ADA 15 - 10am Mon 24 Oct

Cross-Correlation analysis

Introduction to Projects 1 and 2

Cross-correlation

- **Cross-correlation function** (CCF) used to measure the position (and strength) of a feature in the data.
- Pattern *P(x)* matched in width (and shape) to the feature being measured.
- Shift the pattern by an offset *s*, and scale it to fit the data D_i with error bars σ_i measured at positions X_i :

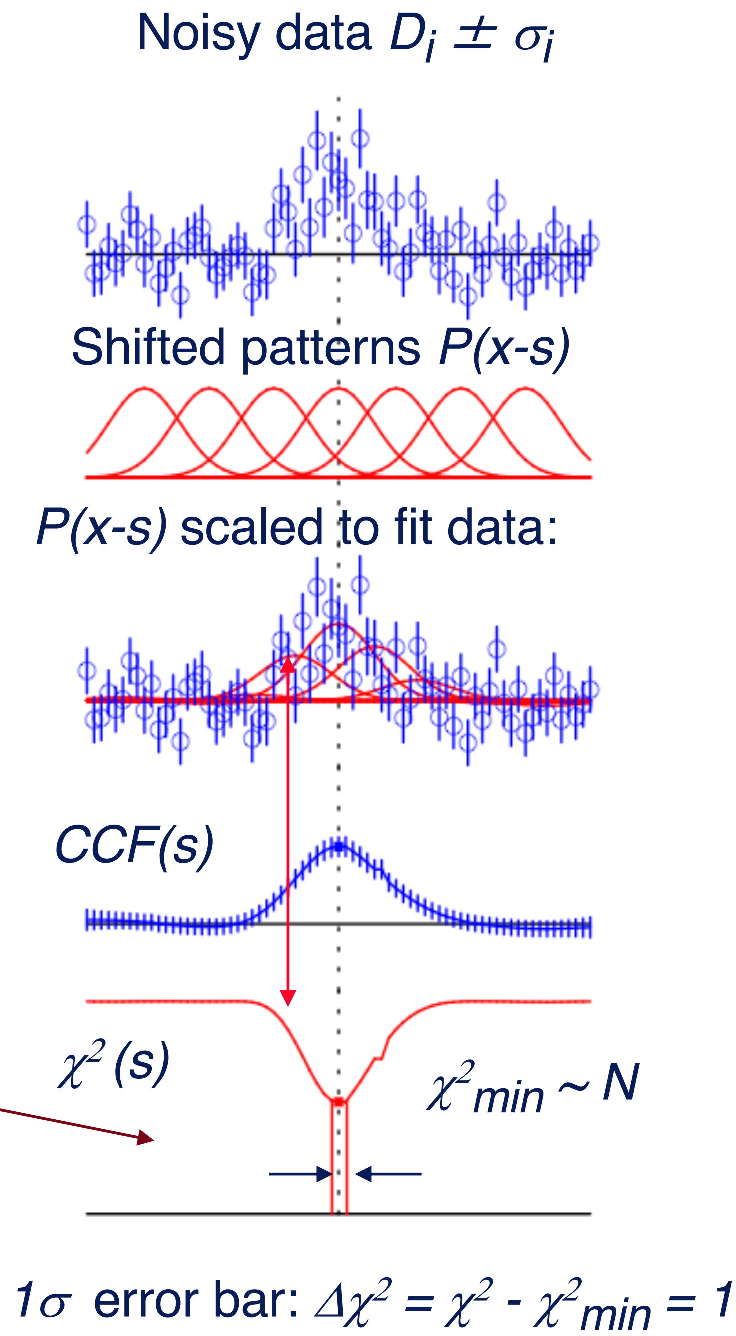
$$CCF(s) \equiv \frac{\sum_i P(X_i - s) D_i / \sigma_i^2}{\sum_i P^2(X_i - s)/\sigma_i^2}$$

Optimal scaling, yet again!

$$\mathrm{Var}\left[CCF(s)\right] = \frac{1}{\sum_i P^2(X_i - s)/\sigma_i^2}$$

Note CCF errors are correlated.

- Good fit: $\chi^2_{min} \sim N$. Get $\sigma(s)$ from $\Delta\chi^2 = 1$.
- CCF analysis fits a non-linear model to the data. **Should minimise χ^2, rather than maximising CCF.**



Pattern too narrow

- Gaussian feature and pattern:

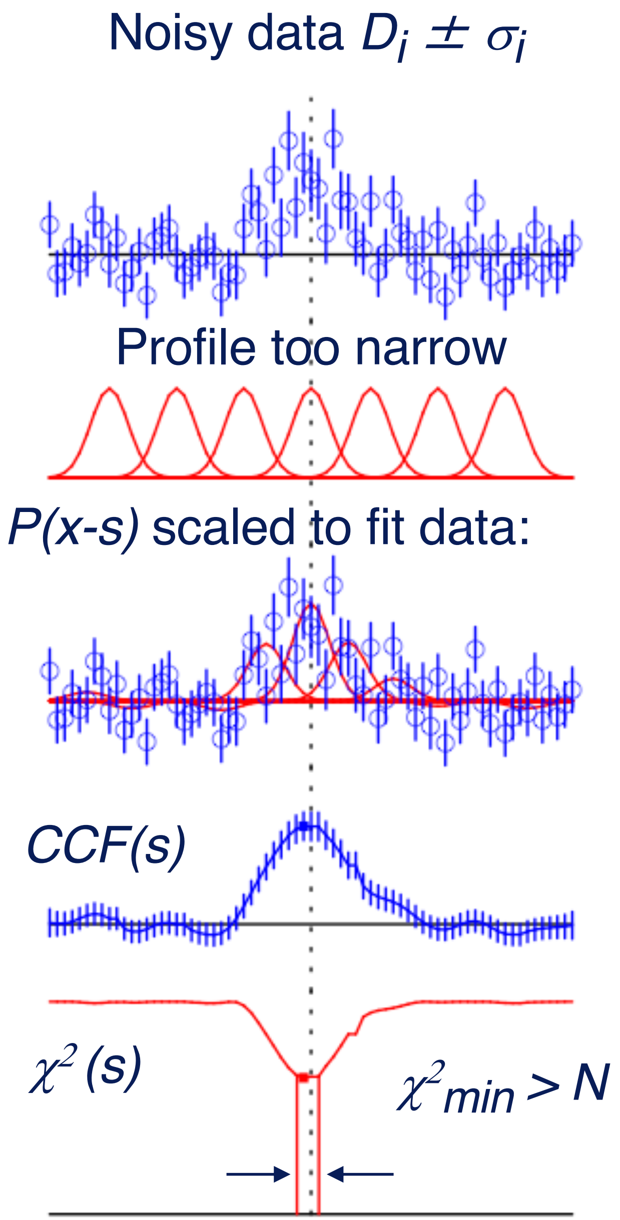
$$\mu_i = \langle D_i \rangle = A \exp\left\{ -\frac{X_i^2}{2\Delta_0^2} \right\} \qquad P(x) = \exp\left\{ -\frac{x^2}{2\Delta^2} \right\}$$

- Pattern width Δ **narrower** than the width Δ_0 of the feature in the data.

$$\langle CCF(s) \rangle \approx \left(\frac{2A^2}{1 + (\Delta/\Delta_0)^2} \right)^{1/2} \exp\left\{ -\frac{s^2}{2\left(\Delta^2 + \Delta_0^2\right)} \right\}$$

- CCF then has **larger error bars**, a **shorter correlation length,** and a **higher** but **narrower peak.**

- Poor fit: χ^2 minimum shallow.
- **Larger error bar** on s.



