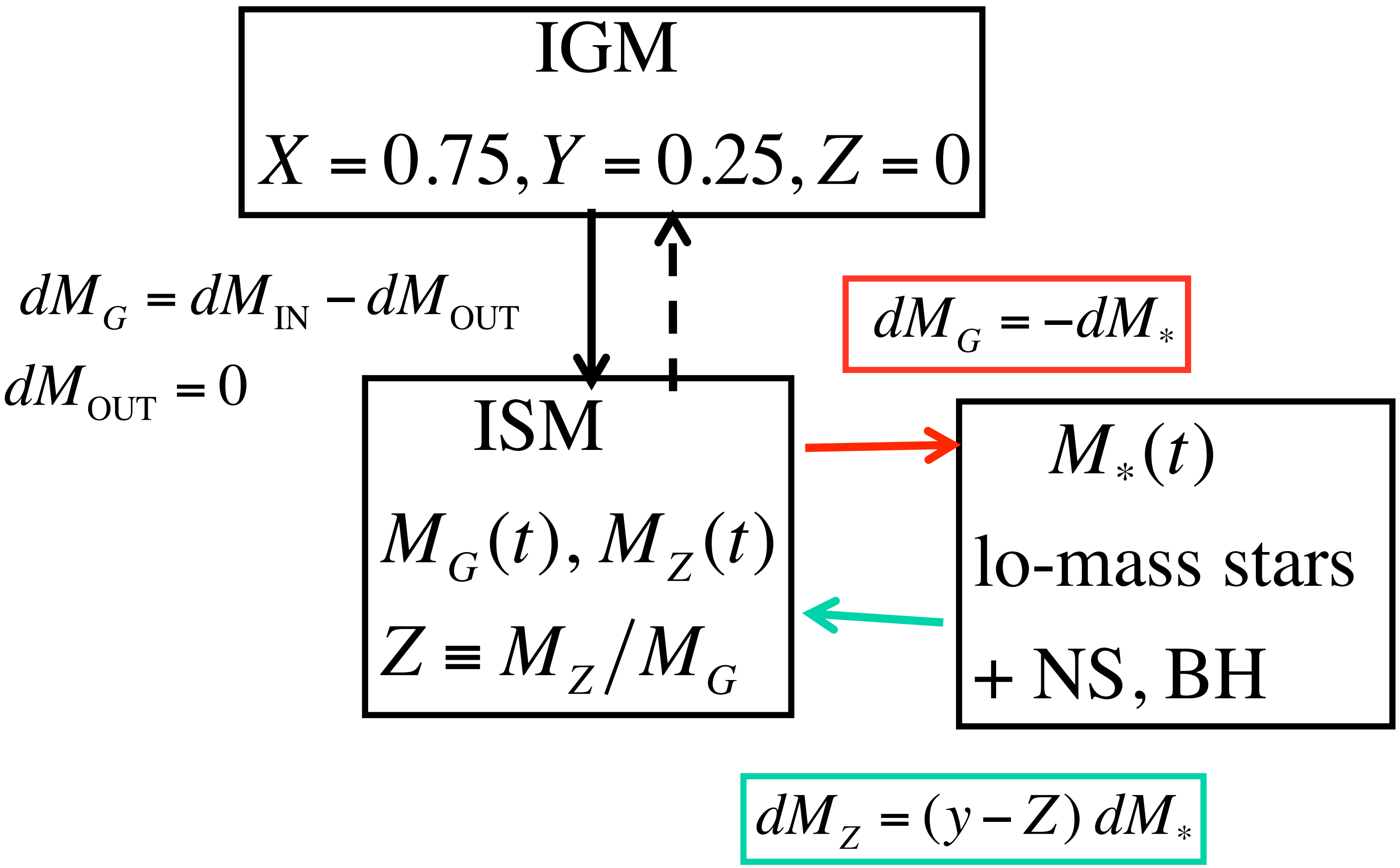
What caused the rapid initial enrichment?

What caused the initial enrichment?

IGM somehow enriched before galaxies form?

First generation (Pop III) $Z = 0$ stars all high mass?

Accreting Box model with low initial gas mass and $Z \Rightarrow y$?



