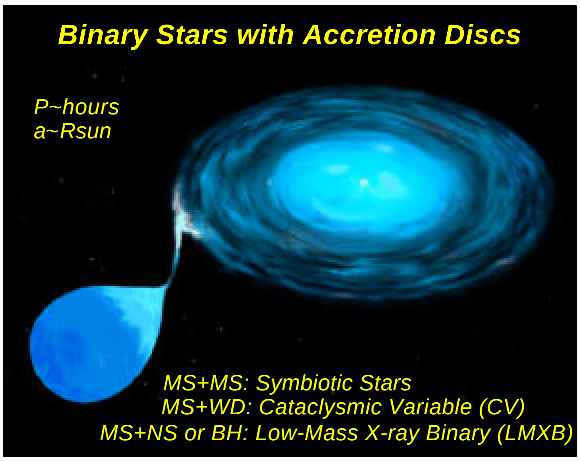


Binary Stars with Accretion Discs

P~hours
a~Rsun

MS+MS: Symbiotic Stars
MS+WD: Cataclysmic Variable (CV)
MS+NS or BH: Low-Mass X-ray Binary (LMXB)

Importance of Accretion Discs

- **Formation of compact objects**
  - friction moves angular momentum outward
  - allows matter to spiral inward
  - build up compact object at centre
- **Generation of light**
  - gravitational potential energy
  - converted by friction to heat
  - radiated as light

3 Classic Papers

- **Black hole accretion discs**

  Shakura, Sunyaev 1973 A&A 24 337

- **Time-dependent discs**

  Pringle, Lynden-Bell 1974 MNRAS 168 603

- **Gas streams**

  Lubow, Shu 1975 ApJ 198 383

Accretion Energy

- **Nuclear energy (H -> He)** **Efficiency:**

$$\frac{\Delta E_{nuc}}{m} \approx 6\times10^{14}\ \frac{\mathrm{J}}{\mathrm{kg}} \qquad h = \frac{\Delta E_{nuc}}{m\,c^2} = 0.7\%$$

- **Accretion energy**

$$\frac{\Delta E_{acc}}{m} \approx \frac{G\,M}{R} \approx 1.7\times10^{11}\left(\frac{M}{M_{sun}}\right)\left(\frac{R}{R_{\rm sun}}\right)^{-1}\frac{\mathrm{J}}{\mathrm{kg}}$$

$$\approx 1.7\times10^{16}\left(\frac{M}{1.4\ M_{sun}}\right)\left(\frac{R}{10\ \mathrm{km}}\right)^{-1}\frac{\mathrm{J}}{\mathrm{kg}}$$

- **more efficient for compact objects**

Efficiency

- **Nuclear energy (H -> He)** **Efficiency:**

$$\Delta E_{nuc} = h\ m\,c^2 \qquad h = 0.7\%$$

- **Accretion efficiency**

$$L \approx \frac{G\,M\ \dot M}{R} = h\ \dot M\ c^2 \qquad h = \frac{G\,M}{R\,c^2} = \frac{R_H}{2\,R} \qquad R_H \approx 3\,\mathrm{km}\left(\frac{M}{M_{sun}}\right)$$

- **Compactness --- M / R**
- **Black hole - smallest stable orbit**
  - non-rotating
  - maximally-rotating

$$R = 6\ R_H \quad h = \frac{1}{12} \approx 8\%$$

$$R = 3\ R_H \quad h = \frac{1}{} \approx 17\%$$

- **White Dwarf**

$$M \approx M_{sun} \quad R \approx 10^4\ \mathrm{km} \quad h \sim 10^{-4}$$

Radiation Opposes Gravity

- **Coulomb attraction ties electrons and protons**
- **gravity pulls in**

$$F_{grav} = -\frac{G\,M\ m}{r^2}$$

- **radiation pushes electrons out**

$$F_{rad} = \frac{L\ s_T}{\quad 2} \qquad L_{acc} = -\frac{G\,M\ \dot M}{}$$

Thompson electron scattering cross-section

$$-21 \quad 2$$

- **Notes:**
  - radiation pressure opposes gravity
  - same R scaling
  - radiation pressure > gravity at high Mdot

Eddington Luminosity

- Photon momentum — Density of photons

$$p_\gamma = \frac{h\nu}{c} \qquad n_\gamma = \frac{L/h\nu}{4\pi R^2 c}$$

- Radiative force (per electron)

$$F_{rad} = n_\gamma\, p_\gamma\, \sigma_T\, c = \frac{L\,\sigma_T}{4\pi R^2 c} \qquad \sigma_T = 6.7\times10^{-25}\,\mathrm{cm}^2$$

- Total force (per electron + proton pair)

$$F_{rad} + F_{grav} = \frac{L\,\sigma_T}{4\pi R^2 c} - \frac{G M \left(m_p + m_e\right)}{R^2}$$