Wider 1σ error bar: $\Delta\chi^2 = \chi^2 - \chi^2_{min} = 1$

Pattern too wide

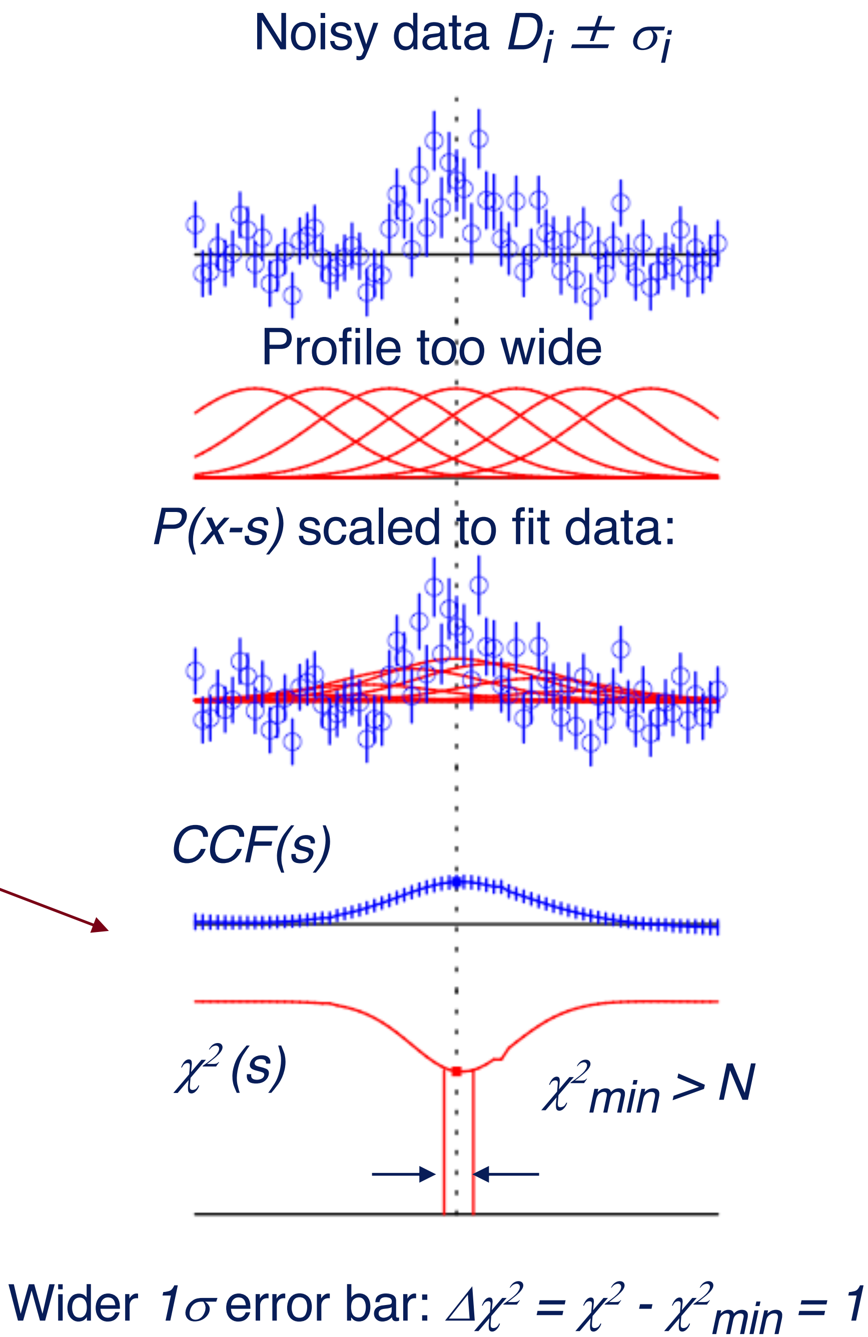
- Gaussian feature and pattern:

$$\mu_i = \langle D_i \rangle = A \exp\left\{ -\frac{X_i^{\ 2}}{2\Delta_0^{\ 2}} \right\} \qquad P(x) = \exp\left\{ -\frac{x^2}{2\Delta^2} \right\}$$

- Pattern width Δ **wider** than the width Δ_0 of the feature in the data.

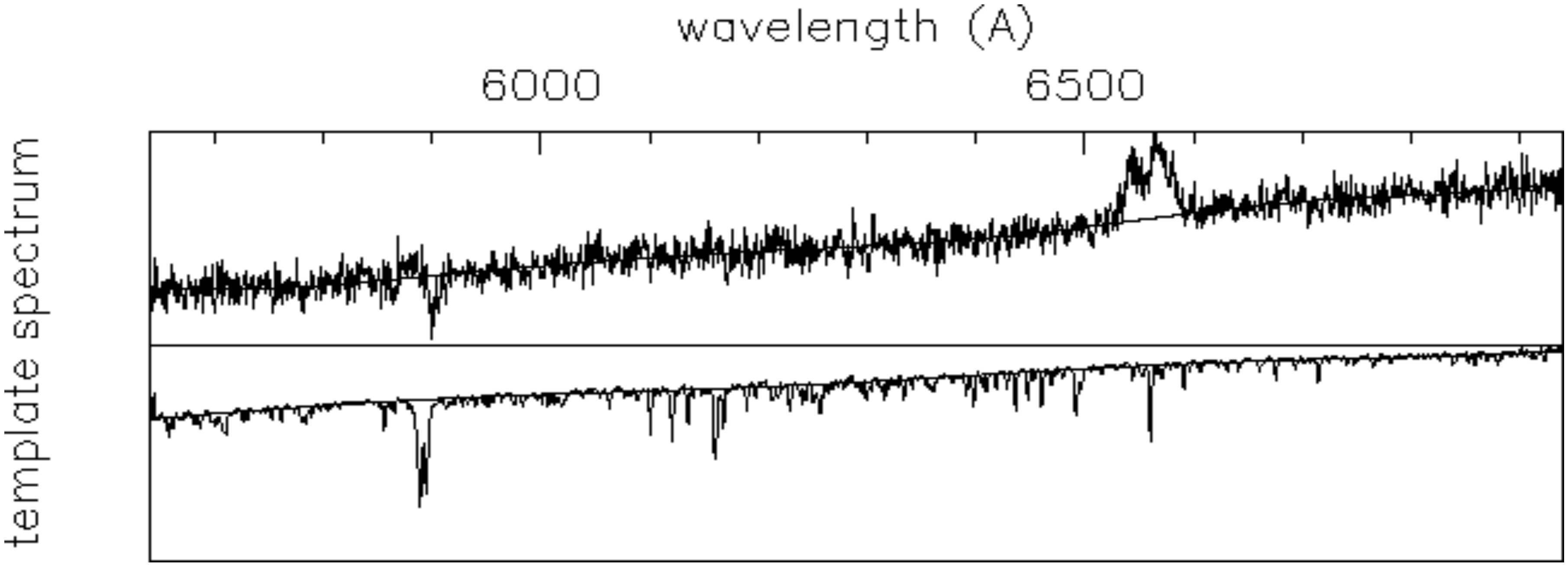
$$\langle CCF(s) \rangle \approx \left(\frac{2A^2}{1 + (\Delta/\Delta_0)^2} \right)^{1/2} \exp\left\{ -\frac{s^2}{2\left(\Delta^2 + \Delta_0^{\ 2}\right)} \right\}$$

- CCF then has **smaller error bars**, **stronger correlations**, and **lower** but **wider peak**.
- Poor fit: χ^2 minimum wide and shallow.
- **Larger error bar** on *s*.