Accreting Box varying Yield

$$\text{Yield} = y \equiv \frac{(Z_{SN} - Z)(1-\alpha)}{\alpha}$$

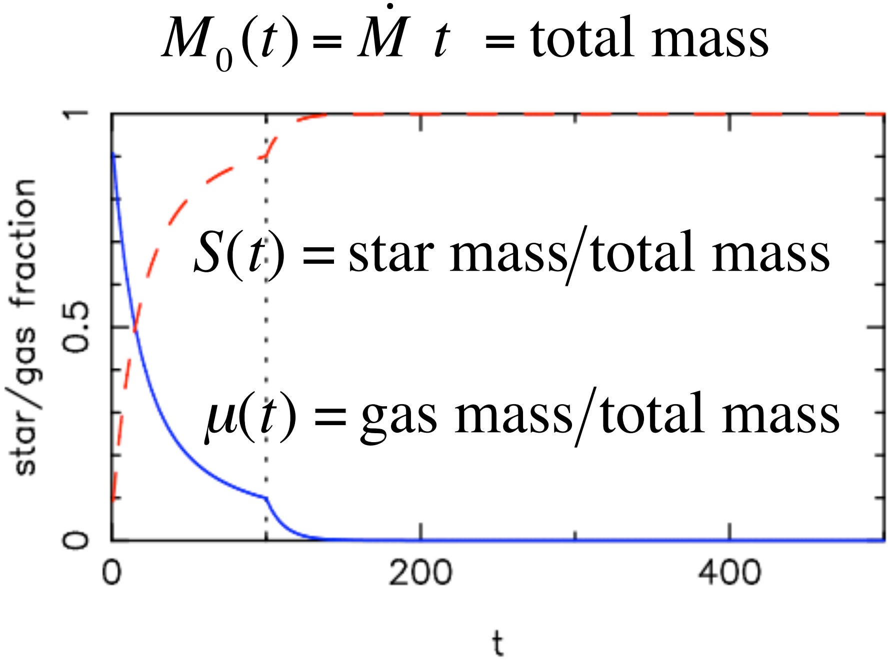
$$\alpha = 0.93 \qquad Z_{SN} = 0.13$$

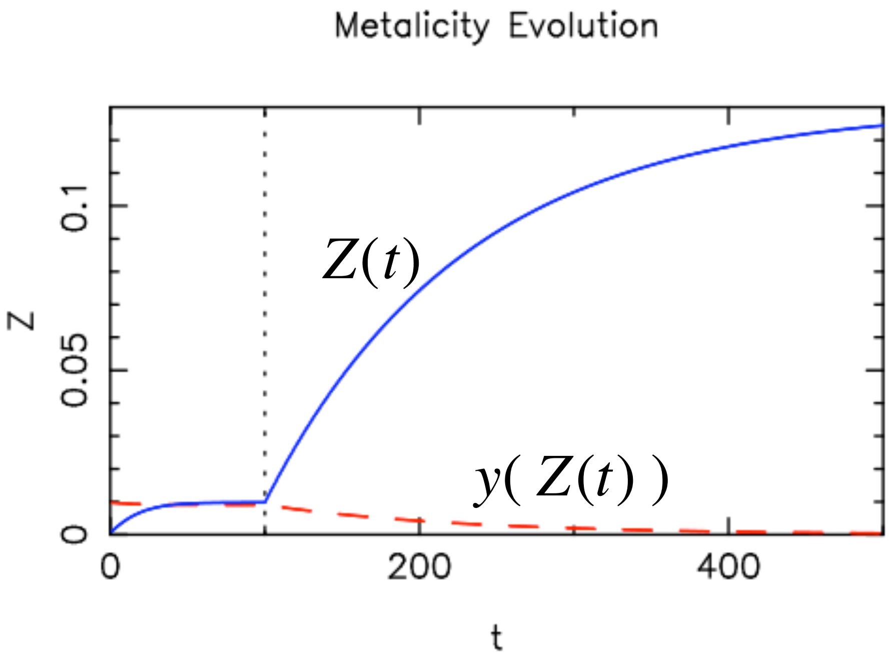
Assume star formation proportional to gas mass (e.g. Elliptical galaxy)

Accrete $Z = 0$ gas, constant dM_{IN} / dt , until $t = 100$.