- Eddington Luminosity

$$\boxed{L_{Edd} = \frac{4\pi G M \left(m_p + m_e\right) c}{\sigma_T} = 1.3\times10^{38}\left(\frac{M}{M_{sun}}\right)\mathrm{erg\,s^{-1}}}$$

Eddington Accretion Rate

- Eddington Luminosity $L_{Edd} = \dfrac{4\pi G M m_p c}{\sigma_T}$
- Accretion Luminosity $L_{acc} = \dfrac{G M \dot{M}}{R}$
- Eddington Accretion Rate $\dfrac{L_{acc}}{L_{Edd}} = \dfrac{\sigma_T \dot{M}}{4\pi m_p c R} = \dfrac{\dot{M}}{\dot{M}_{Edd}}$

$$\boxed{\begin{aligned}\dot{M}_{Edd} &= \frac{4\pi m_p c R}{\sigma_T}\\ &\approx 10^{-5}\left(\frac{R}{R_{sun}}\right)\frac{M_{sun}}{\mathrm{yr}}\end{aligned}}$$

	kg/s	M_{sun}/yr
WD	6×10^{19}	10^{-7}
NS	10^{12}	1.4×10^{-12}

Rough Temperatures

- Optically thick
  - Blackbody radiates accretion luminosity

$$\sigma T_b^{\,4} = \frac{L_{acc}}{4\pi R^2} \qquad T_b = \left(\frac{L_{acc}}{4\pi R^2 \sigma}\right)^{1/4} \approx \left(\frac{G M \dot{M}}{4\pi R^3 \sigma}\right)^{1/4}$$

- Optically thin
  - potential energy released
  - = thermal energy of shocked gas

$$\frac{G M m_p}{R} = 2\times\frac{3}{2}\,k\,T_{th} \qquad T_{th} \approx \frac{G M m_p}{3 k R}$$

- Radiation temperature and photon energy

$$T_b < T_{rad} < T_{th} \qquad k\,T_b < h\nu < k\,T_{th}$$

Rough Temperatures

- Neutron Star or Black Hole

$$L_{acc} \sim L_{Edd} \sim 10^{38}\,\mathrm{erg\,s^{-1}}$$
$$10^7\,\mathrm{K} < T_{rad} < 10^{11}\,\mathrm{K}$$
$$1\,\mathrm{keV} < h\nu < 50\,\mathrm{MeV}$$

  - mid to hard X-rays
- White Dwarf

$$L_{acc} \sim 10^{33}\,\mathrm{erg\,s^{-1}}$$
$$6\times10^4\,\mathrm{K} < T_{rad} < 10^9\,\mathrm{K}$$
$$6\,\mathrm{eV} < h\nu < 100\,\mathrm{keV}$$

  - optical - uv - soft X-ray

Roche Lobe Overflow

- Initial stream velocity

$$V_\parallel \sim c_S \sim 10\,\mathrm{km\,s^{-1}}$$

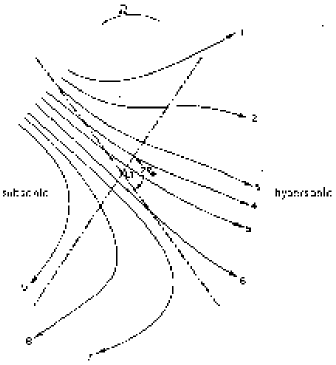
- L1 velocity relative to centre of mass

$$V_\perp = \frac{2\pi}{P} R_{L1} \approx \frac{2\pi a}{P}\left(0.5 - 0.23\log q\right)$$
$$\sim 100\,\mathrm{km\,s^{-1}}\left(\frac{M}{M_{sun}}\right)^{1/3}\left(\frac{P}{\mathrm{day}}\right)^{-1/3}$$

- subsonic --> supersonic transition at nozzle
- ballistic trajectory in Roche potential
  - (neglect pressure forces)