Cross-Correlation Radial Velocities

- Data: spectrum of black-hole binary candidate GRO J0422+32
- Pattern: "template" spectrum of normal K5V star of known radial velocity.
- Mask Hα emission line. Fit continuum (e.g. splines, polynomial, running optimal average) with $\pm$ 2 σ clipping to reject lines.



Wavelength and Velocity shifts

- Target spectrum D_i is measured at wavelengths λ_i and has associated errors σ_i.
- Template spectrum $P(\lambda)$ is measured on same (or very similar) wavelength grid. Errors negligible.
- For small velocity shift v :

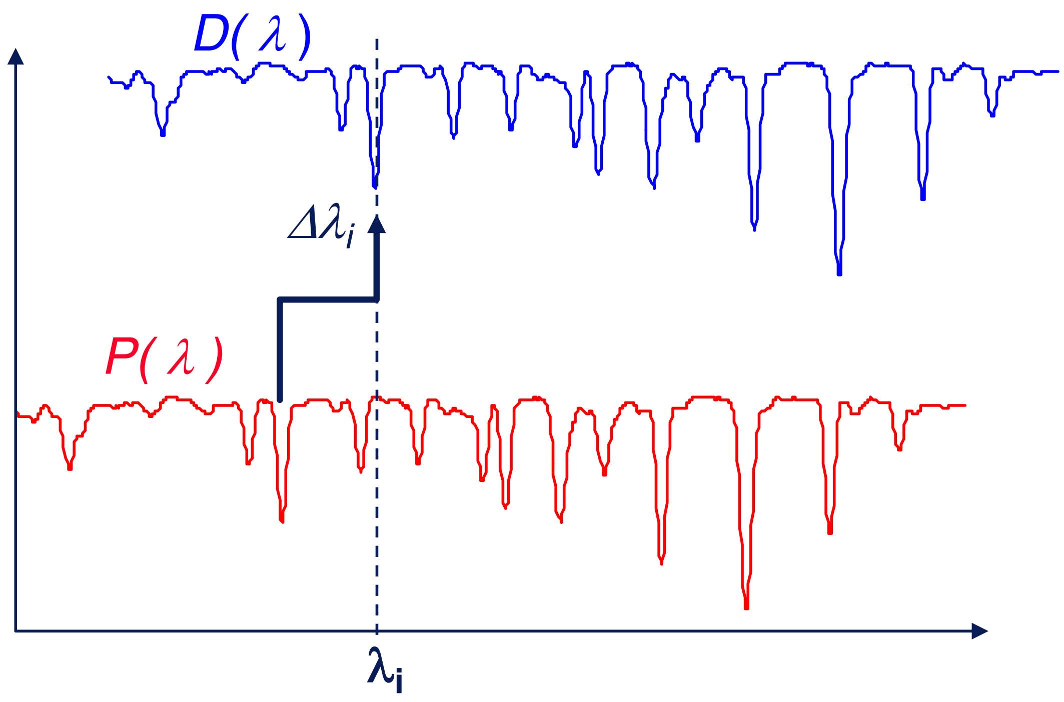
$$\Delta\lambda_i = \lambda_i \frac{v}{c}$$

$$P_i(v) = P(\lambda_i - \Delta\lambda_i) = P(\lambda_i(1-(v/c)))$$

$$CCF(v) = \frac{\sum_i P_i(v)\, D_i / \sigma_i^2}{\sum_i P_i^2(v) / \sigma_i^2}$$

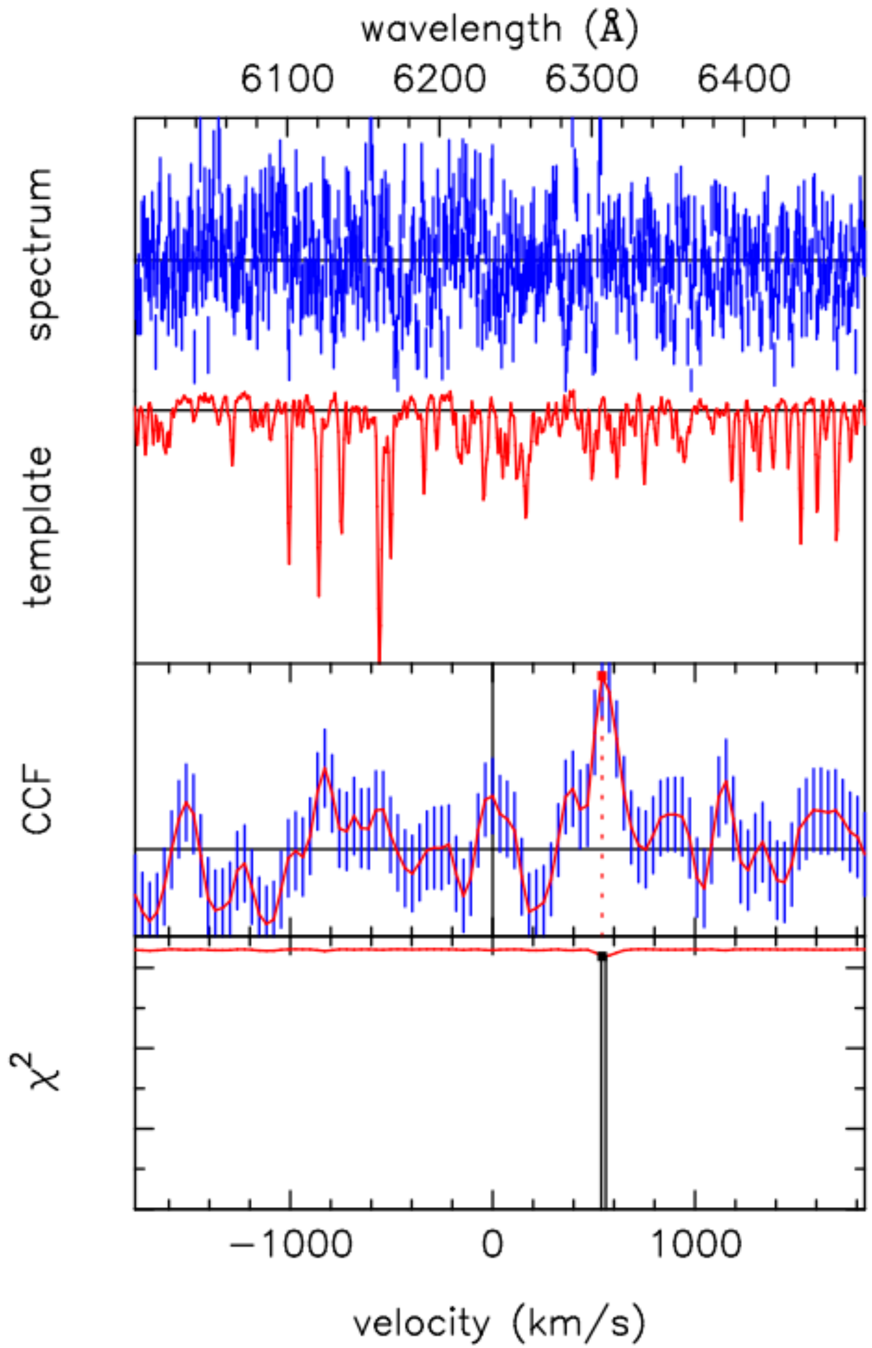
Note that since D is redshifted relative to P in this example, $CCF(v)$ will produce a peak at positive v.