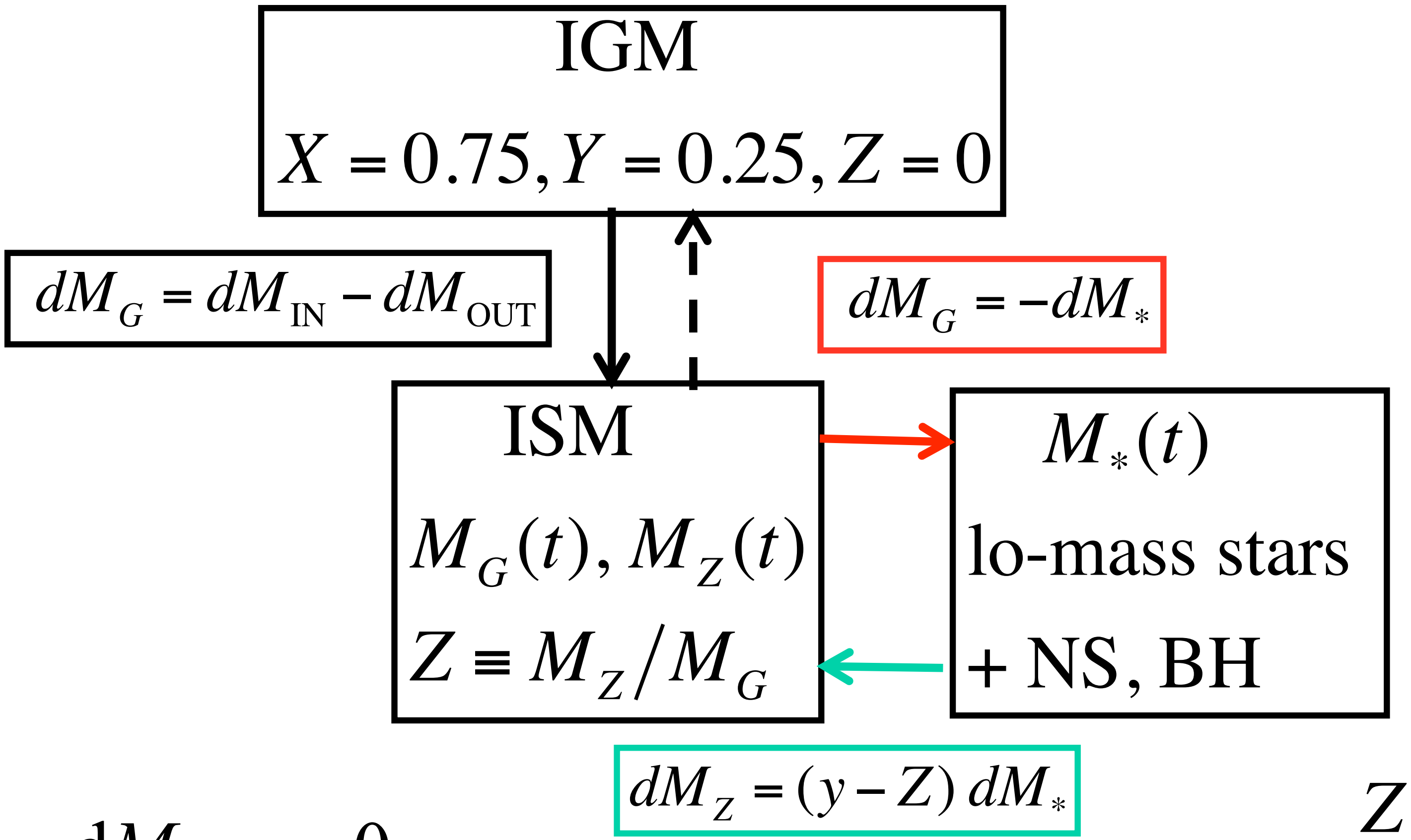
Closed box for $t > 100$.

Result: $Z(t)$ rises until $Z \sim y(Z(t))$





Accreting Box, constant gas mass

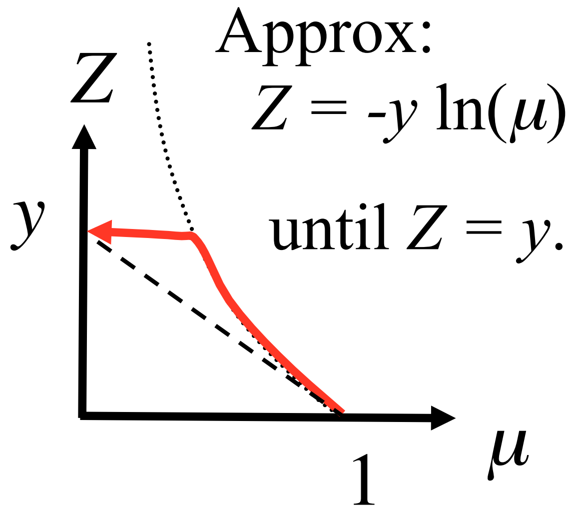


$dM_{OUT} = 0$

$dM_{IN} = dM_*$ => M_G = constant

$dZ = dM_Z / M_G = (y - Z)\, dM_*$

Z stops increasing when Z => y



Insert $\mu(t)$ for each galaxy type into

$$Z(t) = -y \ln\left(\mu(t)\right) \quad \text{for } Z < y$$

Ellipticals:

$$\mu(t) = e^{\left(-t/t_*\right)}$$

$$Z(t) = -y \ln\left(e^{-t/t_*}\right) = y\frac{t}{t_*} \quad \text{for} \quad Z \le y$$

$$Z(t) = y \quad \text{otherwise}$$

Spirals:

$$\mu(\mathrm{t}) = 1 - \frac{\alpha \dot{M} t}{M_0}$$

$$Z(t) = -y \ln\left(1 - \frac{\alpha \dot{M} t}{M_0}\right) \quad \text{for} \quad Z \le y$$

$$Z(t) = \mathrm{y} \quad \text{otherwise}$$

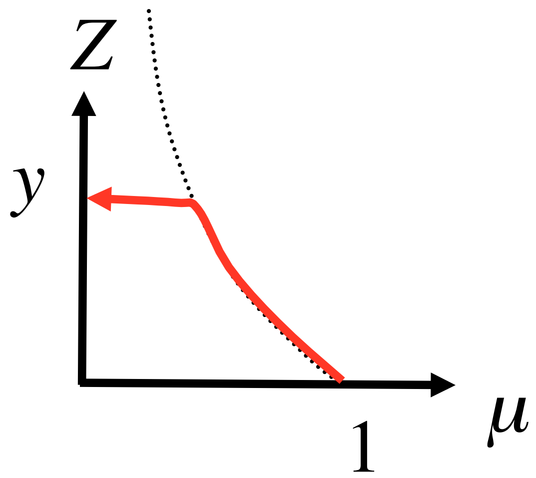
Irregulars:

$$\langle \mu(t) \rangle = f\,\frac{\alpha M t}{M_0}$$

$$Z(t) = -y \ln\left(1 - f\,\frac{\alpha \dot{M} t}{M_0}\right) \quad \text{for} \quad Z \le y$$

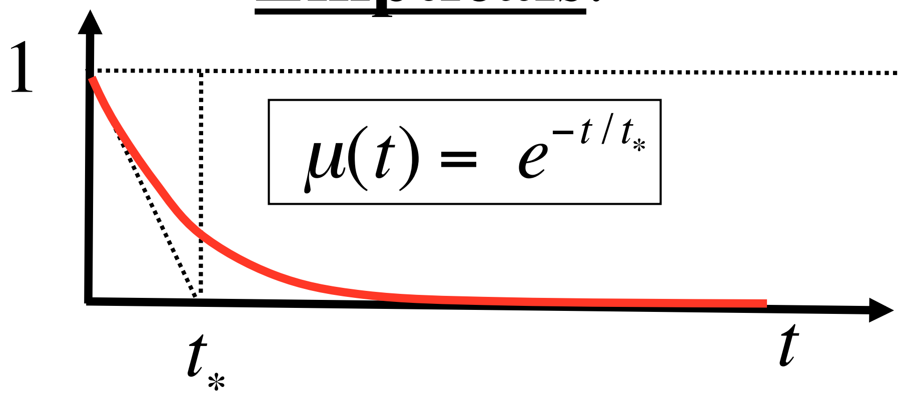
$$Z(t) = \mathrm{y} \quad \text{otherwise}$$