Flow thru the L1 nozzle

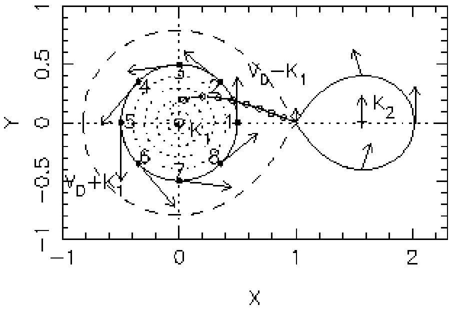
subsonic -> hypersonic



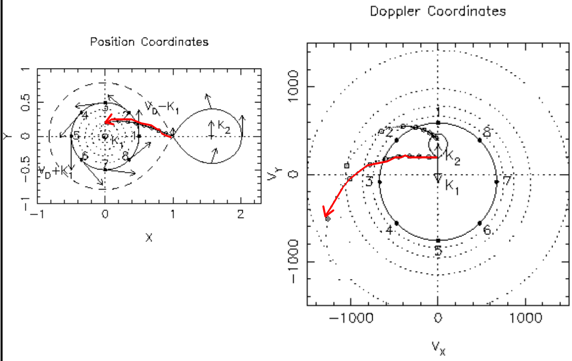
Lubow, Shu 1975 ApJ 198 383

Balistic Stream Trajectory

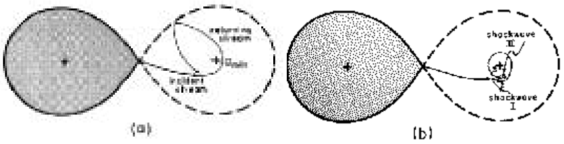
Position Coordinates



Balistic Stream Trajectory

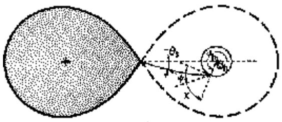


Formation of a Ring



Circularisation Radius

same angular momentum as L1



$$(G\ M\ R_{circ})^{1/2} = \frac{2p}{P} R_{L1}{}^2$$

$$\frac{R_{circ}}{a} = (1+q)\left(\frac{R_{L1}}{a}\right)^4 \approx (1+q)\,(0.5 - 0.23\log q)^4$$

Friction -> Spreading

- **Kepler Velocity**

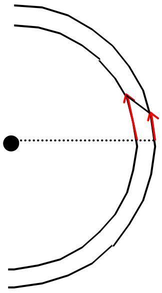
$$V = \sqrt{\frac{G\,M}{R}}$$

- **Differential rotation (Shear)**

$$\Delta V = \frac{d}{dR}\left(\sqrt{\frac{G\,M}{R}}\right)\Delta R$$

$$= \frac{V}{2}\,\frac{\Delta R}{R}$$

- **Friction**
  - **opposes shear**
  - **causes ring to spread**
  - **inward + outward**
  - **diffusion**

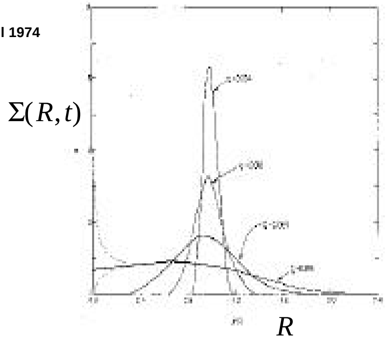


Ring spreads to form a disc

Pringle, Lynden-Bell 1974
MNRAS 168 603

Surface density
evolution

diffusion in radius



Accretion Discs

- **Keplerian Orbits**

$$V_f = V_K = \left(\frac{G\,M}{R}\right)^{1/2} \qquad \Omega \equiv \frac{V_f}{R} = \sqrt{G\,M\ R^3}$$

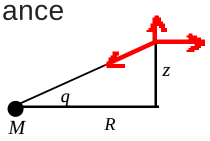
- **Vertical thickness: thin <--> supersonic**

$$\frac{H}{R} \approx \frac{c_S}{V_f}$$

- **sound speed for perfect gas**

$$c_S{}^2 \equiv \overline{dP} = g\,\overline{P} = \overline{g\,k\,T}$$

Force Balance



- **horizontal:**
  - gravity in
  - centrifugal out

$$\frac{V^2}{R} = \frac{G\ M}{R^2 + z^2}\cos q = \frac{G\ M\ R}{\left(R^2 + z^2\right)^{3/2}} \approx \frac{G\ M}{R^2} \qquad (\ z << R\ )$$

- **vertical:**
  - gravity down
  - pressure gradient up

$$\frac{dP}{dz} = -r\ g_z$$

$$g_z = \frac{G\ M}{R^2 + z^2}\sin q = \frac{G\ M\ z}{\left(R^2 + z^2\right)^{3/2}} \approx \frac{G\ M\ z}{R^3}$$

Vertical Hydrostatic Equilibrium

- **Assume vertically isothermal**
- **vertical structure (Gaussian if iso-thermal)**

$$\frac{dP}{dz} = -r\ g_z = -\frac{g\ P}{c_S{}^2}\frac{G\ M\ z}{R^3} = -\frac{P\ z}{H^2}$$

$$\frac{dP}{P} = -\frac{z\ dz}{H^2}$$

$$\ln P = \ln P_0 - \frac{1}{2}\left(\frac{z}{H}\right)^2$$

$$P = P_0 \exp\left\{-\frac{1}{2}\left(\frac{z}{H}\right)^2\right\}$$

$$\boxed{\begin{aligned} H^2 &= \frac{c_S{}^2\ R^3}{g\ G\ M} = \frac{k\ T\ r^3}{m\ m_H\ G\ M} \\ \frac{H^2}{R^2} &= \frac{1}{g}\frac{c_S{}^2}{V_f{}^2} \end{aligned}}$$