Interpolate the Template P(λ), rather than the noisy data D(λ).



Radial velocity of GRO J0422+32

- Subtract continuum fit.
- Cross-correlate data with template spectrum.
- Compute CCF for shifts in range ± 1800 km s^{-1}.
- CCF shows peak between 500 and 600 km s^{-1}.
- Use $\Delta\chi^2 = 1$ for 1σ error bar on radial velocity.

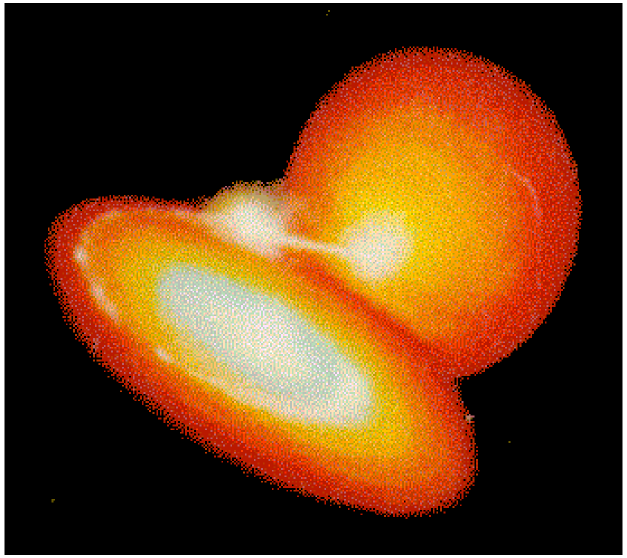


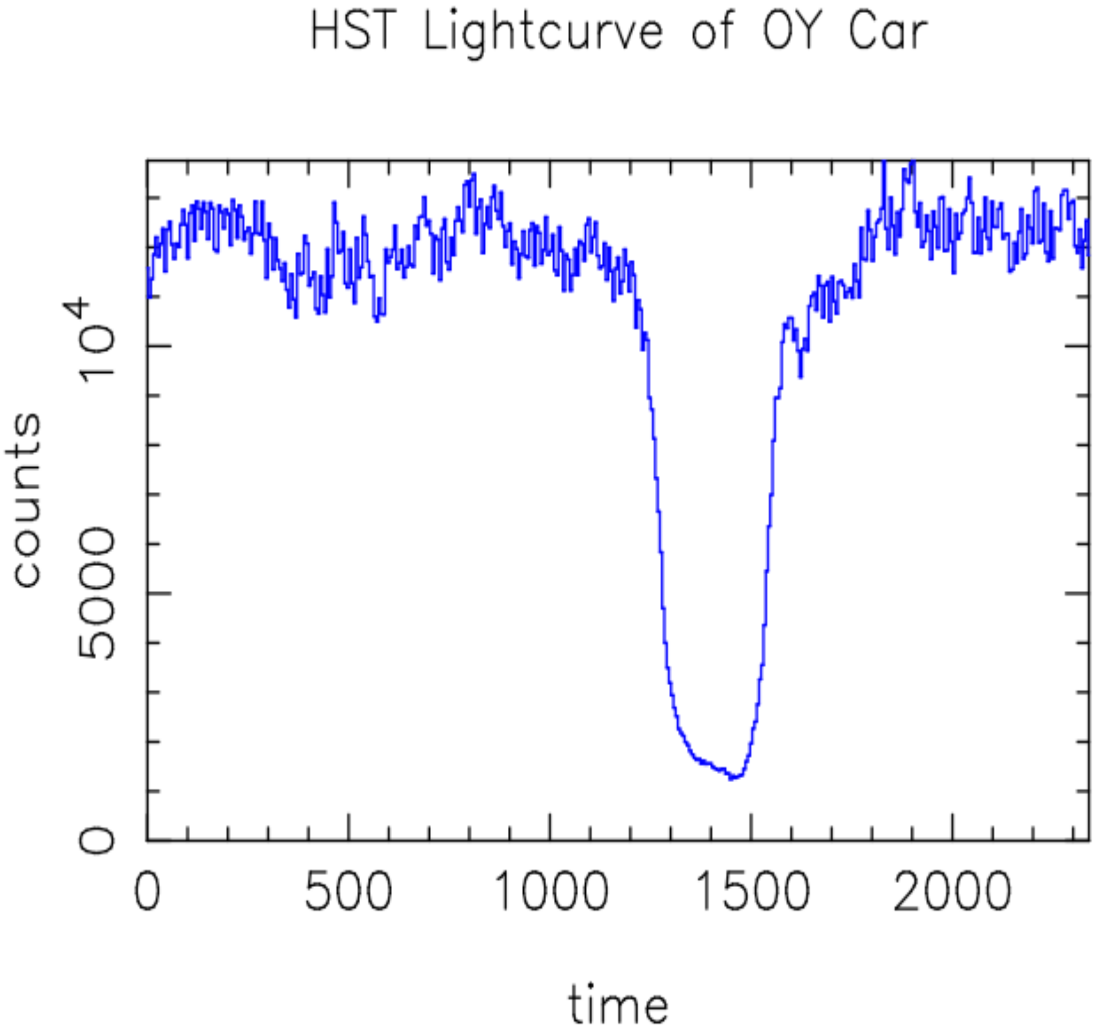
Project 1 = HST lightcurve of OY Car

2 oscillations present.

Spinning magnetised white dwarf.

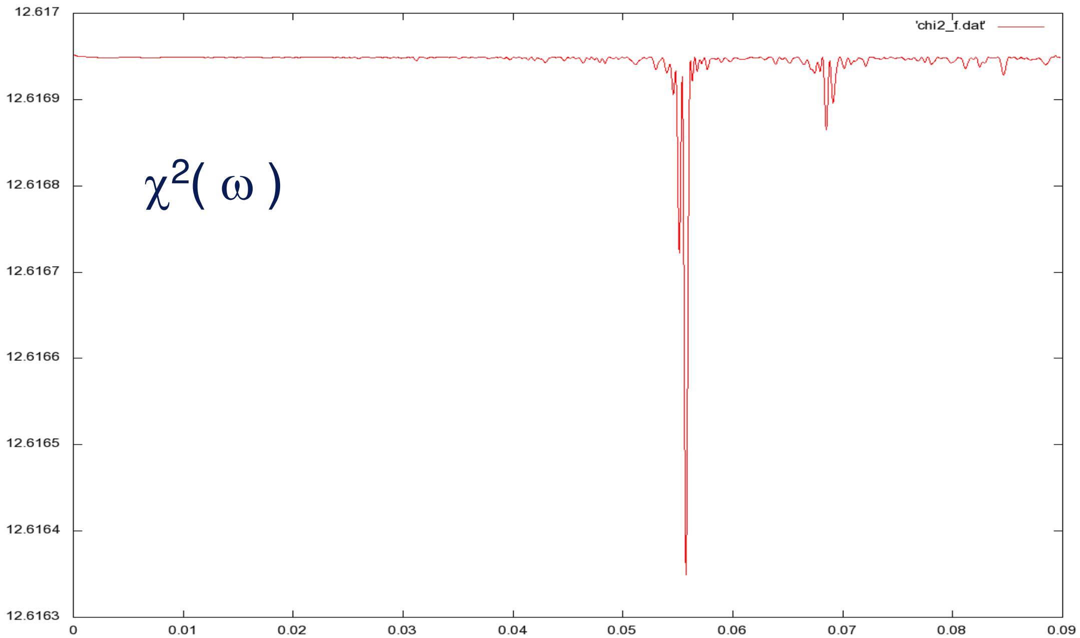
Amplitude and phase modulated by eclipse.