$$Z(t) = -y\ln\left(\mu(t)\right) \quad \text{for } Z < y$$

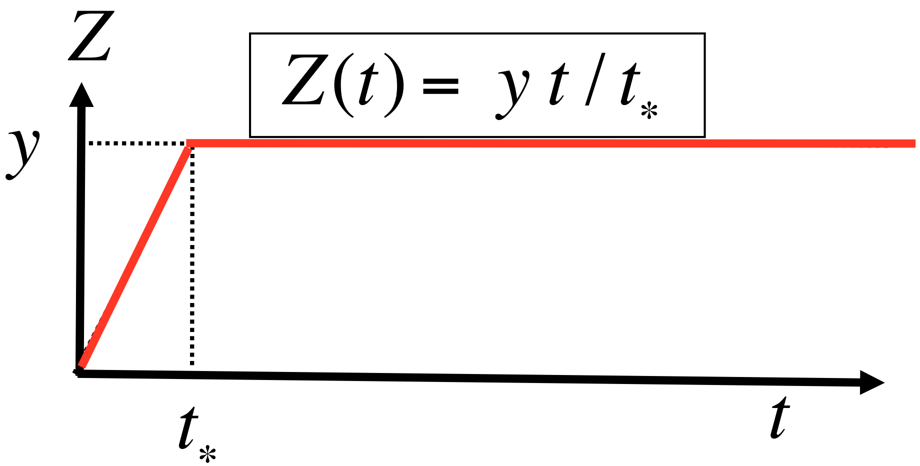


Ellipticals:

$$\mu(t) = e^{-t/t_*}$$



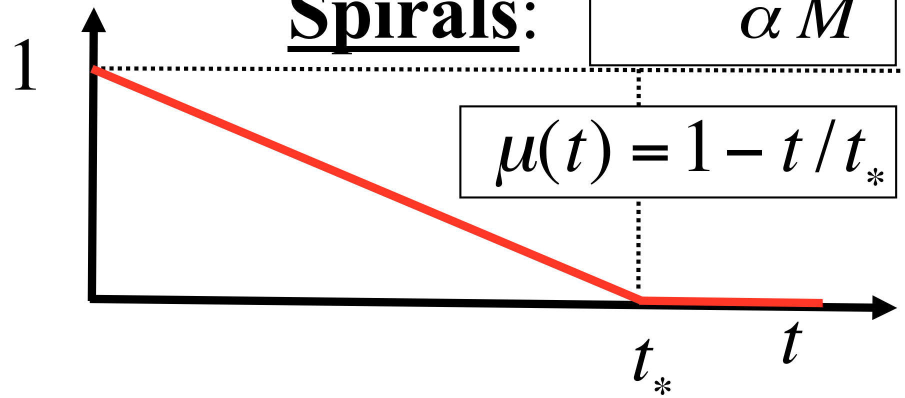
$$Z(t) = y\,t/t_*$$



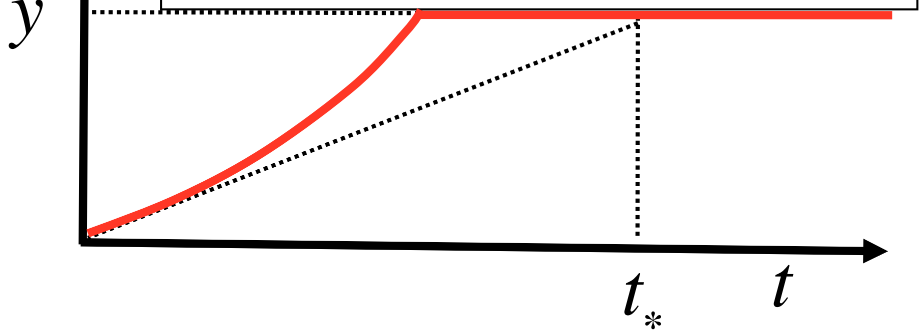
Spirals:

$$t_* = \frac{M_0}{\alpha \dot{M}}$$

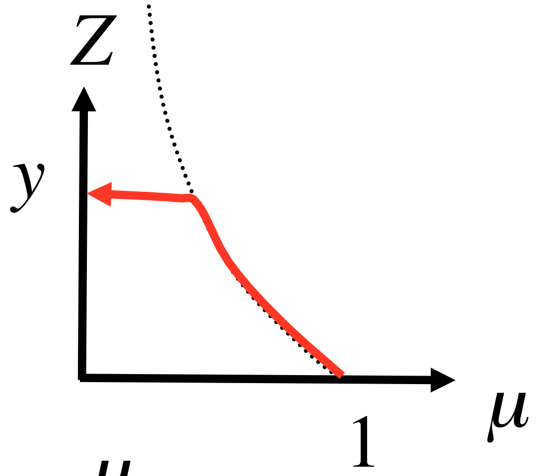
$$\mu(t) = 1 - t/t_*$$



$$Z(t) = -y\ln\left(1 - t/t_*\right)$$

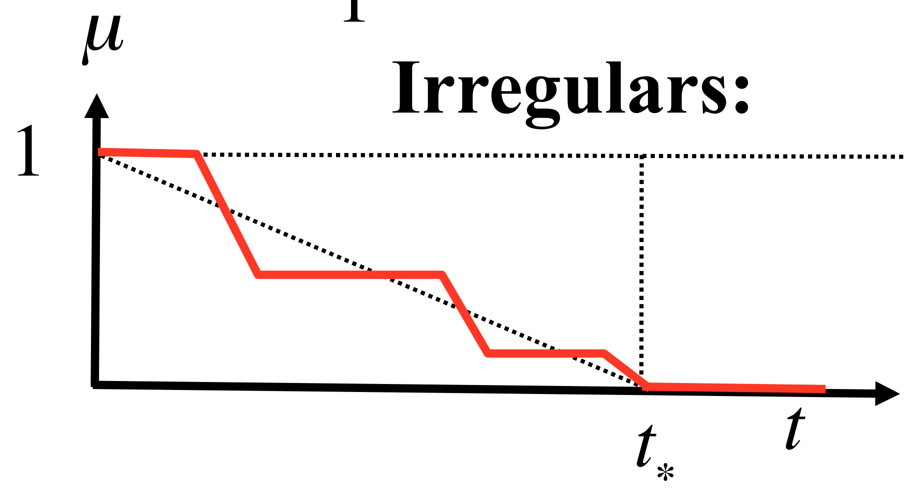


$$Z(t) = -y \ln(\mu(t)) \quad \text{for } Z < y$$

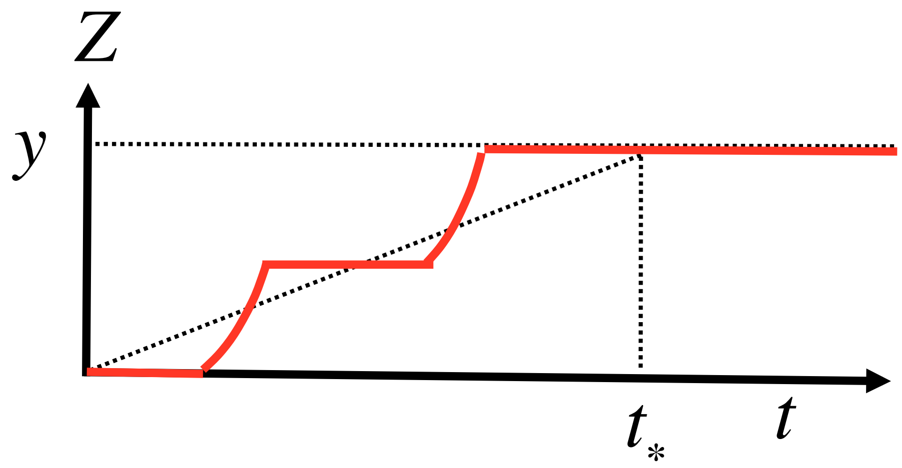


$$\langle \mu(t) \rangle \approx 1 - t/t_*$$

Irregulars:



$$t_* = f \frac{M_0}{\alpha \dot{M}_{\text{burst}}} \qquad f < 1$$



$$Z(t) \approx -y \ln(1 - t/t_*)$$

Initial and Effective Yield

$$y \equiv -\frac{\delta Z}{\delta(\ln \mu)} = (Z_{SN} - Z)\,\frac{1-\alpha}{\alpha}$$

First generation: $Z = 0$ later generations $Z << Z_{SN}$:

From Salpeter IMF and SN 1987A: $\alpha = 0.93$

From SN 1987A: $Z_{SN} = 0.13$

$\Longrightarrow$ Initial yield $= y_0 \approx 0.13\,\frac{0.07}{0.93} = 0.01$

Solar metals : $Z_{\odot} \approx 0.02$

Milky Way has used about 90% of its gas:

$$y_{\text{eff}} \equiv \frac{Z_{\text{obs}}}{\ln(1/\mu)} \sim \frac{0.02}{\ln(10)} = 0.01$$

