Mass transfer in binary systems

- **Mass transfer occurs when**
  - star expands to fill Roche-lobe
    - due to stellar evolution
  - orbit, and thus Roche-lobe, shrinks till $R_* < R_L$
    - due to angular momentum loss
    - e.g. magnetic braking, gravitational radiation

- **Three cases**
  - Case A: mass transfer while donor is on main sequence
  - Case B: donor star is in (or evolving to) Red Giant phase
  - Case C: SuperGiant phase

- **Mass transfer changes mass ratio**
  - changes Roche-lobe sizes
  - can drive further mass transfer

Orbit evolution

$$\text{Kepler} \quad a^3 \propto P^2 \, M \quad \rightarrow \quad 3\,\frac{\dot{a}}{a} = 2\,\frac{\dot{P}}{P} + \frac{\dot{M}}{M}$$

$$M \equiv m_1 + m_2$$

orbital angular momentum

$$J = \frac{m_1 \, m_2}{M} \left(\frac{2\boldsymbol{p}\, a}{P} \right) \left(1 - e^2 \right)^{1/2}$$

$$= m_1 \, m_2 \left(\frac{G \, a \left(1 - e^2 \right)}{M} \right)^{1/2}$$

$$\rightarrow \quad \frac{\dot{J}}{J} = \frac{\dot{m}_1}{m_1} + \frac{\dot{m}_2}{m_2} + \frac{1}{2}\,\frac{\dot{a}}{a} - \frac{1}{2}\,\frac{\dot{M}}{M} - \frac{1}{2}\,\frac{2\,e\,\dot{e}}{1-e^2}$$

$$\boxed{\frac{\dot{a}}{a} = \frac{\dot{M}}{M} + 2\,\frac{\dot{J}}{J} - 2\left(\frac{\dot{m}_1}{m_1} + \frac{\dot{m}_2}{m_2} \right) - \frac{e\,\dot{e}}{1-e^2}}$$

orbit size

$$\frac{\dot{a}}{a} = \frac{\dot{M}}{M} + 2\,\frac{\dot{J}}{J} - 2\left(\frac{\dot{m}_1}{m_1} + \frac{\dot{m}_2}{m_2}\right) - \frac{e\,\dot{e}}{1-e^2}$$

period

$$\frac{\dot{P}}{P} = \frac{3}{2}\,\frac{\dot{a}}{a} - \frac{1}{2}\,\frac{\dot{M}}{M}$$

$$= \frac{\dot{M}}{M} + 3\,\frac{\dot{J}}{J} - 3\left(\frac{\dot{m}_1}{m_1} + \frac{\dot{m}_2}{m_2}\right) - \frac{3}{2}\,\frac{e\,\dot{e}}{1-e^2}$$

Conservative mass exchange

$$\frac{\dot a}{a} = \frac{\dot M}{M} + 2\,\frac{\dot J}{J} - 2\left(\frac{\dot m_1}{m_1} + \frac{\dot m_2}{m_2} \right) - \frac{e\,\dot e}{1-e^2}$$

circular orbit conservative mass exchange :

$$e = 0 \quad \dot e = 0 \qquad \dot M = 0 \quad \dot J = 0 \quad \dot m_1 = -\dot m_2 > 0$$

$$\frac{\dot a}{a} = -2\left(\frac{-\dot m_2}{m_1} + \frac{\dot m_2}{m_2} \right) = 2\,\frac{-\dot m_2}{m_2}\left(1 - \frac{m_2}{m_1} \right) > 0$$

$$\frac{\dot P}{P} = \frac{3}{2}\frac{\dot a}{a} > 0$$

Orbit expands
period increases
if m1>m2

Shrinks if m1<m2

Roche Lobe size

Eggelton 1983 Paczynski $0.1 < q < 0.8$

$$\frac{R_L}{a} \approx \frac{0.49\, q^{2/3}}{0.69\, q^{2/3} + \ln\left(1 + q^{1/3}\right)} \approx 0.462 \left(\frac{q}{1+q}\right)^{1/3}$$