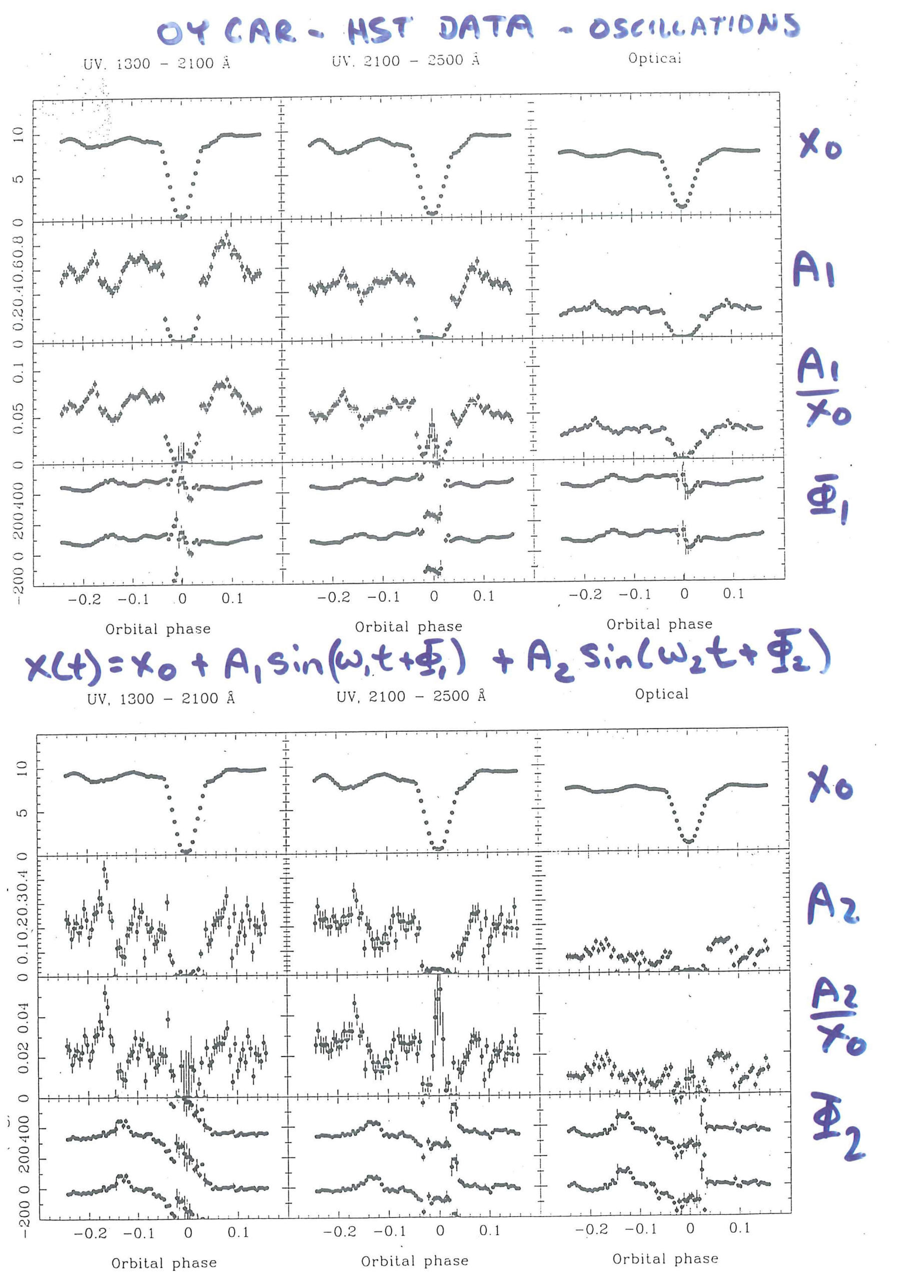




ADA-P1 : OY Car Oscillations

Periodogram Analysis





Measure the periods and amplitudes of the oscillations.

Project 2 = Keck Spectra of a Black-Hole Binary

13 spectra from Keck 10m on Mauna Kea.

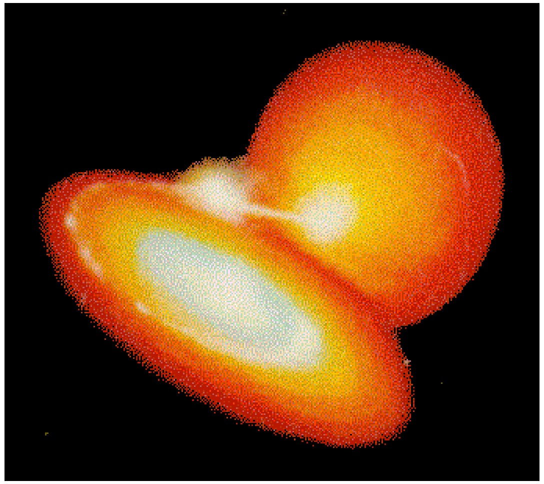
Fit continuum.

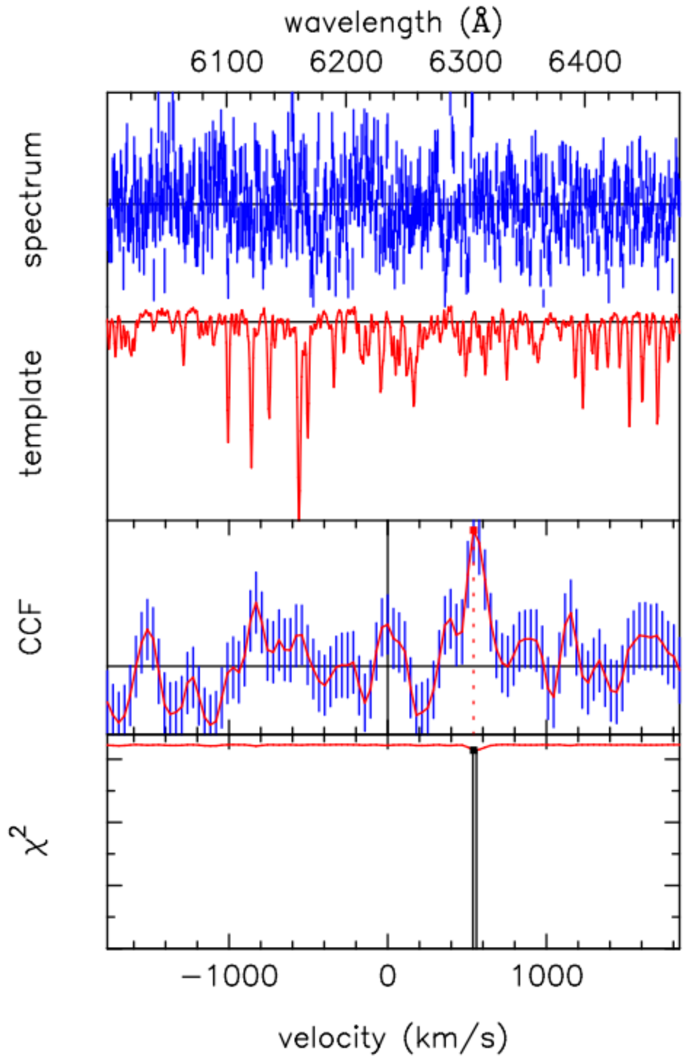
Cross-correlate with template star spectra.