$$\mu \approx \frac{M_G}{M_* + M_G} \sim 0.1$$

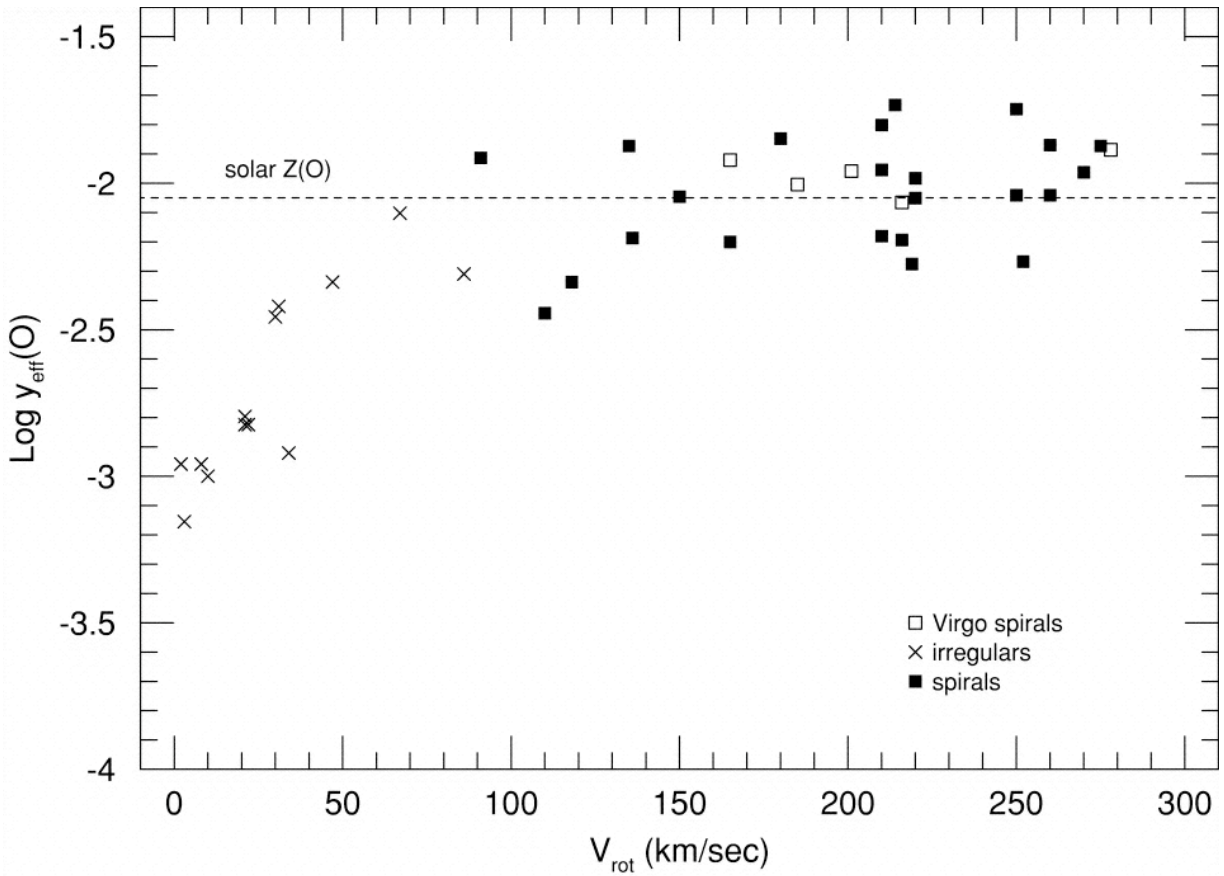
Effective Yield vs Galaxy Mass

Tully-Fisher : $(M / 10^{11} M_\odot) \sim (V_{rot} / 200 \text{ km/s})^4$

Lower yield in small galaxies because SN ejecta excape.

$$y_{eff} \equiv \frac{Z_{obs}}{\ln(1/\mu)}$$

$$\mu \approx \frac{M_G}{M_* + M_G}$$



Garnett 2002

Summary

- Simple models for $Z(\mu(t))$
  (Closed Box, Accreting Box, Leaky Box)
- Yield: y = mass of metals returned to ISM per mass turned into low-mass stars and remnants

  $$Z = -y \ln(\mu) = y \ln(1/\mu)$$
- "G dwarf problem" Closed Box model fails, predicts too many low-Z stars.
- Infall of $Z = 0$ material causes $Z \Rightarrow y$.
- $y_{\rm eff} = Z_{\rm obs} / \ln(1/\mu) \sim 0.01$
- 0.001 for small Galaxies (SN ejecta escape)