Star 2 fills Roche Lobe :

$$\frac{\dot{R}_L}{R_L} = \frac{\dot{a}}{a} + \frac{1}{3}\frac{\dot{m}_2}{m_2}$$

$$\frac{\dot{a}}{a} = 2\,\frac{-\dot{m}_2}{m_2}\left(1 - \frac{m_2}{m_1}\right)$$

$$= 2\,\frac{-\dot{m}_2}{m_2}\left(\frac{5}{6} - \frac{m_2}{m_1}\right)$$

Critical mass ratio:

Lobe shrinks if q = m2 / m1 > 5 / 6

expands if q < 5 / 6

Timescales

- **Dynamical timescale**
  - timescale for star to establish hydrostatic equilibrium

$$t_{dyn} \sim \left(\frac{R^3}{G\,m} \right)^{1/2} \sim 30\,\text{min} \left(\frac{R}{R_\odot} \right)^{3/2} \left(\frac{m}{M_\odot} \right)^{-1/2}$$

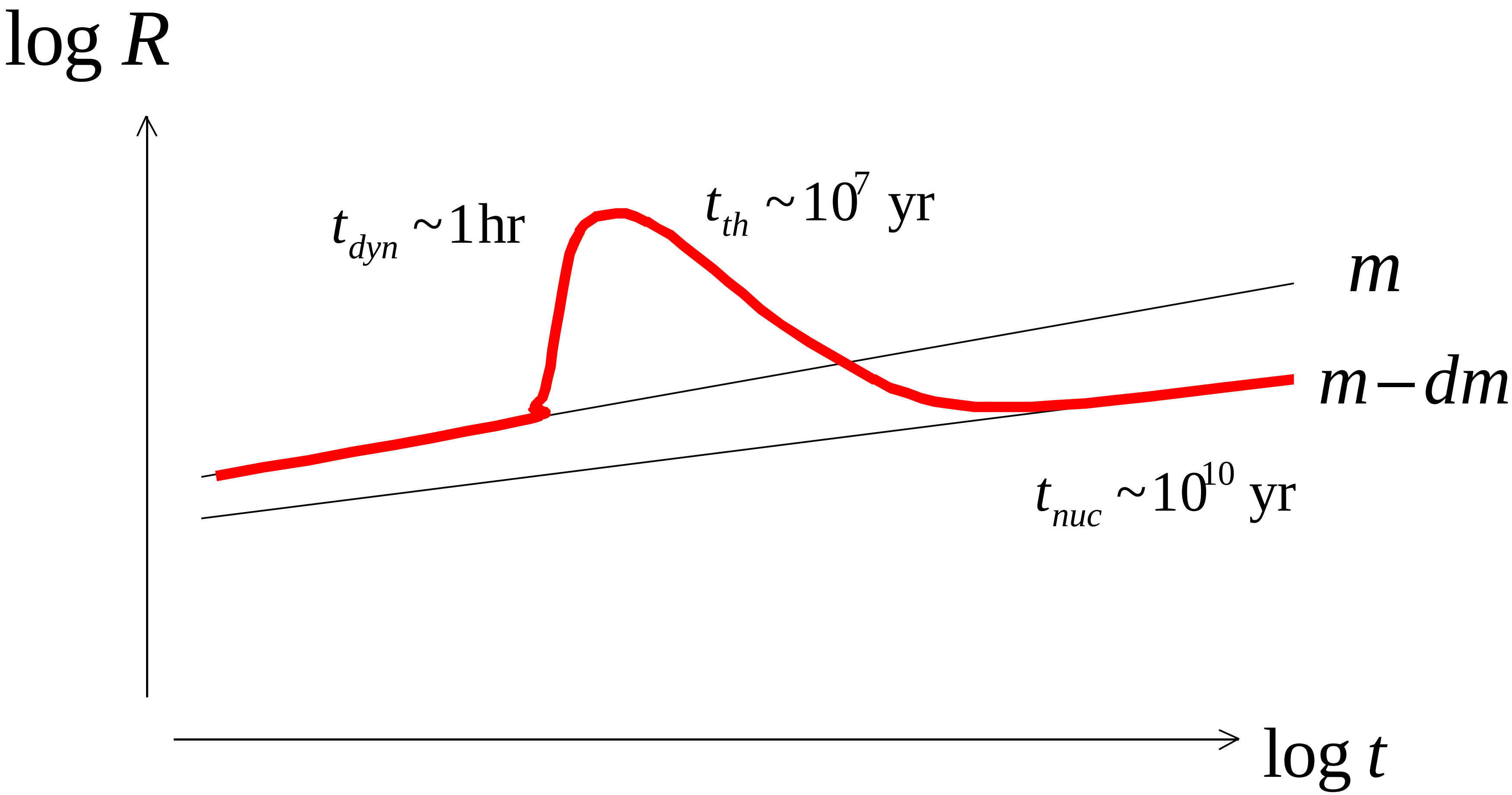
- **Thermal timescale**
  - timescale for star to establish thermal equilibrium