Measure 13 radial velocities.

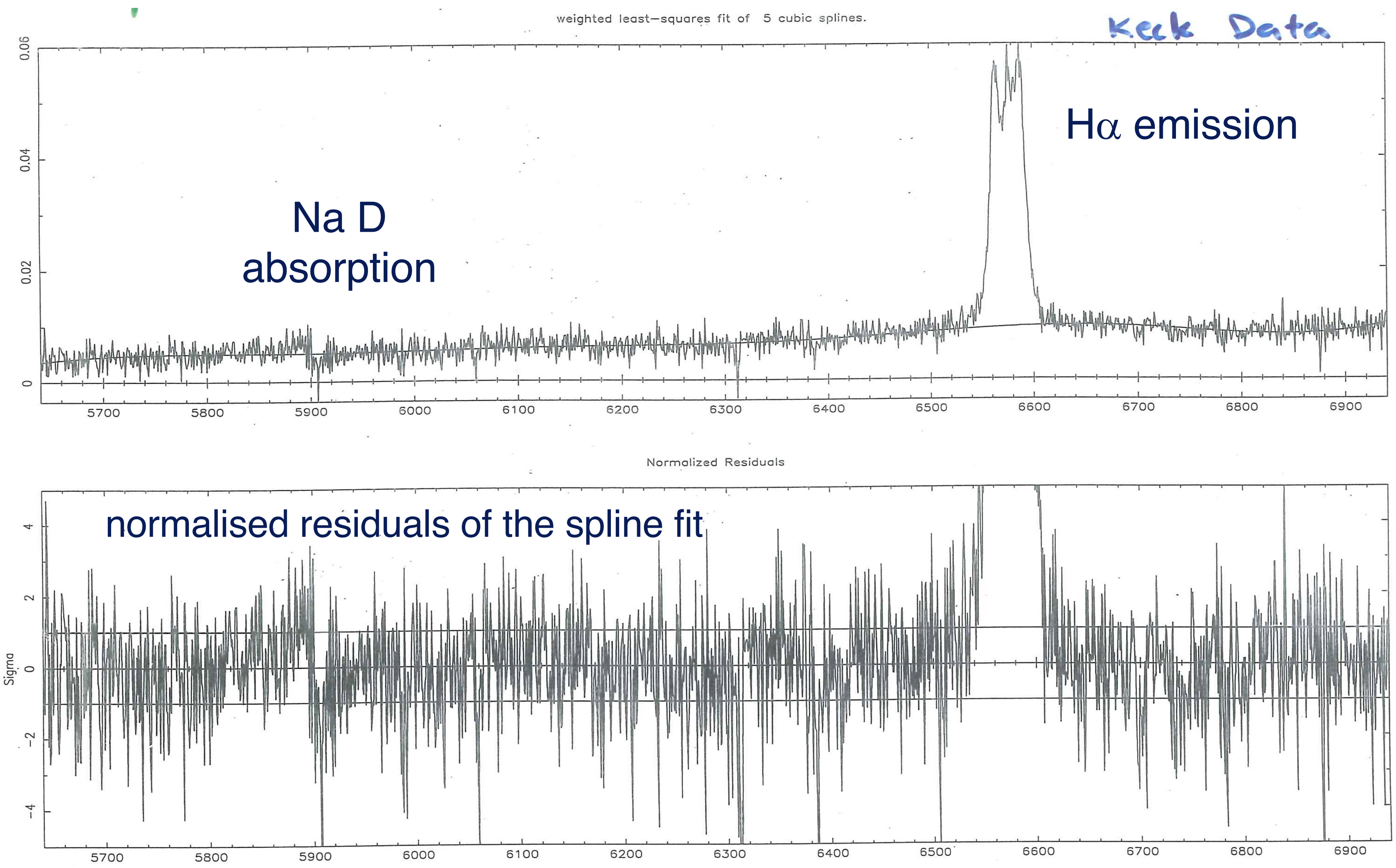
Fit sine curve to measure velocity semi-amplitude.

Work out constraints on the black hole mass.



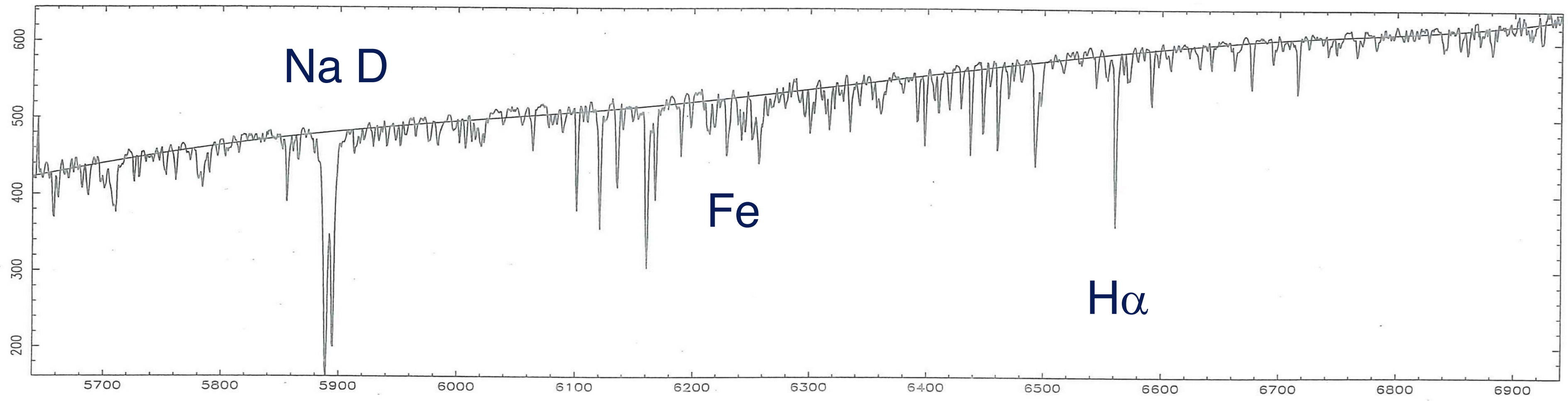


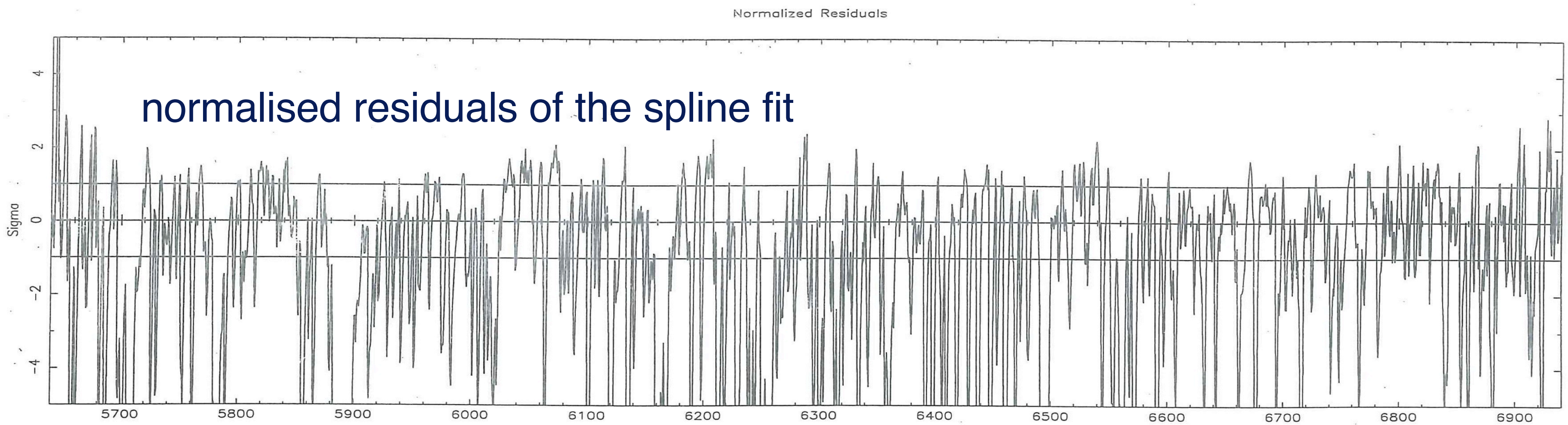
ADA-P2 : Continuum Fit



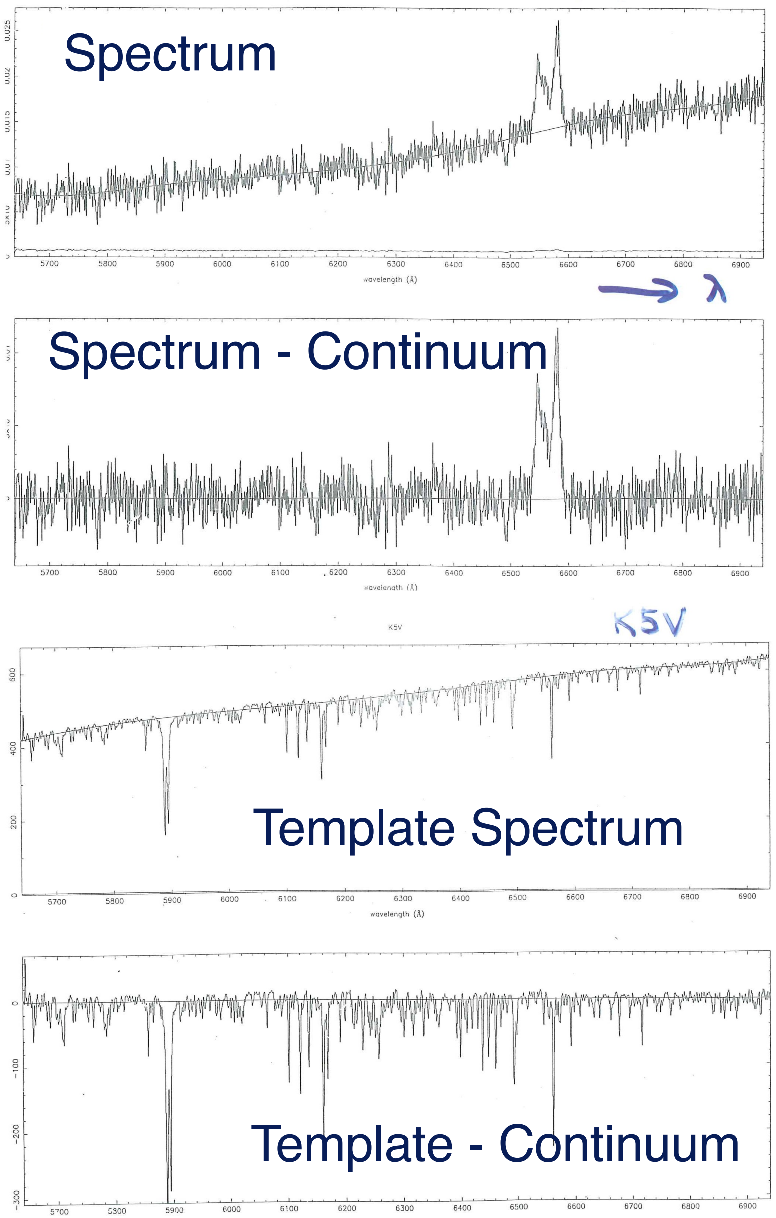
ADA-P2 : Template Continuum Fit

Clip at -2σ to exclude absorption lines

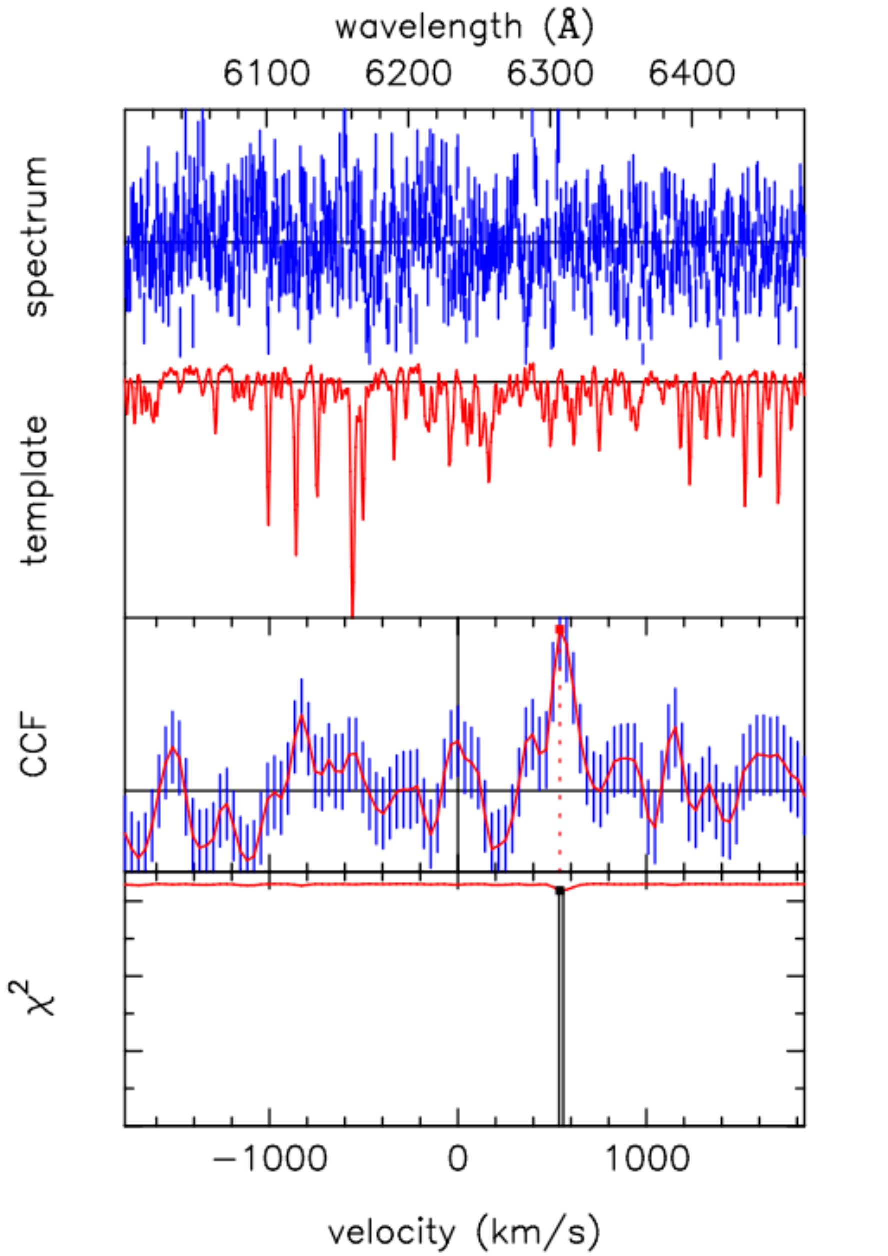




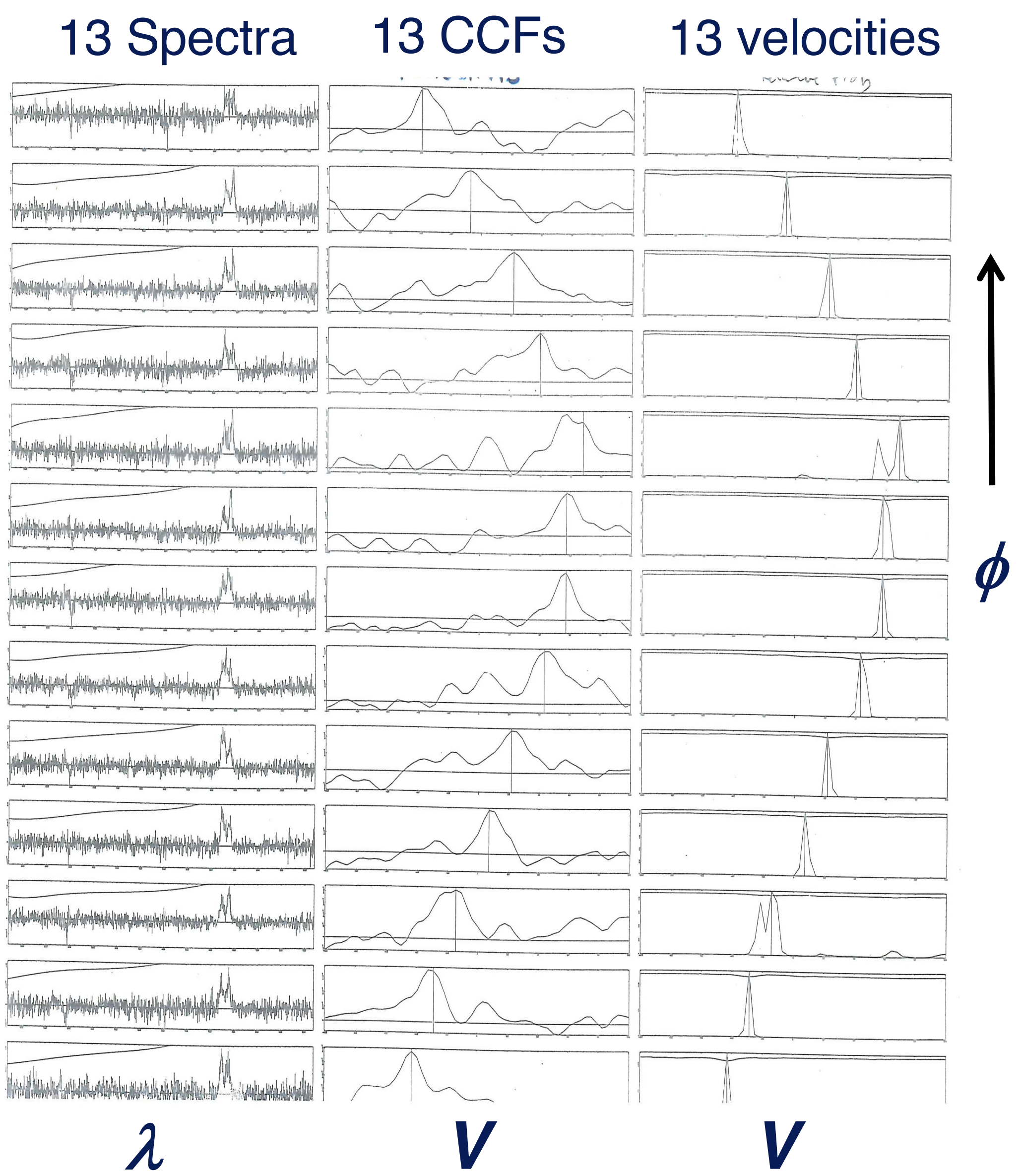
ADA-P2 :
spectra => velocities



Cross-Correlation Radial Velocities



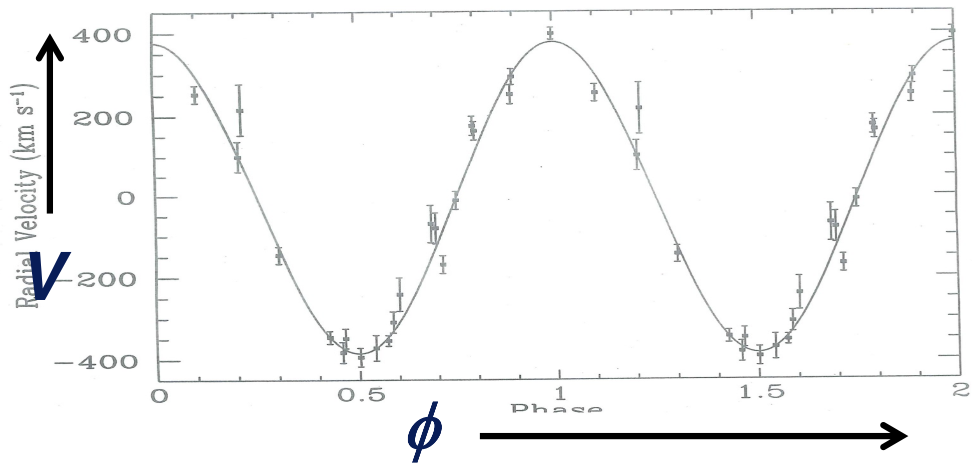
Spectra => Velocities => Orbit=> Masses



Circular orbit model:

$$V(\phi) = \gamma + K_X \sin(2\pi\phi) + K_Y \cos(2\pi\phi)$$

$$K^2 = (K_X)^2 + (K_Y)^2$$



Component masses:

$$K, P => M_X, M_c$$

ADA lectures are now finished ☺ .

We've come a long way. You now have all the tools you need to tackle challenging data analysis projects.

The 2 Homework sets (done) and 2 Projects (to do) let you build expertise by putting these concepts and techniques into practice.

Thanks for listening !

Fini -- ADA 15

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