$$t_{th} \sim \frac{G\,m^2}{R\,L} \sim 3\times10^7\ \text{yr} \left(\frac{m}{M_\odot} \right)^2 \left(\frac{R}{R_\odot} \frac{L}{L_\odot} \right)^{-1}$$

- **Nuclear timescale**
  - timescale of energy source of star
  - ie main sequence lifetime

$$t_{nuclear} \approx 7\times10^9\ \text{yr} \frac{m}{M_\odot} \left(\frac{L}{L_\odot} \right)^{-1}$$

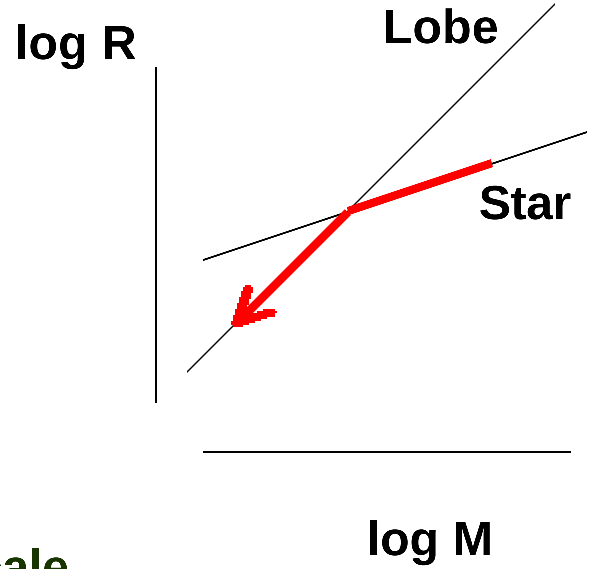
Star reacts to mass loss



Reaction to mass loss

- **Star reacts to mass loss**
  - expands / contracts
  - Roche-lobe also expands or contracts
- **Define**

$$\zeta \equiv \frac{d \ln R}{d \ln m}$$



If $\zeta_L > \zeta_{dyn}$

- star transfers mass on dynamical timescale
  - star stripped down too fast to adjust

If $\zeta_{dyn} > \zeta_L > \zeta_{th}$

- hydrostatic equilirbium easily maintained
  - star transfers mass on thermal timescale

If $\zeta_{dyn}, \zeta_{th} > \zeta_L$

- stable on thermal timescale
  - mass transfer due to stellar evolution, nuclear timescale

Mass loss timescales

$$\text{e.g. for } \dot{M} = 0, \quad V_L \approx 2\left(q - \frac{5}{6} \right)$$

thermal timescale

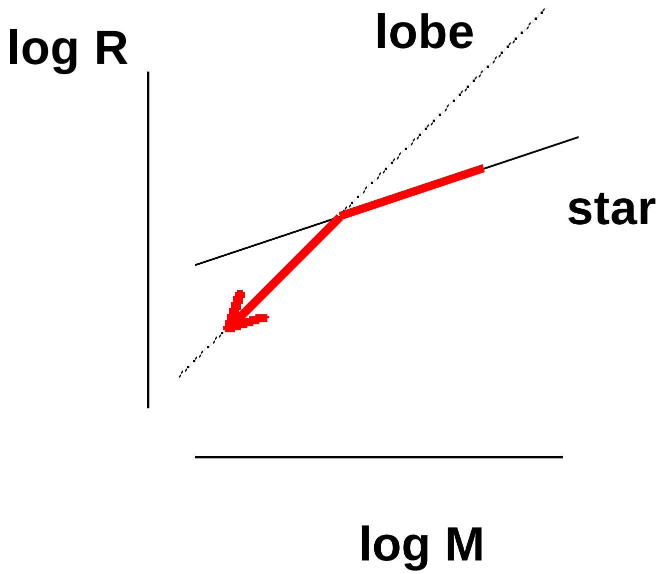
$$V_{dyn} < V_L < V_{th\,L}$$

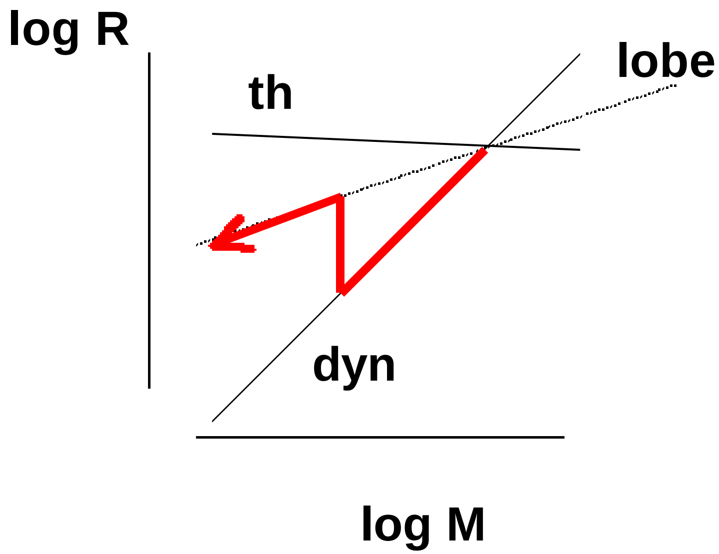
Dynamical timescale

$$V_L > V_{dyn}$$

nuclear timescale

$$V_{dyn}\,, \quad V_{th} < V_L < V_{nuc}$$





q > 5/6 -- unstable mass transfer

- Unstable (runaway) mass transfer
- q > 5/6
- Roche lobe shrinks down around the star, stripping it down.
- Rapid (dynamical)
- violent
- rare because very fast
- must occur (more massive stars evolve first)

- Possible example
- super-soft x-ray binaries:
  - wd primary accreting from main sequence secondary with q > 5/6
  - orbit and Roche lobe shrink from orbital angular momentum loss
  - high accretion rate (e.g. 1e-4 Msun / yr) allows steady burning of H on white dwarf surface

q< 5/6 -- stable mass transfer

- **q < 5/6**
- **conservative mass transfer makes Roche lobe expand.**
  - **cuts off mass transfer**
- **Mass transfer if**
- **1) star expands**
  - **nuclear evolution**
- **2) angular momentum lost**
  - **winds**
  - **gravitational radiation**
- **donor star fills Roche lobe**

- **Case A,B,C mass transfer**
- **Many interacting binaries of this type**

$$\frac{1}{t_{nuc}} \approx \frac{\dot{R}_2}{R_2} = \frac{-\dot{m}_2}{m_2}\left(\frac{5}{3} - 2q\right)$$

$$-\dot{m}_2 = \frac{m_2}{\left(\frac{5}{3} - 2q\right)t_{nuc}}$$

$$\frac{\dot{a}}{a} = \frac{2}{3}\frac{\dot{P}}{P} = \frac{1-q}{\left(\frac{5}{3} - 2q\right)t_{nuc}} > 0$$

Orbit expands

period increases

on nuclear timescale

Angular momentum loss

- **Magnetic braking**
- **gravitational radiation**
- **Stable q < 5/6**
- **donor stays on main-seq**
- **R ~ M**

- **Cataclysmic variables**
  - white dwarf primary
  - late K or M donors

$$\frac{\dot{m}_2}{m_2} \approx \frac{\dot{R}_2}{R_2} = 2\,\frac{\dot{J}}{J} + \frac{-\dot{m}_2}{m_2}\left(\frac{5}{3} - 2\,q\right)$$

$$\dot{J} < 0$$

$$\frac{-\dot{m}_2}{m_2} = \frac{-\dot{J}/J}{\left(\frac{4}{3} - q\right)}$$

$$\frac{\dot{a}}{a} = \frac{2}{3}\,\frac{\dot{P}}{P} = \frac{2\dot{J}/3J}{\left(\frac{4}{3} - q\right)} < 0$$

orbit shrinks

period decreases